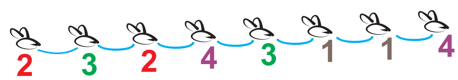
KWANTLEN POLYTECHNIC UNIVERSITY

Mathematics Problem of the Week

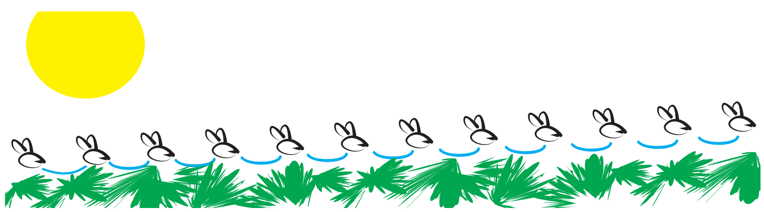
Problem 285 Solution

Twelve Bunnies

Playing “Hop Game” is very popular in bunnies world. An even number of bunnies on a line want to see if they can be partitioned in pairs such that the first pair are one hop apart, the second pair are two hops apart, the third pair are three hops apart and so on. For example, here is an answer to the game of 8 bunnies. The bunnies labelled 1 are one hop apart, the pair labelled 2 are two hops apart, the pair labelled 3 are three hops apart and the last pair, labelled 4, are four hops apart.



How about the game with 12 bunnies? Can a set of 12 bunnies play the hop game?



Solution: A set of 12 bunnies cannot play the hop game.

While there is a rigorous mathematical proof for this puzzle, it can also be shown in cases.

Case 1: the left most bunny is labelled 6: 6 - - - - - 6 - - - - - .

Then we have the following cases for labels 5: (a) 6 - 5 - - - 6 5 - - - - , (b) 6 - - 5 - - 6 - 5 - - -, (c) 6 - - - 5 - 6 - - 5 - -, (d) 6 - - - - 5 6 - - - 5.

Now we show that case (a) does not work and the rest of the cases can be analyzed similarly.

(a) 6 - 5 - - - 6 5 - - - -

Now if we assign labels 1 as follows: 6 - 5 1 1 - 6 5 - - - -, then 6 4 5 1 1 4 6 5 - - - - is the only option and then it is not possible to assign labels 3 and 2 to the blank positions.

If we assign labels 1 as follows: 6 - 5 - 1 1 6 5 - - - -, then it is not possible to assign label 4 to the blank positions.

If we assign labels 1 as 6 - 5 - - - 6 5 1 1 - -, then again it is not possible to assign the rest of the labels to the blank positions.

If we assign labels 1 as 6 - 5 - - - 6 5 - 1 1 -, then again it is not possible to assign the remaining labels, 2, 3 and 4 to the blank positions.

If we assign labels 1 as 6 - 5 - - - 6 5 - - 1 1, then again it is not possible to assign both labels 3 and 4 to the blank positions.

Similarly, it can be shown that none of the cases works.

Winner: This week winner is Enguang Shen (Kirby). Congratulations Kirby!