



openstax™

Alg- ebra

and

Trig- onom- etry

Chapter 1
Prerequisites

1.1 Real Numbers: Algebra Essentials

Verbal

1. Is $\sqrt{2}$ an example of a rational terminating, rational repeating, or irrational number? Tell why it fits that category.

Irrational number. The square root of two does not terminate, and it does not repeat a pattern. It can't be written as a quotient of two integers, so it is irrational.

2. -

3. What do the Associative Properties allow us to do when following the order of operations? Explain your answer.

The Associative Properties state that the sum or product of multiple numbers can be grouped differently without affecting the result. This is because the same operation is performed (either addition or subtraction), so the terms can be re-ordered.

Numeric

For the following exercises, simplify the given expression.

4. -

5. $6 \div 2 - (81 \div 3^2)$

$$6 \div 2 - (81 \div 9)$$

$$6 \div 2 - (9)$$

$$3 - 9$$

$$-6$$

6. -

7. $-2 \times [16 \div (8 - 4)^2]^2$

$$-2 \times [16 \div (4)^2]^2$$

$$-2 \times [16 \div 16]^2$$

$$-2 \times 1^2$$

$$-2 \times 1$$

$$-2$$

8. -

9. $3(5 - 8)$

$$3(-3)$$

$$-9$$

Section 1.1

10. -

$$11. 12 \div (36 \div 9) + 6$$

$$12 \div (4) + 6$$

$$3 + 6$$

$$9$$

12. -

$$13. 3 - 12 \times 2 + 19$$

$$3 - 24 + 19$$

$$-21 + 19$$

$$-2$$

14. -

$$15. 5 + (6 + 4) - 11$$

$$5 + (10) - 11$$

$$15 - 11$$

$$4$$

16. -

$$17. 14 \times 3 \div 7 - 6$$

$$42 \div 7 - 6$$

$$6 - 6$$

$$0$$

18. -

$$19. 6 + 2 \times 2 - 1$$

$$6 + 4 - 1$$

$$10 - 1$$

$$9$$

20. -

$$21. 9 + 4(2^2)$$

$$9 + 4(4)$$

$$9 + 16$$

$$25$$

22. -

$$23. 25 \div 5^2 - 7$$

$$25 \div 25 - 7$$

$$1 - 7$$

$$-6$$

24. -

25. $2 \times 4 - 9(-1)$

$8 - 9(-1)$

$8 + 9$

17

26. -

27. $12(3-1) \div 6$

$12(2) \div 6$

$24 \div 6$

4

Algebraic

For the following exercises, solve for the variable.

28. -

29. $4y + 8 = 2y$

$4y + 8 = 2y$ Subtract $2y$ from both sides

$2y + 8 = 0$ Subtract 8 from both sides

$2y = -8$ Divide both sides by 2

$y = -4$

30. -

31. $4z - 2z(1+4) = 36$

$4z - 2z(1+4) = 36$ Combine terms in parenthesis

$4z - 2z(5) = 36$ Multiply $2z$ and 5

$4z - 10z = 36$ Combine like terms

$-6z = 36$ Divide both sides by -6

$z = -6$

32. -

33. $-(2x)^2 + 1 = -3$

$-(2x)^2 + 1 = -3$ Square the $2x$

$-4x^2 + 1 = -3$ Subtract -1 from both sides

$-4x^2 = -4$ Divide both sides by -4

$x^2 = 1$ Take the square root of both sides

$\sqrt{x^2} = \sqrt{1}$ Evaluate

$x = \pm 1$

34. -

35. $2(11c - 4) = 36$

$2(11c - 4) = 36$ Distribute the 2

$22c - 8 = 36$ Add 8 to both sides

$22c = 44$ Divide both sides by 22

$c = 2$

36. -

37. $\frac{1}{4}(8w - 4^2) = 0$

$\frac{1}{4}(8w - 4^2) = 0$ Square the 4

$\frac{1}{4}(8w - 16) = 0$ Distribute the 1/4

$2w - 4 = 0$ Add 4 to both sides

$2w = 4$ Divide both sides by 2

$w = 2$

For the following exercises, simplify the expression.

38. -

39. $2y - (4)^2 y - 11$

$2y - (4)^2 y - 11$ Square the 4

$2y - 16y - 11$ Combine like terms

$-14y - 11$

40. -

41. $8b - 4b(3) + 1$

$8b - 4b(3) + 1$ Multiply the coefficients of b

$8b - 12b + 1$ Combine like terms

$-4b + 1$

42. -

43. $7z - 3 + z \times 6^2$

$7z - 3 + z \times 6^2$ Square the 6

$7z - 3 + z \times 36$ Multiply 36 and z

$7z - 3 + 36z$ Combine like terms

$43z - 3$

44. -

45. $9(y+8) - 27$

$9(y+8) - 27$ Distribute the 9

$9y + 72 - 27$ Combine like terms

$9y + 45$

46. -

47. $6 + 12b - 3 \times 6b$

$6 + 12b - 3 \times 6b$ Multiply 3 and 6b

$6 + 12b - 18b$ Combine like terms

$-6b + 6$

48. -

49. $\left(\frac{4}{9}\right)^2 \times 27x$

$\left(\frac{4}{9}\right)^2 \times 27x$ Square the expression in parenthesis

$\frac{16}{81} \times 27x$ Note that $27 \times 3 = 81$. Simplify

$\frac{16x}{3}$

50. -

51. $9x + 4x(2+3) - 4(2x+3x)$

$9x + 4x(2+3) - 4(2x+3x)$ Combine terms inside of the parenthesis

$9x + 4x(5) - 4(5x)$ Distribute the 4x and the -4

$9x + 20x - 20x$ Combine like terms

$9x$

52. -

Real-World Applications

For the following exercises, consider this scenario: Fred earns \$40 mowing lawns. He spends \$10 on mp3s, puts half of what is left in a savings account, and gets another \$5 for washing his neighbor's car.

Section 1.1

53. Write the expression that represents the number of dollars Fred keeps (and does not put in his savings account). Remember the order of operations.

Note that Fred earns 40 then spends 10 before putting half inside of his bank account.

The additional 5 comes after the deposit, so we write the expression:

$$\frac{1}{2}(40 - 10) + 5$$

54. -

55. According to the U.S. Mint, the diameter of a quarter is 0.955 inches. The circumference of the quarter would be the diameter multiplied by π . Is the circumference of a quarter a whole number, a rational number, or an irrational number?

π is irrational and 0.955 is rational, so their product is an irrational number.

56. -

For the following exercises, consider this scenario: There is a mound of g pounds of gravel in a quarry. Throughout the day, 400 pounds of gravel is added to the mound. Two orders of 600 pounds are sold and the gravel is removed from the mound. At the end of the day, the mound has 1,200 pounds of gravel.

57. Write the equation that describes the situation.

$$g + 400 - 2(600) = 1200$$

58. -

59. Ramon runs the marketing department at his company. His department gets a budget every year, and every year, he must spend the entire budget without going over. If he spends less than the budget, then his department gets a smaller budget the following year. At the beginning of this year, Ramon got \$2.5 million for the annual marketing budget. He must spend the budget such that $2,500,000 - x = 0$. What property of addition tells us what the value of x must be?

inverse property of addition

Technology

For the following exercises, use a graphing calculator to solve for x . Round the answers to the nearest hundredth.

60. -

61. $(0.25 - 0.75)^2 x - 7.2 = 9.9$

68.4

Extensions

Section 1.1

62. -

63. Determine whether the statement is true or false: The multiplicative inverse of a rational number is also rational.

true

64. -

65. Determine whether the simplified expression is rational or irrational: $\sqrt{-18 - 4(5)(-1)}$.

$$\sqrt{-18 - 4(5)(-1)} \quad \text{Multiply 4 and 5}$$

$$\sqrt{-18 - 20(-1)} \quad \text{Multiply 20 and -1}$$

$$\sqrt{-18 + 20} \quad \text{Add -18 and 20}$$

$$\sqrt{2}$$

Irrational

66. -

67. The division of two whole numbers will always be what type of number?

rational

68. -

This file is copyright 2015, Rice University. All Rights Reserved.

Chapter 1
Prerequisites

1.2 Exponents and Scientific Notation

Verbal

1. Is 2^3 the same as 3^2 ? Explain.
No, the two expressions are not the same. An exponent tells how many times you multiply the base. So 2^3 is the same as $2 \times 2 \times 2$, which is 8. 3^2 is the same as 3×3 , which is 9.
2. -
3. What is the purpose of scientific notation?
It is a method of writing very small and very large numbers.
4. -

Numeric

For the following exercises, simplify the given expression. Write answers with positive exponents.

5. 9^2
 $9 \cdot 9$
81
6. -
7. $3^2 \times 3^3$
 $(3 \cdot 3) \times (3 \cdot 3 \cdot 3)$
 9×27
243
8. -
9. $(2^2)^{-2}$
 $\frac{1}{(2^2)^2}$
 $\frac{1}{4^2}$
 $\frac{1}{16}$
10. -

Section 1.2

$$11. 11^3 \div 11^4$$

$$(11 \cdot 11 \cdot 11) \div (11 \cdot 11 \cdot 11 \cdot 11)$$

$$\frac{1}{11}$$

$$12. -$$

$$13. (8^0)^2$$

$$1^2$$

$$1$$

$$14. -$$

For the following exercises, write each expression with a single base. Do not simplify further. Write answers with positive exponents.

$$15. 4^2 \times 4^3 \div 4^{-4}$$

$$4^5 \div 4^{-4}$$

$$4^5 \div \frac{1}{4^4}$$

$$4^5 \times 4^4$$

$$4^9$$

$$16. -$$

$$17. (12^3 \times 12)^{10}$$

$$(12^4)^{10}$$

$$12^{40}$$

$$18. -$$

$$19. 7^{-6} \times 7^{-3}$$

$$7^{-6+-3}$$

$$7^{-9}$$

$$\frac{1}{7^9}$$

$$20. -$$

For the following exercises, express the decimal in scientific notation.

$$21. 0.0000314$$

$$3.14 \times 10^{-5}$$

Section 1.2

22. -

For the following exercises, convert each number in scientific notation to standard notation.

23. 1.6×10^{10}
 16,000,000,000

24. -

Algebraic

For the following exercises, simplify the given expression. Write answers with positive exponents.

25. $\frac{a^3 a^2}{a}$
 $\frac{a^5}{a}$
 a^{5-1}
 a^4

26. -

27. $(b^3 c^4)^2$
 $b^6 c^8$

28. -

29. $ab^2 \div d^{-3}$
 $ab^2 \div \frac{1}{d^3}$
 $ab^2 \times d^3$
 $ab^2 d^3$

30. -

31. $\frac{m^4}{n^0}$
 $\frac{m^4}{1}$
 m^4

32. -

Section 1.2

$$33. \frac{p^{-4}q^2}{p^2q^{-3}}$$

$$p^{-4-2}q^{2-(-3)}$$

$$p^{-6}q^5$$

$$\frac{q^5}{p^6}$$

34. -

$$35. (y^7)^3 \div x^{14}$$

$$y^{21} \div x^{14}$$

$$\frac{y^{21}}{x^{14}}$$

36. -

$$37. 5^2m \div 5^0m$$

$$25m \div m$$

$$\frac{25m}{m}$$

$$25$$

38. -

$$39. \frac{2^3}{(3a)^{-2}}$$

$$\frac{8}{\frac{1}{(3a)^2}}$$

$$8 \cdot 9a^2$$

$$72a^2$$

40. -

$$41. (b^{-3}c)^3$$

$$\left(\frac{c}{b^3}\right)^3$$

$$\frac{c^3}{b^9}$$

42. -

$$43. (9z^3)^{-2} y$$

$$\frac{y}{(9z^3)^2}$$

$$\frac{y}{81z^6}$$

Real-World Applications

44. -

45. A dime is the thinnest coin in U.S. currency. A dime's thickness measures 1.35×10^{-3} meters. Write the number in standard notation.

0.00135 meter

46. -

47. A terabyte is made of approximately 1,099,500,000,000 bytes. Rewrite in scientific notation.

 1.0995×10^{12}

48. -

49. One picometer is approximately 3.397×10^{-11} inches. Rewrite the number of inches in a picometer using standard notation.

0.00000000003397 inches

50. -

Technology

For the following exercises, use a graphing calculator to simplify. Round the answers to the nearest hundredth.

$$51. \left(\frac{12^3 m^{33}}{4^{-3}} \right)^2$$

$$12,230,590,464 m^{66}$$

52. -

Extensions

For the following exercises, simplify the given expression. Write answers with positive exponents.

Section 1.2

This file is copyright 2015, Rice University. All Rights Reserved.

Chapter 1
Prerequisites

1.3 Radicals and Rational Expressions

Verbal

1. What does it mean when a radical does not have an index? Is the expression equal to the radicand? Explain.

When there is no index, it is assumed to be 2 or the square root. The expression would only be equal to the radicand if the index were 1.

2. -

3. Every number will have two square roots. What is the principal square root?

The principal square root is the nonnegative root of the number.

4. -

Numeric

For the following exercises, simplify each expression.

5. $\sqrt{256}$

$\sqrt{256}$ Note that 256 is 16^2

$\sqrt{16 \cdot 16}$ Evaluate

16

6. -

7. $\sqrt{4(9+16)}$

$\sqrt{4(9+16)}$ Combine terms inside of parenthesis

$\sqrt{4(25)}$ Separate into two different square roots

$\sqrt{4} \sqrt{25}$ Evaluate square roots

$2 \cdot 5$ Multiply

10

8. -

9. $\sqrt{196}$

$\sqrt{196}$ Note that 196 is 14^2

$\sqrt{14 \cdot 14}$ Evaluate square root

14

10. -

Section 1.3

11. $\sqrt{98}$
 $\sqrt{98}$ 98 has the factors 2 and 49
 $\sqrt{49 \cdot 2}$ Multiplication property of radicals
 $\sqrt{49} \sqrt{2}$ 49 is a square root; 2 is not a perfect square
 $7\sqrt{2}$
12. -
13. $\sqrt{\frac{81}{5}}$
 $\sqrt{\frac{81}{5}}$ Division property of radicals
 $\frac{\sqrt{81}}{\sqrt{5}}$ 81 is a perfect square; rationalize the denominator
 $\frac{9}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ Note that $\sqrt{5^2} = 5$ and multiply
 $\frac{9\sqrt{5}}{5}$
14. -
15. $\sqrt{169} + \sqrt{144}$
 $\sqrt{169} + \sqrt{144}$ 169 is 13^2 and 144 is 12^2
 $\sqrt{13 \cdot 13} + \sqrt{12 \cdot 12}$ Evaluate square roots
 $13 + 12$ Add
 25
16. -
17. $\frac{18}{\sqrt{162}}$

Section 1.3

$$\frac{18}{\sqrt{162}} \quad 162 \text{ has factors } 81 \text{ and } 2$$

$$\frac{18}{\sqrt{81 \cdot 2}} \quad \text{Multiplication property of addition}$$

$$\frac{18}{\sqrt{81}\sqrt{2}} \quad 81 \text{ is a perfect square; } 2 \text{ is not a perfect square}$$

$$\frac{18}{9\sqrt{2}} \quad \text{Divide}$$

$$\frac{2}{\sqrt{2}} \quad \text{Rationalize the denominator}$$

$$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{Multiply}$$

$$\frac{2\sqrt{2}}{2} = \sqrt{2}$$

18. -

19. $14\sqrt{6} - 6\sqrt{24}$

$$14\sqrt{6} - 6\sqrt{24} \quad 24 \text{ has factors } 4 \text{ and } 6$$

$$14\sqrt{6} - 6\sqrt{4 \cdot 6} \quad \text{Multiplication property of radicals}$$

$$14\sqrt{6} - 6\sqrt{4}\sqrt{6} \quad 4 \text{ is a perfect square; } 6 \text{ is not a perfect square}$$

$$14\sqrt{6} - 6(2)\sqrt{6} \quad \text{Multiply}$$

$$14\sqrt{6} - 12\sqrt{6} \quad \text{Subtract}$$

$$2\sqrt{6}$$

20. -

21. $\sqrt{150}$

$$\sqrt{150} \quad 150 \text{ has factors } 25 \text{ and } 6$$

$$\sqrt{25 \cdot 6} \quad \text{Multiplication property of radicals}$$

$$\sqrt{25}\sqrt{6} \quad 25 \text{ is a perfect square; } 6 \text{ is not a perfect square}$$

$$5\sqrt{6}$$

22. -

23. $(\sqrt{42})(\sqrt{30})$

Section 1.3

$$(\sqrt{42})(\sqrt{30}) \quad 42 \text{ has factors } 7 \text{ and } 6; 30 \text{ has factors } 5 \text{ and } 6$$

$$(\sqrt{7 \cdot 6})(\sqrt{5 \cdot 6}) \quad \text{Multiplication property of radicals}$$

$$\sqrt{7 \cdot 5 \cdot 6 \cdot 6} \quad \text{Multiplication property of radicals}$$

$$\sqrt{7 \cdot 5} \sqrt{6 \cdot 6} \quad \text{Evaluate and rewrite}$$

$$6\sqrt{35}$$

24. -

$$25. \quad \sqrt{\frac{4}{225}}$$

$$\sqrt{\frac{4}{225}} \quad \text{Division property of radicals}$$

$$\frac{\sqrt{4}}{\sqrt{225}} \quad 4 \text{ and } 225 \text{ are perfect squares}$$

$$\frac{2}{15}$$

26. -

$$27. \quad \sqrt{\frac{360}{361}}$$

$$\sqrt{\frac{360}{361}} \quad \text{Division property of radicals}$$

$$\frac{\sqrt{360}}{\sqrt{361}} \quad 360 \text{ has factors } 36 \text{ and } 10; 361 \text{ is a perfect square}$$

$$\frac{\sqrt{36 \cdot 10}}{19} \quad \text{Multiplication property of radicals}$$

$$\frac{\sqrt{36} \sqrt{10}}{19} \quad 36 \text{ is a perfect square; } 10 \text{ is not a perfect square}$$

$$\frac{6\sqrt{10}}{19}$$

28. -

$$29. \quad \frac{8}{1 - \sqrt{17}}$$

Section 1.3

$$\frac{8}{1-\sqrt{17}}$$

Multiply top and bottom by the conjugate of the denominator

$$\frac{8}{1-\sqrt{17}} \cdot \frac{1+\sqrt{17}}{1+\sqrt{17}}$$

Use the FOIL method in the denominator

$$\frac{8(1+\sqrt{17})}{1+\sqrt{17}-\sqrt{17}-\sqrt{17}^2}$$

Simplify

$$-\frac{8(1+\sqrt{17})}{16}$$

Simplify

$$-\frac{1+\sqrt{17}}{2}$$

30. -

31. $\sqrt[3]{128} + 3\sqrt[3]{2}$

$$\sqrt[3]{128} + 3\sqrt[3]{2}$$

128 has factors 64 and 2

$$\sqrt[3]{64 \cdot 2} + 3\sqrt[3]{2}$$

Multiplication property of radicals

$$\sqrt[3]{64}\sqrt[3]{2} + 3\sqrt[3]{2}$$

64 is a perfect cube; 2 is not a perfect cube

$$4\sqrt[3]{2} + 3\sqrt[3]{2}$$

Add

$$7\sqrt[3]{2}$$

32. -

33. $\frac{15\sqrt[4]{125}}{\sqrt[4]{5}}$

$$\frac{15\sqrt[4]{125}}{\sqrt[4]{5}}$$

Division property of radicals

$$15\sqrt[4]{\frac{125}{5}}$$

Divide $15\sqrt{5}$

$$15\sqrt[4]{25}$$

$25 = 5^2$ therefore $5^{\frac{2}{4}} = \sqrt{5}$

$$15\sqrt{5}$$

34. -

Algebraic

For the following exercises, simplify each expression.

Section 1.3

35. $\sqrt{400x^4}$
 $\sqrt{400x^4}$ Multiplication property of radicals
 $\sqrt{400}\sqrt{x^4}$ 400 is a perfect square; x^4 is a perfect square
 $20x^2$
36. -
37. $\sqrt{49p}$
 $\sqrt{49p}$ Multiplication property of radicals
 $\sqrt{49}\sqrt{p}$ 49 is a perfect square; p is not a perfect square
 $7\sqrt{p}$
38. -
39. $m^{\frac{5}{2}}\sqrt{289}$
 $m^{\frac{5}{2}}\sqrt{289}$ Use properties of exponents to rewrite $m^{\frac{5}{2}}$
 $m^{\frac{4}{2}}m^{\frac{1}{2}}\sqrt{289}$ $m^{\frac{4}{2}} = m^2$; 289 is a perfect square
 $17m^2\sqrt{m}$
40. -
41. $3\sqrt{ab^2} - b\sqrt{a}$
 $3\sqrt{ab^2} - b\sqrt{a}$ Multiplication property of addition
 $3\sqrt{a}\sqrt{b^2} - b\sqrt{a}$ b^2 is a perfect square
 $3b\sqrt{a} - b\sqrt{a}$ Subtract
 $2b\sqrt{a}$
42. -

Section 1.3

43. $\sqrt{\frac{225x^3}{49x}}$
 $\sqrt{\frac{225x^3}{49x}}$ Division property of radicals
 $\frac{\sqrt{225x^3}}{\sqrt{49x}}$ Multiplication property of radicals
 $\frac{\sqrt{225}\sqrt{x^2}\sqrt{x}}{\sqrt{49}\sqrt{x}}$ 225, 49 and x^2 are perfect squares
 $\frac{15x\sqrt{x}}{7\sqrt{x}}$ Simplify
 $\frac{15x}{7}$
44. -
45. $\sqrt{50y^8}$
 $\sqrt{50y^8}$ Multiplication property of radicals
 $\sqrt{50}\sqrt{y^8}$ 50 has factors 25 and 2; y^4 is a perfect square
 $\sqrt{25 \cdot 2}y^4$ Multiplication property of radicals
 $\sqrt{25}\sqrt{2}y^4$ 25 is a perfect square
 $5y^4\sqrt{2}$
46. -
47. $\sqrt{\frac{32}{14d}}$
 $\sqrt{\frac{32}{14d}}$ Simplify
 $\sqrt{\frac{16}{7d}}$ Division property of radicals
 $\frac{\sqrt{16}}{\sqrt{7d}}$ 16 is a perfect square
 $\frac{4}{\sqrt{7d}} \cdot \frac{\sqrt{7d}}{\sqrt{7d}}$ Rationalize the denominator
 $\frac{4\sqrt{7d}}{7d}$
48. -

Section 1.3

49. $\frac{\sqrt{8}}{1-\sqrt{3x}}$

$\frac{\sqrt{8}}{1-\sqrt{3x}}$ Multiply top and bottom by the conjugate of the denominator

$\frac{\sqrt{8}}{1-\sqrt{3x}} \cdot \frac{1+\sqrt{3x}}{1+\sqrt{3x}}$ $\sqrt{8} = 2\sqrt{2}$; Use the FOIL method

$\frac{2\sqrt{2} + 2\sqrt{2}\sqrt{3x}}{1-\sqrt{3x} + \sqrt{3x} - 3x}$ Multiplication property of radicals; simplify

$\frac{2\sqrt{2} + 2\sqrt{6x}}{1-3x}$

50. -

51. $w^{\frac{3}{2}}\sqrt{32} - w^{\frac{3}{2}}\sqrt{50}$

$w^{\frac{3}{2}}\sqrt{32} - w^{\frac{3}{2}}\sqrt{50}$ Rewrite $w^{\frac{3}{2}}$

$w^{\frac{2}{2}}w^{\frac{1}{2}}\sqrt{32} - w^{\frac{2}{2}}w^{\frac{1}{2}}\sqrt{50}$ Simplify

$w\sqrt{w}\sqrt{32} - w\sqrt{w}\sqrt{50}$ Multiplication property of radicals

$w\sqrt{32w} - w\sqrt{50w}$ $32w$ has factors 16 and $2w$; $50w$ has factors 25 and $2w$

$w\sqrt{16 \cdot 2w} - w\sqrt{25 \cdot 2w}$ Multiplication property of radicals

$w\sqrt{16}\sqrt{2w} - w\sqrt{25}\sqrt{2w}$ 16 and 25 are perfect squares

$4w\sqrt{2w} - 5w\sqrt{2w}$ Subtract

$-w\sqrt{2w}$

52. -

53. $\frac{\sqrt{12x}}{2+2\sqrt{3}}$

Section 1.3

$$\frac{\sqrt{12x}}{2+2\sqrt{3}}$$

12 has factors 4 and 3

$$\frac{\sqrt{4 \cdot 3x}}{2+2\sqrt{3}}$$

Multiplication property of radicals

$$\frac{\sqrt{4}\sqrt{3x}}{2+2\sqrt{3}}$$

4 is a perfect square

$$\frac{2\sqrt{3x}}{2+2\sqrt{3}} \cdot \frac{2-2\sqrt{3}}{2-2\sqrt{3}}$$

Multiply top and bottom by the conjugate of the denominator

$$\frac{4\sqrt{3x} - 4\sqrt{9x}}{4 - 4\sqrt{3} + 4\sqrt{3} - 12}$$

Simplify

$$-\frac{4\sqrt{3x} - 12\sqrt{x}}{8}$$

Simplify

$$\frac{3\sqrt{x} - \sqrt{3x}}{2}$$

54. -

55. $\sqrt{125n^{10}}$

$$\sqrt{125n^{10}}$$

125 has factors 25 and 5

$$\sqrt{25n^{10}} \cdot 5$$

Multiplication property of radicals

$$\sqrt{25n^{10}} \sqrt{5}$$

Multiplication property of radicals

$$\sqrt{25}\sqrt{n^{10}}\sqrt{5}$$

25 and n^{10} are perfect squares

$$5n^5\sqrt{5}$$

56. -

57. $\sqrt{\frac{81m}{361m^2}}$

$$\sqrt{\frac{81m}{361m^2}}$$

Division property of radicals

$$\frac{\sqrt{81m}}{\sqrt{361m^2}}$$

Multiplication property of radicals

$$\frac{\sqrt{81}\sqrt{m}}{\sqrt{361}\sqrt{m^2}}$$

81, 361 and m^2 are perfect squares

$$\frac{9\sqrt{m}}{19m}$$

58. -

59. $\sqrt{\frac{144}{324d^2}}$

$\sqrt{\frac{144}{324d^2}}$ Division property of radicals

$\frac{\sqrt{144}}{\sqrt{324d^2}}$ 144 is a perfect square; Multiplication property of radicals

$\frac{12}{\sqrt{324}\sqrt{d^2}}$ 324 and d^2 are perfect squares

$\frac{12}{18d}$ Simplify

$\frac{2}{3d}$

60. -

61. $\sqrt[4]{\frac{162x^6}{16x^4}}$

$\sqrt[4]{\frac{162x^6}{16x^4}}$ Division property of radicals

$\frac{\sqrt[4]{162x^6}}{\sqrt[4]{16x^4}}$ Multiplication property of radicals; rewrite x^6

$\frac{\sqrt[4]{162}\sqrt[4]{x^4x^2}}{\sqrt[4]{16}\sqrt[4]{x^4}}$ 162 has factors 81 and 2; simplify; multiplication property of radicals

$\frac{\sqrt[4]{81}\cdot 2\sqrt[4]{x^4}\sqrt[4]{x^2}}{2x}$ Multiplication property of radicals; simplify

62. $\frac{-x^4\sqrt[4]{81}\sqrt[4]{2}\sqrt[4]{x^2}}{81 = 3^4}$

63. $\frac{\sqrt[3]{128z^{2x}} - \sqrt[3]{-16z^3}}{3x^4\sqrt{2x^2}}$ Simplify

$\frac{2x}{3\sqrt[4]{2x^2}}$

$\frac{2}{3}$

Section 1.3

$\sqrt[3]{128z^3} - \sqrt[3]{-16z^3}$	Multiplication property of radicals
$\sqrt[3]{128z^3} - \sqrt[3]{-16} \sqrt[3]{z^3}$	128 has factors 64 and 2; -16 has factors -8 and 2
$\sqrt[3]{64 \cdot 2} \sqrt[3]{z^3} - \sqrt[3]{-8 \cdot 2} \sqrt[3]{z^3}$	Multiplication property of radicals; z^3 is a perfect cube
$z \sqrt[3]{64} \sqrt[3]{2} - z \sqrt[3]{-8} \sqrt[3]{2}$	64 and -8 are perfect cubes
$4z \sqrt[3]{2} - (-2)z \sqrt[3]{2}$	Simplify and subtract
$6z \sqrt[3]{2}$	

64. -

Real-World Applications

65. A guy wire for a suspension bridge runs from the ground diagonally to the top of the closest pylon to make a triangle. We can use the Pythagorean Theorem to find the length of guy wire needed. The square of the distance between the wire on the ground and the pylon on the ground is 90,000 feet. The square of the height of the pylon is 160,000 feet. So the length of the guy wire can be found by evaluating

$\sqrt{90,000 + 160,000}$. What is the length of the guy wire?

$\sqrt{90,000 + 160,000}$ Add

$\sqrt{250,000}$ Note: $250,000 = 500^2$

500 feet

66. -

Extensions

For the following exercises, simplify each expression.

Section 1.3

67. $\frac{\sqrt{8} - \sqrt{16}}{4 - \sqrt{2}} - 2^{\frac{1}{2}}$

$$\frac{\sqrt{8} - \sqrt{16}}{4 - \sqrt{2}} - 2^{\frac{1}{2}}$$

16 is a perfect square; Note that 8 is $4 \cdot 2$

$$\frac{2\sqrt{2} - 4}{4 - \sqrt{2}} \cdot \frac{4 + \sqrt{2}}{4 + \sqrt{2}} - \sqrt{2}$$

Multiply the top and bottom by the conjugate of the denom.

$$\frac{8\sqrt{2} + 4 - 16 - 4\sqrt{2}}{16 + 4\sqrt{2} - 4\sqrt{2} - 2} - \sqrt{2}$$

Simplify

$$\frac{-12 + 4\sqrt{2}}{14} - \frac{14\sqrt{2}}{14}$$

Give $\sqrt{2}$ a common denominator

$$\frac{-12 - 10\sqrt{2}}{14}$$

Simplify

$$-\frac{6 + 5\sqrt{2}}{7}$$

68. -

69. $\frac{\sqrt{mn^3}}{a^2\sqrt{c^{-3}}} \cdot \frac{a^{-7}n^{-2}}{\sqrt{m^2c^4}}$

$$\frac{\sqrt{mn^3}}{a^2\sqrt{c^{-3}}} \cdot \frac{a^{-7}n^{-2}}{\sqrt{m^2c^4}}$$

Rewrite all negative exponents

$$\frac{\sqrt{mn^3c^3}}{a^2} \cdot \frac{1}{a^7n^2\sqrt{m^2c^4}}$$

Multiplication property of radicals; multiply

$$\frac{\sqrt{m}\sqrt{n^3}\sqrt{c^3}}{a^9n^2\sqrt{m^2}\sqrt{c^4}}$$

Evaluate square roots and combine like terms

$$\frac{nc\sqrt{mnc}}{a^9n^2mc^2}$$

Simplify

$$\frac{\sqrt{mnc}}{a^9mnc}$$

70. -

$$\begin{aligned}
 71. \quad & \frac{x\sqrt{64y} + 4\sqrt{y}}{\sqrt{128y}} \\
 & \frac{x\sqrt{64}\sqrt{y} + 4\sqrt{y}}{\sqrt{128}\sqrt{y}} \\
 & \frac{8x + 4\sqrt{y}}{\sqrt{64 \cdot 2y}} \\
 & \frac{8x + 4\sqrt{y}}{\sqrt{64}\sqrt{2y}} \\
 & \frac{8x + 4\sqrt{y}}{8\sqrt{2y}} \cdot \frac{\sqrt{2y}}{\sqrt{2y}} \\
 & \frac{8x\sqrt{2y} + 4y\sqrt{2}}{16y} \\
 & \frac{2x\sqrt{2y} + y\sqrt{2}}{2y}
 \end{aligned}$$

72. -

$$\begin{aligned}
 73. \quad & \sqrt{\frac{\sqrt[3]{64} + \sqrt[4]{256}}{\sqrt{64} + \sqrt{256}}} \\
 & \frac{\sqrt{3}}{3}
 \end{aligned}$$

This file is copyright 2015, Rice University. All Rights Reserved.

Chapter 1
Prerequisites
1.4 Polynomials

Verbal

1. Evaluate the following statement: the degree of a polynomial in standard form is the exponent of the leading term. Explain why the statement is true or false.
 The statement is true. In standard form, the polynomial with the highest value exponent is placed first and is the leading term. The degree of a polynomial is the value of the highest exponent, which in standard form is also the exponent of the leading term.

2. -

3. You can multiply polynomials with any number of terms and any number of variables using four basic steps over and over until you reach the expanded polynomial. What are the four steps?

Use the distributive property, multiply, combine like terms, and simplify.

4. -

Algebraic

For the following exercises, identify the degree of the polynomial.

5. $7x - 2x^2 + 13$

2

6. -

7. $-625a^8 + 16b^4$

8

8. -

9. $x^2 + 4x + 4$

2

10. -

For the following exercises, find the sum or difference.

11. $(12x^2 + 3x) - (8x^2 - 19)$

$$(12x^2 + 3x) - (8x^2 - 19) \quad \text{Distribute the minus symbol}$$

$$12x^2 + 3x - 8x^2 + 19 \quad \text{Combine like terms}$$

$$4x^2 + 3x + 19$$

12. -

13. $(6w^2 + 24w + 24) - (3w^2 - 6w + 3)$

Section 1.4

$$(6w^2 + 24w + 24) - (3w^2 - 6w + 3) \quad \text{Distribute the minus symbol}$$

$$6w^2 + 24w + 24 - 3w^2 + 6w - 3 \quad \text{Combine like terms}$$

$$3w^2 + 30w + 21$$

14. -

$$15. (11b^4 - 6b^3 + 18b^2 - 4b + 8) - (3b^3 + 6b^2 + 3b)$$

16. -

For the following exercises, find the product.

$$17. (4x + 2)(6x - 4)$$

$$(4x + 2)(6x - 4) \quad \text{Use the FOIL method}$$

$$24x^2 - 16x + 12x - 8 \quad \text{Combine like terms}$$

$$24x^2 - 4x - 8$$

18. -

$$19. (6b^2 - 6)(4b^2 - 4)$$

$$(6b^2 - 6)(4b^2 - 4) \quad \text{Use the FOIL method}$$

$$24b^4 - 24b^2 - 24b^2 + 24 \quad \text{Combine like terms}$$

$$24b^4 - 48b^2 + 24$$

20. -

$$21. (9v - 11)(11v - 9)$$

$$(9v - 11)(11v - 9) \quad \text{Use the FOIL method}$$

$$99v^2 - 81v - 121v + 99 \quad \text{Combine like terms}$$

$$99v^2 - 202v + 99$$

22. -

$$23. (8n - 4)(n^2 + 9)$$

$$(8n - 4)(n^2 + 9) \quad \text{Use the FOIL method}$$

$$8n^3 + 72n - 4n^2 - 36 \quad \text{Rewrite}$$

$$8n^3 - 4n^2 + 72n - 36$$

For the following exercises, expand the binomial.

24. -

$$25. (3y - 7)^2$$

Section 1.4

$(3y - 7)^2$ Expand the expression

$(3y - 7)(3y - 7)$ Use the FOIL method

$9y^2 - 21y - 21y + 49$ Combine like terms

$9y^2 - 42y + 49$

26. -

27. $(4p + 9)^2$

$(4p + 9)^2$ Expand the expression

$(4p + 9)(4p + 9)$ Use the FOIL method

$16p^2 + 36p + 36p + 81$ Combine like terms

$16p^2 + 72p + 81$

28. -

29. $(3y - 6)^2$

$(3y - 6)^2$ Expand the expression

$(3y - 6)(3y - 6)$ Use the FOIL method

$9y^2 - 18y - 18y + 36$ Combine like terms

$9y^2 - 36y + 36$

30. -

For the following exercises, multiply the binomials.

31. $(4c + 1)(4c - 1)$

$(4c + 1)(4c - 1)$ Use the FOIL method

$16c^2 - 4c + 4c - 1$ Combine like terms

$16c^2 - 1$

32. -

33. $(15n - 6)(15n + 6)$

$(15n - 6)(15n + 6)$ Use the FOIL method

$225n^2 + 90n - 90n - 36$ Combine like terms

$225n^2 - 36$

34. -

Section 1.4

$$35. (4+4m)(4-4m)$$

$$(4+4m)(4-4m) \quad \text{Use the FOIL method}$$

$$16-16m+16m-16m^2 \quad \text{Combine like terms}$$

$$-16m^2+16$$

36. -

$$37. (11q-10)(11q+10)$$

$$(11q-10)(11q+10) \quad \text{Use the FOIL method}$$

$$121q^2+110q-110q-100 \quad \text{Combine like terms}$$

$$121q^2-100$$

For the following exercises, multiply the polynomials.

38. -

$$39. (4t^2+t-7)(4t^2-1)$$

$$(4t^2+t-7)(4t^2-1) \quad \text{Use the distributive property}$$

$$16t^4+4t^3-28t^2-4t^2-t+7 \quad \text{Combine like terms}$$

$$16t^4+4t^3-32t^2-t+7$$

40. -

$$41. (y-2)(y^2-4y-9)$$

$$(y-2)(y^2-4y-9) \quad \text{Use the distributive property}$$

$$y^3-4y^2-9y-2y^2+8y+18 \quad \text{Combine like terms}$$

$$y^3-6y^2-y+18$$

42. -

$$43. (3p^2+2p-10)(p-1)$$

$$(3p^2+2p-10)(p-1) \quad \text{Use the distributive property}$$

$$3p^3-3p^2+2p^2-2p-10p+10 \quad \text{Combine like terms}$$

$$3p^3-p^2-12p+10$$

44. -

$$45. (a+b)(a-b)$$

$$(a+b)(a-b) \quad \text{Use the FOIL method}$$

$$a^2-ab+ab-b^2 \quad \text{Combine like terms}$$

$$a^2-b^2$$

46. -

47. $(4t - 5u)^2$

$(4t - 5u)^2$ Expand the expression

$(4t - 5u)(4t - 5u)$ Use the FOIL method

$16t^2 - 20tu - 20tu + 25u^2$ Combine like terms

$16t^2 - 40tu + 25u^2$

48. -

49. $(4t - x)(t - x + 1)$

$(4t - x)(t - x + 1)$ Use the distributive property

$4t^2 - 4tx + 4t - tx + x^2 - x$ Combine like terms

$4t^2 + x^2 + 4t - 5tx - x$

50. -

51. $(4r - d)(6r + 7d)$

$(4r - d)(6r + 7d)$ Use the FOIL method

$24r^2 + 28rd - 6rd - 7d^2$ Combine like terms

$24r^2 + 22rd - 7d^2$

52. -

Real-World Applications

53. A developer wants to purchase a plot of land to build a house. The area of the plot can be described by the following expression: $(4x + 1)(8x - 3)$ where x is measured in meters.

Multiply the binomials to find the area of the plot in standard form.

$(4x + 1)(8x - 3)$ Use the FOIL method

$32x^2 - 12x + 8x - 3$ Combine like terms

$32x^2 - 4x - 3$ square meters

54. -

Extensions

For the following exercises, perform the given operations.

Section 1.4

55. $(4t - 7)^2(2t + 1) - (4t^2 + 2t + 11)$
 $(4t - 7)^2(2t + 1) - (4t^2 + 2t + 11)$ Expand the first expression
 $(4t - 7)(4t - 7)(2t + 1) - (4t^2 + 2t + 11)$ Use the FOIL method
 $16t^2 - 28t - 28t + 49(2t + 1) - (4t^2 + 2t + 11)$ Combine like terms
 $(16t^2 - 56t + 49)(2t + 1) - (4t^2 + 2t + 11)$ Use the distributive property
 $32t^3 + 16t^2 - 112t^2 - 56t + 98t + 49 - (4t^2 + 2t + 11)$ Combine like terms
 $32t^3 - 96t^2 + 42t + 49 - 4t^2 - 2t - 11$ Combine like terms
 $32t^3 - 100t^2 + 40t + 38$

56. -

57. $(a^2 + 4ac + 4c^2)(a^2 - 4c^2)$
 $(a^2 + 4ac + 4c^2)(a^2 - 4c^2)$ Use the distributive property
 $a^4 - 4a^2c^2 + 4a^3c - 16ac^3 + 4a^2c^2 - 16c^4$ Combine like terms
 $a^4 + 4a^3c - 16ac^3 - 16c^4$

This file is copyright 2015, Rice University. All Rights Reserved.

Chapter 1

Prerequisites

1.5 Factoring Polynomials

Verbal

1. If the terms of a polynomial do not have a greatest common factor, then does that mean it is not factorable? Explain.

The terms of a polynomial don't have to have a common factor for the entire polynomial to be factorable. For example, $4x^2$ and $-9y^2$ don't have a common factor, but the whole polynomial is still factorable $4x^2 - 9y^2 = (2x + 3y)(2x - 3y)$.

2. -
3. How do you factor by grouping?

Divide the x term into the sum of two terms, factor each portion of the expression separately, and then factor out the GCF of the entire expression.

Algebraic

For the following exercises, find the greatest common factor.

4. -
5. $49mb^2 - 35m^2ba + 77ma^2$
 $7m$
6. -
7. $200p^3m^3 - 30p^2m^3 + 40m^3$
 $10m^3$
8. -
9. $6y^4 - 2y^3 + 3y^2 - y$
 y

For the following exercises, factor by grouping.

10. -

Section 1.5

11. $2a^2 + 9a - 18$

$$2a^2 + 9a - 18$$

Rewrite $9a$ as $12a - 3a$

$$(2a^2 + 12a) + (-3a - 18)$$

Factor out the GCF from each expression

$$2a(a + 6) - (3a + 6)$$

Use the distributive property

$$(2a - 1)(3a + 6)$$

12. -

13. $6n^2 - 19n - 11$

$$6n^2 - 19n - 11$$

Rewrite $-19n$ as $3n - 22n$

$$(6n^2 + 3n) + (-22n - 11)$$

Factor out the GCF from each expression

$$3n(2n + 1) - 11(2n + 1)$$

Use the distributive property

$$(3n - 11)(2n + 1)$$

14. -

15. $2p^2 - 5p - 7$

$$2p^2 - 5p - 7$$

Rewrite $-5p$ as $2p - 7p$

$$(2p^2 + 2p) + (-7p - 7)$$

Factor out the GCF of each expression

$$2p(p + 1) - 7(p + 1)$$

Use the distributive property

$$(2p - 7)(p + 1)$$

For the following exercises, factor the polynomial.

16. -

17. $10h^2 - 9h - 9$

$$10h^2 - 9h - 9$$

Rewrite $-9h$ as $-15h + 6h$

$$(10h^2 - 15h) + (6h - 9)$$

Factor out the GCF of each expression

$$5h(2h - 3) + 3(2h - 3)$$

Use the distributive property

$$(5h + 3)(2h - 3)$$

18. -

19. $9d^2 - 73d + 8$

Section 1.5

$$9d^2 - 73d + 8$$

Rewrite $-73d$ as $-72d - d$

$$(9d^2 - 72d) + (-d + 8)$$

Factor out the GCF of each expression

$$9d(d - 8) - (d - 8)$$

Use the distributive property

$$(9d - 1)(d - 8)$$

20. -

21. $12t^2 + t - 13$

$$12t^2 + t - 13$$

Rewrite t as $13t - 12t$

$$(12t^2 + 13t) + (-12t - 13)$$

Factor out the GCF of each expression

$$t(12t + 13) - (12t + 13)$$

Use the distributive property

$$(t - 1)(12t + 13)$$

22. -

23. $16x^2 - 100$

$$16x^2 - 100$$

Rewrite $0x$ as $-40x + 40x$

$$(16x^2 - 40x) + (40x - 100)$$

Factor out the GCF of each expression

$$4x(4x - 10) + 10(4x - 10)$$

Use the distributive property

$$(4x + 10)(4x - 10)$$

24. -

25. $121p^2 - 169$

$$121p^2 - 169$$

Rewrite $0x$ as $143p - 143p$

$$(121p^2 + 143p) + (-143p - 169)$$

Factor out the GCF of each expression

$$11p(11p + 13) - 13(11p - 13)$$

Use the distributive property

$$(11p + 13)(11p - 13)$$

26. -

27. $361d^2 - 81$

$$361d^2 - 81$$

Rewrite $0d$ as $171d - 171d$

$$(361d^2 + 171d) + (-171d - 81)$$

Factor out the GCF of each expression

$$19d(19d + 9) - 9(19d - 9)$$

Use the distributive property

$$(19d - 9)(19d + 9)$$

28. -

Section 1.5

29. $144b^2 - 25c^2$

$144b^2 - 25c^2$	Rewrite $0bc$ as $60bc - 60bc$
$(144b^2 + 60bc) + (-60bc - 25c^2)$	Factor out the GCF of each expression
$12b(12b + 5c) - 5c(12b - 5c)$	Use the distributive property
$(12b - 5c)(12b + 5c)$	

30. -

31. $49n^2 + 168n + 144$

$49n^2 + 168n + 144$	Rewrite $168n$ as $84n + 84n$
$(49n^2 + 84n) + (84n + 144)$	Factor out the GCF of each expression
$7n(7n + 12) + 12(7n + 12)$	Use the distributive property
$(7n + 12)(7n + 12)$	Rewrite
$(7n + 12)^2$	

32. -

33. $225y^2 + 120y + 16$

$225y^2 + 120y + 16$	Rewrite $120y$ as $60y + 60y$
$(225y^2 + 60y) + (60y + 16)$	Factor out the GCF of each expression
$15y(15y + 4) + 4(15y + 4)$	Use the distributive property
$(15y + 4)(15y + 4)$	Rewrite
$(15y + 4)^2$	

34. -

35. $25p^2 - 120p + 144$

$25p^2 - 120p + 144$	Rewrite $-120p$ as $-60p - 60p$
$(25p^2 - 60p) + (-60p + 144)$	Factor out the GCF of each expression
$5p(5p - 12) - 12(5p - 12)$	Use the distributive property
$(5p - 12)(5p - 12)$	Rewrite
$(5p - 12)^2$	

36. -

Section 1.5

For the following exercises, factor the polynomials.

37. $x^3 + 216$

$$\begin{aligned} x^3 + 216 & \quad \text{Sum of cubes; } 216 = 6^3 \\ (x+6)(x^2 - 6x + 36) \end{aligned}$$

38. -

39. $125a^3 + 343$

$$\begin{aligned} 125a^3 + 343 & \quad \text{Sum of cubes; } 125 = 5^3 \text{ and } 343 = 7^3 \\ (5a+7)(25a^2 - 35a + 49) \end{aligned}$$

40. -

41. $64x^3 - 125$

$$\begin{aligned} 64x^3 - 125 & \quad \text{Difference of cubes; } 64 = 4^3 \text{ and } 125 = 5^3 \\ (4x-5)(16x^2 + 20x + 25) \end{aligned}$$

42. -

43. $125r^3 + 1728s^3$

$$\begin{aligned} 125r^3 + 1728s^3 & \quad \text{Sum of cubes; } 125 = 5^3 \text{ and } 1728 = 12^3 \\ (5r+12s)(25r^2 - 60rs + 144s^2) \end{aligned}$$

44. -

45. $3c(2c+3)^{\frac{1}{4}} - 5(2c+3)^{\frac{3}{4}}$

$$3c(2c+3)^{\frac{1}{4}} - 5(2c+3)^{\frac{3}{4}} \quad \text{Factor out the GCF of the expression}$$

$$(2c+3)^{\frac{1}{4}} [3c - 5(2c+3)] \quad \text{Distribute the -5}$$

$$(2c+3)^{\frac{1}{4}} (3c - 10c - 15) \quad \text{Combine like terms}$$

$$(2c+3)^{\frac{1}{4}} (-7c - 15)$$

46. -

47. $14x(x+2)^{-\frac{2}{5}} + 5(x+2)^{\frac{3}{5}}$

$14x(x+2)^{-\frac{2}{5}} + 5(x+2)^{\frac{3}{5}}$ Factor out the GCF of the expression

$(x+2)^{-\frac{2}{5}}[14x + 5(x+2)]$ Distribute the 5

$(x+2)^{-\frac{2}{5}}(14x + 5x + 10)$ Combine like terms

$(x+2)^{-\frac{2}{5}}(19x + 10)$

48. -

49. $5z(2z-9)^{-\frac{3}{2}} + 11(2z-9)^{-\frac{1}{2}}$

$5z(2z-9)^{-\frac{3}{2}} + 11(2z-9)^{-\frac{1}{2}}$ Factor out the GCF of the expression

$(2z-9)^{-\frac{3}{2}}[5z + 11(2z-9)]$ Distribute the 11

$(2z-9)^{-\frac{3}{2}}(5z + 22z - 99)$ Combine like terms

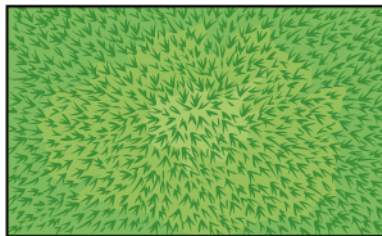
$(2z-9)^{-\frac{3}{2}}(27z - 99)$

50. -

Real-World Applications

For the following exercises, consider this scenario:

Charlotte has appointed a chairperson to lead a city beautification project. The first act is to install statues and fountains in one of the city's parks. The park is a rectangle with an area of $98x^2 + 105x - 27$ square meters, as shown. The length and width of the park are perfect factors of the area.



$$l \times w = 98x^2 + 105x - 27$$

Section 1.5

51. Factor by grouping to find the length and width of the park.

$$98x^2 + 105x - 27 \quad \text{Rewrite } 105x \text{ as } 126x - 21x$$

$$(98x^2 + 126x) + (-21x - 27) \quad \text{Factor out the GCF of each expression}$$

$$14x(7x + 9) - 3(7x + 9) \quad \text{Use the distributive property}$$

$$(14x - 3)(7x + 9)$$

52. -

53. At the northwest corner of the park, the city is going to install a fountain. The area of the base of the fountain is $9x^2 - 25$. Factor the area to find the lengths of the sides of the fountain.

$$9x^2 - 25 \quad \text{Rewrite } 0x \text{ as } 15x - 15x$$

$$(9x^2 + 15x) + (-15x - 25) \quad \text{Factor out the GCF of each expression}$$

$$3x(3x + 5) - 5(3x + 5) \quad \text{Use the distributive property}$$

$$(3x + 5)(3x - 5)$$

54. -

Extensions

For the following exercises, factor the polynomials completely.

55. $16x^4 - 200x^2 + 625$

$$16x^4 - 200x^2 + 625 \quad \text{Rewrite } -200x^2 \text{ as } -100x^2 - 100x^2$$

$$(16x^4 - 100x^2) + (-100x^2 + 625) \quad \text{Factor out the GCF of each expression}$$

$$4x^2(4x^2 - 25) - 25(4x^2 - 25) \quad \text{Use the distributive property}$$

$$(4x^2 - 25)(4x^2 - 25) \quad \text{Rewrite}$$

$$(4x^2 - 25)^2 \quad \text{Rewrite } 0x \text{ as } 10x - 10x$$

$$((4x^2 + 10x) + (-10x - 25))^2 \quad \text{Factor out the GCF of each expression}$$

$$(2x(2x + 5) - 5(2x + 5))^2 \quad \text{Use the distributive property}$$

$$((2x - 5)(2x + 5))^2 \quad \text{Distribute the exponent}$$

$$(2x - 5)^2(2x + 5)^2$$

56. -

Section 1.5

57. $16z^4 - 2401a^4$

$$16z^4 - 2401a^4$$

$$(16z^4 - 196z^2a^2) + (196z^2a^2 - 2401a^4)$$

$$4z^2(4z^2 - 49a^2) + 49a^2(4z^2 - 49a^2)$$

$$(4z^2 + 49a^2)(4z^2 - 49a^2)$$

$$(4z^2 + 49a^2)[(4z^2 - 14za) + (14za - 49a^2)]$$

$$(4z^2 + 49a^2)[2z(2z - 7a) + 7a(2z - 7a)]$$

$$(4z^2 + 49a^2)(2z + 7a)(2z - 7a)$$

Rewrite $0z^2a^2$ as $-196z^2a^2 + 196z^2a^2$

Factor out the GCF of each expression

Use the distributive property

Rewrite $0za$ as $-14za + 14za$

Factor out the GCF of each expression

Use the distributive property

58. -

59. $(32x^3 + 48x^2 - 162x - 243)^{-1}$

$$(32x^3 + 48x^2 - 162x - 243)^{-1}$$

$$((4x + 9)(8x^2 - 6x - 27))^{-1}$$

$$((4x + 9)((8x^2 - 18x) + (12x - 27)))^{-1}$$

$$((4x + 9)(2x(4x - 9) + 3(4x - 9)))^{-1}$$

$$((4x + 9)(2x + 3)(4x - 9))^{-1}$$

$$\frac{1}{(4x + 9)(4x - 9)(2x + 3)}$$

Note that $(4x + 9)$ is a factor of the expression

Rewrite $-6x$ as $-18x + 12x$

Factor out the GCF of each expression

Use the distributive property

Rewrite

This file is copyright 2015, Rice University. All Rights Reserved.

Chapter 1
Prerequisites
1.6 Rational Expressions

Verbal

1. How can you use factoring to simplify rational expressions?
 You can factor the numerator and denominator to see if any of the terms can cancel one another out.
2. -
3. Tell whether the following statement is true or false and explain why: You only need to find the Least Common Denominator when adding or subtracting rational expressions.
 True, multiplication and division do not require finding the LCD because the denominators can be combined through those operations, whereas addition and subtraction require like terms.

Algebraic

For the following exercises, simplify the rational expressions.

4. -

$$5. \frac{y^2 + 10y + 25}{y^2 + 11y + 30}$$

$$\frac{y^2 + 10y + 25}{y^2 + 11y + 30}$$

Rewrite $10y$ as $5y + 5y$; Rewrite $11y$ as $6y + 5y$

$$\frac{(y^2 + 5y) + (5y + 25)}{(y^2 + 6y) + (5y + 30)}$$

Factor out the GCF of each expression

$$\frac{y(y + 5) + 5(y + 5)}{y(y + 6) + 5(y + 6)}$$

Use the distributive property

$$\frac{(y + 5)(y + 5)}{(y + 5)(y + 6)}$$

Simplify

$$\frac{y + 5}{y + 6}$$

6. -

Section 1.6

$$7. \frac{9b^2 + 18b + 9}{3b + 3}$$

$\frac{9b^2 + 18b + 9}{3b + 3}$ Rewrite $18b$ as $9b + 9b$

$$\frac{(9b^2 + 9b) + (9b + 9)}{3(b + 1)}$$

Factor out the GCF of each expression

$$\frac{9b(b + 1) + 9(b + 1)}{3(b + 1)}$$

Use the distributive property

$$\frac{(9b + 9)(b + 1)}{3(b + 1)}$$

Factor out the 9

$$\frac{9(b + 1)(b + 1)}{3(b + 1)}$$

Simplify

$$3(b + 1) = 3b + 3$$

8. -

$$9. \frac{2x^2 + 7x - 4}{4x^2 + 2x - 2}$$

$$\frac{2x^2 + 7x - 4}{4x^2 + 2x - 2}$$

Rewrite $7x$ as $8x - x$; Rewrite $2x$ as $4x - 2x$

$$\frac{(2x^2 + 8x) + (-x - 4)}{(4x^2 + 4x) + (-2x - 2)}$$

Factor out the GCF of the expressions

$$\frac{2x(x + 4) - (x + 4)}{4x(x + 1) - 2(x + 1)}$$

Use the distributive property

$$\frac{(2x - 1)(x + 4)}{(4x - 2)(x + 1)}$$

Factor out the 2

$$\frac{(2x - 1)(x + 4)}{2(2x - 1)(x + 1)}$$

Simplify

$$\frac{x + 4}{2(x + 1)}$$

Simplify

$$\frac{x + 4}{2x + 2}$$

10. -

Section 1.6

$$11. \frac{a^2 + 9a + 18}{a^2 + 3a - 18}$$

Rewrite $9a$ as $6a + 3a$; Rewrite $3a$ as $6a - 3a$

$$\frac{a^2 + 9a + 18}{a^2 + 3a - 18}$$

$$\frac{(a^2 + 6a) + (3a + 18)}{(a^2 + 6a) + (-3a - 18)}$$

Factor out the GCF of the expressions

$$\frac{a(a + 6) + 3(a + 6)}{a(a + 6) - 3(a + 6)}$$

Use the distributive property

$$\frac{(a + 3)(a + 6)}{(a - 3)(a + 6)}$$

Simplify

$$\frac{a + 3}{a - 3}$$

12. -

$$13. \frac{12n^2 - 29n - 8}{28n^2 - 5n - 3}$$

Rewrite $-29n$ as $3n - 32n$; Rewrite $-5n$ as $-12n + 7n$

$$\frac{12n^2 - 29n - 8}{28n^2 - 5n - 3}$$

$$\frac{(12n^2 + 3n) + (-32n - 8)}{(28n^2 - 12n) + (7n - 3)}$$

Factor out the GCF of the expressions

$$\frac{3n(4n + 1) - 8(4n + 1)}{4n(7n - 3) + (7n - 3)}$$

Use the distributive property

$$\frac{(3n - 8)(4n + 1)}{(4n + 1)(7n - 3)}$$

Simplify

$$\frac{3n - 8}{7n - 3}$$

For the following exercises, multiply the rational expressions and express the product in simplest form.

14. -

$$15. \frac{c^2 + 2c - 24}{c^2 + 12c + 36} \cdot \frac{c^2 - 10c + 24}{c^2 - 8c + 16}$$

Section 1.6

$$\frac{c^2 + 2c - 24}{c^2 + 12c + 36} \cdot \frac{c^2 - 10c + 24}{c^2 - 8c + 16}$$

Rewrite the expressions

$$\frac{(c^2 + 6c) + (-4c - 24)}{(c^2 + 6c) + (6c + 36)} \cdot \frac{(c^2 - 6c) + (-4c + 24)}{(c^2 - 4c) + (-4c + 16)}$$

Factor out the GCF of the expressions

$$\frac{c(c + 6) - 4(c + 6)}{c(c + 6) + 6(c + 6)} \cdot \frac{c(c - 6) - 4(c - 6)}{c(c - 4) - 4(c - 4)}$$

Use the distributive property

$$\frac{(c - 4)(c + 6)}{(c + 6)(c + 6)} \cdot \frac{(c - 4)(c - 6)}{(c - 4)(c - 4)}$$

Simplify

$$\frac{c - 6}{c + 6}$$

16. -

$$17. \frac{10h^2 - 9h - 9}{2h^2 - 19h + 24} \cdot \frac{h^2 - 16h + 64}{5h^2 - 37h - 24}$$

$$\frac{10h^2 - 9h - 9}{2h^2 - 19h + 24} \cdot \frac{h^2 - 16h + 64}{5h^2 - 37h - 24}$$

Rewrite the expressions

$$\frac{(10h^2 - 15h) + (6h - 9)}{(2h^2 - 16h) + (-3h + 24)} \cdot \frac{(h^2 - 8h) + (-8h + 64)}{(5h^2 - 40h) + (3h - 24)}$$

Factor out the GCF of the expressions

$$\frac{5h(2h - 3) + 3(2h - 3)}{2h(h - 8) - 3(h - 8)} \cdot \frac{h(h - 8) - 8(h - 8)}{5h(h - 8) + 3(h - 8)}$$

Use the distributive property

$$\frac{(5h + 3)(2h - 3)}{(2h - 3)(h - 8)} \cdot \frac{(h - 8)(h - 8)}{(5h + 3)(h - 8)}$$

Simplify

$$1$$

18.

$$19. \frac{2d^2 + 15d + 25}{4d^2 - 25} \cdot \frac{2d^2 - 15d + 25}{25d^2 - 1}$$

Section 1.6

$$\frac{2d^2 + 15d + 25}{4d^2 - 25} \cdot \frac{2d^2 - 15d + 25}{25d^2 - 1}$$

Rewrite the expressions

$$\frac{(2d^2 + 10d) + (5d + 25)}{(2d - 5)(2d + 5)} \cdot \frac{(2d^2 - 10d) + (-5d + 25)}{(5d - 1)(5d + 1)}$$

Factor out the GCF in the expressions

$$\frac{2d(d + 5) + 5(d + 5)}{(2d - 5)(2d + 5)} \cdot \frac{2d(d - 5) - 5(d - 5)}{(5d - 1)(5d + 1)}$$

Use the distributive property

$$\frac{(2d + 5)(d + 5)}{(2d - 5)(2d + 5)} \cdot \frac{(2d - 5)(d - 5)}{(5d - 1)(5d + 1)}$$

Simplify

$$\frac{(d + 5)(d - 5)}{(5d - 1)(5d + 1)}$$

Multiply

$$\frac{d^2 - 25}{25d^2 - 1}$$

20. -

$$21. \frac{t^2 - 1}{t^2 + 4t + 3} \cdot \frac{t^2 + 2t - 15}{t^2 - 4t + 3}$$

$$\frac{t^2 - 1}{t^2 + 4t + 3} \cdot \frac{t^2 + 2t - 15}{t^2 - 4t + 3}$$

Rewrite the expressions; Difference of squares

$$\frac{(t - 1)(t + 1)}{(t^2 + 3t) + (t + 3)} \cdot \frac{(t^2 + 5t) + (-3t - 15)}{(t^2 - 3t) + (-t + 3)}$$

Factor out the GCF of the expressions

$$\frac{(t - 1)(t + 1)}{t(t + 3) + (t + 3)} \cdot \frac{t(t + 5) - 3(t + 5)}{t(t - 3) - (t - 3)}$$

Use the distributive property

$$\frac{(t - 1)(t + 1)}{(t + 1)(t + 3)} \cdot \frac{(t - 3)(t + 5)}{(t - 1)(t - 3)}$$

Simplify

$$\frac{t + 5}{t + 3}$$

22. -

$$23. \frac{36x^2 - 25}{6x^2 + 65x + 50} \cdot \frac{3x^2 + 32x + 20}{18x^2 + 27x + 10}$$

Section 1.6

$$\frac{36x^2 - 25}{6x^2 + 65x + 50} \cdot \frac{3x^2 + 32x + 20}{18x^2 + 27x + 10}$$

Rewrite the expressions

$$\frac{(6x-5)(6x+5)}{(6x^2+5x)+(60x+50)} \cdot \frac{(3x^2+30x)+(2x+20)}{(18x^2+15x)+(12x+10)}$$

Factor out the GCF of the expressions

$$\frac{(6x-5)(6x+5)}{x(6x+5)+10(6x+5)} \cdot \frac{3x(x+10)+2(x+10)}{3x(6x+5)+2(6x+5)}$$

Use the distributive property

$$\frac{(6x-5)(6x+5)}{(x+10)(6x+5)} \cdot \frac{(3x+2)(x+10)}{(3x+2)(6x+5)}$$

Simplify

$$\frac{6x-5}{6x+5}$$

For the following exercises, divide the rational expressions.

24. -

$$25. \frac{6p^2 + p - 12}{8p^2 + 18p + 9} \div \frac{6p^2 - 11p + 4}{2p^2 + 11p - 6}$$

$$\frac{6p^2 + p - 12}{8p^2 + 18p + 9} \div \frac{6p^2 - 11p + 4}{2p^2 + 11p - 6}$$

Rewrite the expressions; divide

$$\frac{(6p^2 + 9p) + (-8p - 12)}{(8p^2 + 12p) + (6p + 9)} \cdot \frac{(2p^2 + 12p) + (-p - 6)}{(6p^2 - 3p) + (-8p + 4)}$$

Factor out the GCF of each expression

$$\frac{3p(2p+3) - 4(2p+3)}{4p(2p+3) + 3(2p+3)} \cdot \frac{2p(p+6) - (p+6)}{3p(2p-1) - 4(2p-1)}$$

Use the distributive property

$$\frac{(3p-4)(2p+3)}{(4p+3)(2p+3)} \cdot \frac{(2p-1)(p+6)}{(3p-4)(2p-1)}$$

Simplify

$$\frac{p+6}{4p+3}$$

26. -

$$27. \frac{18d^2 + 77d - 18}{27d^2 - 15d + 2} \div \frac{3d^2 + 29d - 44}{9d^2 - 15d + 4}$$

28. -

$$29. \frac{144b^2 - 25}{72b^2 - 6b - 10} \div \frac{18b^2 - 21b + 5}{36b^2 - 18b - 10}$$

$$\frac{144b^2 - 25}{72b^2 - 6b - 10} \div \frac{18b^2 - 21b + 5}{36b^2 - 18b - 10}$$

Rewrite the expressions; divide

$$\frac{(12b-5)(12b+5)}{(72b^2+24b)+(-30b-10)} \cdot \frac{(36b^2-30b)+(12b-10)}{(18b^2-6b)+(-15b+5)}$$

Factor out the GCF of each expression

$$\frac{(12b-5)(12b+5)}{24b(3b+1)-10(3b+1)} \cdot \frac{6b(6b-5)+2(6b-5)}{6b(3b-1)-5(3b-1)}$$

Use the distributive property

$$\frac{(12b-5)(12b+5)}{(24b-10)(3b+1)} \cdot \frac{(6b+2)(6b-5)}{(6b-5)(3b-1)}$$

Factor

$$\frac{(12b-5)(12b+5)}{2(12b-5)(3b+1)} \cdot \frac{2(3b+1)(6b-5)}{(6b-5)(3b-1)}$$

Simplify

$$\frac{12b+5}{3b-1}$$

30. -

$$31. \frac{22y^2 + 59y + 10}{12y^2 + 28y - 5} \div \frac{11y^2 + 46y + 8}{24y^2 - 10y + 1}$$

$$\frac{22y^2 + 59y + 10}{12y^2 + 28y - 5} \div \frac{11y^2 + 46y + 8}{24y^2 - 10y + 1}$$

Rewrite the expression; divide

$$\frac{(22y^2 + 55y) + (4y + 10)}{(12y^2 + 30y) + (-2y - 5)} \cdot \frac{(24y^2 - 6y) + (-4y + 1)}{(11y^2 + 44y) + (2y + 8)}$$

Factor out the GCF of each expression

$$\frac{11y(2y+5)+2(2y+5)}{6y(2y+5)-(2y+5)} \cdot \frac{6y(4y-1)-(4y-1)}{11y(y+4)+2(y+4)}$$

Use the distributive property

$$\frac{(11y+2)(2y+5)}{(6y-1)(2y+5)} \cdot \frac{(6y-1)(4y-1)}{(11y+2)(y+4)}$$

Simplify

$$\frac{4y-1}{y+4}$$

32. -

For the following exercises, add and subtract the rational expressions, and then simplify.

$$33. \frac{4}{x} + \frac{10}{y}$$

Section 1.6

$$\frac{4}{x} + \frac{10}{y}$$

The LCD is xy

$$\frac{4}{x} \cdot \frac{y}{y} + \frac{10}{y} \cdot \frac{x}{x}$$

Multiply

$$\frac{4y}{xy} + \frac{10x}{xy}$$

Add

$$\frac{4y + 10x}{xy}$$

34. -

$$35. \frac{4}{a+1} + \frac{5}{a-3}$$

$$\frac{4}{a+1} + \frac{5}{a-3}$$

The LCD is $(a + 1)(a - 3)$

$$\frac{4}{a+1} \cdot \frac{a-3}{a-3} + \frac{5}{a-3} \cdot \frac{a+1}{a+1}$$

Multiply

$$\frac{4(a-3)}{(a+1)(a-3)} + \frac{5(a+1)}{(a+1)(a-3)}$$

Distribute and combine fractions

$$\frac{4a - 12 + 5a + 5}{a^2 - 3a + a - 3}$$

Simplify

$$\frac{9a - 7}{a^2 - 2a - 3}$$

36. -

Section 1.6

$$37. \frac{y+3}{y-2} + \frac{y-3}{y+1}$$

$$\frac{y+3}{y-2} + \frac{y-3}{y+1}$$

$$\frac{y+3}{y-2} \cdot \frac{y+1}{y+1} + \frac{y-3}{y+1} \cdot \frac{y-2}{y-2}$$

$$\frac{(y+3)(y+1)}{(y-2)(y+1)} + \frac{(y-3)(y-2)}{(y+1)(y-2)}$$

$$\frac{y^2 + y + 3y + 3}{y^2 + y - 2y - 2} + \frac{y^2 - 2y - 3y + 6}{y^2 + y - 2y - 2}$$

$$\frac{y^2 + 4y + 3 + y^2 - 5y + 6}{y^2 - y - 2}$$

$$\frac{2y^2 - y + 9}{y^2 - y - 2}$$

The LCD is $(y - 2)(y + 1)$

Multiply

Use the FOIL method

Combine like terms

Combine like terms

38. -

$$39. \frac{3z}{z+1} + \frac{2z+5}{z-2}$$

$$\frac{3z}{z+1} + \frac{2z+5}{z-2}$$

$$\frac{3z}{z+1} \cdot \frac{z-2}{z-2} + \frac{2z+5}{z-2} \cdot \frac{z+1}{z+1}$$

$$\frac{(3z)(z-2)}{(z+1)(z-2)} + \frac{(2z+5)(z+1)}{(z-2)(z+1)}$$

$$\frac{3z^2 - 6z}{z^2 - 2z + z - 2} + \frac{2z^2 + 2z + 5z + 5}{z^2 - 2z + z - 2}$$

$$\frac{3z^2 - 6z + 2z^2 + 7z + 5}{z^2 - z - 2}$$

$$\frac{5z^2 + z + 5}{z^2 - z - 2}$$

The LCD is $(z + 1)(z - 2)$

Multiply

Use the FOIL method

Combine like terms

Combine like terms

40. -

Section 1.6

$$41. \frac{x}{x+1} + \frac{y}{y+1}$$

The LCD is $(x + 1)(y + 1)$

$$\frac{x}{x+1} + \frac{y}{y+1}$$

Multiply

$$\frac{x}{x+1} \cdot \frac{y+1}{y+1} + \frac{y}{y+1} \cdot \frac{x+1}{x+1}$$

Use the FOIL method

$$\frac{x(y+1)}{(x+1)(y+1)} + \frac{y(x+1)}{(x+1)(y+1)}$$

Combine like terms

$$\frac{xy+x}{xy+x+y+1} + \frac{xy+y}{xy+x+y+1}$$

$$\frac{2xy+x+y}{xy+x+y+1}$$

For the following exercises, simplify the rational expression.

42. -

$$43. \frac{\frac{2}{a} + \frac{7}{b}}{b}$$

Rewrite the expression

$$\frac{\frac{2}{a} + \frac{7}{b}}{b}$$

Distribute

$$\frac{1}{b} \left(\frac{2}{a} + \frac{7}{b} \right)$$

The LCD is ab^2

$$\frac{2}{ab} + \frac{7}{b^2}$$

Multiply

$$\frac{2}{ab} \cdot \frac{b}{b} + \frac{7}{b^2} \cdot \frac{a}{a}$$

Add

$$\frac{2b}{ab^2} + \frac{7a}{ab^2}$$

$$\frac{7a+2b}{ab^2}$$

44. -

45. $\frac{\frac{3}{a} + \frac{b}{6}}{\frac{2b}{3a}}$
- $\frac{\frac{3}{a} + \frac{b}{6}}{\frac{2b}{3a}}$ Rewrite the expression
- $\frac{3a\left(\frac{3}{a} + \frac{b}{6}\right)}{2b}$ Distribute
- $\frac{9a}{2ab} + \frac{3ab}{12b}$ Simplify; The LCD is $4b$
- $\frac{9}{2b} \cdot \frac{2}{2} + \frac{ab}{4b}$ Multiply
- $\frac{18}{4b} + \frac{ab}{4b}$ Add
- $\frac{18 + ab}{4b}$
46. -
47. $\frac{\frac{a}{b} - \frac{a}{a+b}}{\frac{ab}{a+b}}$
- $\frac{\frac{a}{b} - \frac{a}{a+b}}{\frac{ab}{a+b}}$ Rewrite the expression
- $\frac{ab}{a+b} \left(\frac{a}{b} - \frac{a}{a+b} \right)$ Distribute
- $\frac{a^2b}{b(a+b)} - \frac{ab^2}{a(a+b)}$ Simplify
- $\frac{a^2}{a+b} - \frac{b^2}{a+b}$ Combine
- $\frac{a^2 - b^2}{a+b}$ Difference of squares
- $\frac{(a-b)(a+b)}{a+b}$ Simplify
- $a - b$
48. -

$$49. \frac{\frac{2c}{c+2} + \frac{c-1}{c+1}}{\frac{2c+1}{c+1}}$$

$$\frac{\frac{2c}{c+2} + \frac{c-1}{c+1}}{\frac{2c+1}{c+1}}$$

Rewrite the expression

$$\frac{c+1}{2c+1} \left(\frac{2c}{c+2} + \frac{c-1}{c+1} \right)$$

Distribute

$$\frac{2c(c+1)}{(2c+1)(c+2)} + \frac{(c+1)(c-1)}{(2c+1)(c+1)}$$

Simplify; The LCD is $(2c + 1)(c + 2)$

$$\frac{2c^2 + 2c}{(2c+1)(c+2)} + \frac{c-1}{2c+1} \cdot \frac{c+2}{c+2}$$

Multiply

$$\frac{2c^2 + 2c}{2c^2 + 4c + c + 2} + \frac{c^2 + 2c - c - 2}{2c^2 + 4c + c + 2}$$

Combine like terms

$$\frac{2c^2 + 2c + c^2 + c - 2}{2c^2 + 5c + 2}$$

Combine like terms

$$\frac{3c^2 + 3c - 2}{2c^2 + 5c + 2}$$

50. -

Real-World Applications

51. Brenda is placing tile on her bathroom floor. The area of the floor is $15x^2 - 8x - 7$ square feet. The area of one tile is $x^2 - 2x + 1$. To find the number of tiles needed, simplify the

rational expression: $\frac{15x^2 - 8x - 7}{x^2 - 2x + 1}$.



Area = $15x^2 - 8x - 7$

Section 1.6

$$\frac{15x^2 - 8x - 7}{x^2 - 2x + 1}$$

Rewrite $-8x$ as $-15x + 7x$; Rewrite $-2x$ as $-x - x$

$$\frac{(15x^2 - 15x) + (7x - 7)}{(x^2 - x) + (-x + 1)}$$

Factor out the GCF of each expression

$$\frac{15x(x - 1) + 7(x - 1)}{x(x - 1) - (x - 1)}$$

Use the distributive property

$$\frac{(15x + 7)(x - 1)}{(x - 1)(x - 1)}$$

Simplify

$$\frac{15x + 7}{x - 1}$$

52. -

53. Aaron wants to mulch his garden. His garden is $x^2 + 18x + 81$ square feet. One bag of mulch covers $x^2 - 81$ square feet. Divide the expressions and simplify to find how many bags of mulch Aaron needs to mulch his garden.

$$\frac{x^2 + 18x + 81}{x^2 - 81}$$

Rewrite $18x$ as $9x + 9x$; difference of squares

$$\frac{(x^2 + 9x) + (9x + 81)}{(x - 9)(x + 9)}$$

Factor out the GCF of each expression

$$\frac{x(x + 9) + 9(x + 9)}{(x - 9)(x + 9)}$$

Use the distributive property

$$\frac{(x + 9)(x + 9)}{(x - 9)(x + 9)}$$

Simplify

$$\frac{x + 9}{x - 9}$$

Extensions

For the following exercises, perform the given operations and simplify.

54. -

$$55. \frac{\frac{3y^2 - 10y + 3}{3y^2 + 5y - 2} \cdot \frac{2y^2 - 3y - 20}{2y^2 - y - 15}}{y - 4}$$

$$\frac{3y^2 - 10y + 3}{3y^2 + 5y - 2} \cdot \frac{2y^2 - 3y - 20}{2y^2 - y - 15}$$

$$y - 4$$

Rewrite the expression

$$\frac{1}{y - 4} \left(\frac{3y^2 - 10y + 3}{3y^2 + 5y - 2} \cdot \frac{2y^2 - 3y - 20}{2y^2 - y - 15} \right)$$

Rewrite the expressions

$$\frac{1}{y - 4} \left(\frac{3y^2 - 9y - y + 3}{3y^2 + 6y - y - 2} \cdot \frac{2y^2 - 8y + 5y - 20}{2y^2 - 6y + 5y - 15} \right)$$

Factor out the GCF of each expression

$$\frac{1}{y - 4} \left(\frac{3y(y - 3) - (y - 3)}{3y(y + 2) - (y + 2)} \cdot \frac{2y(y - 4) + 5(y - 4)}{2y(y - 3) + 5(y - 3)} \right)$$

Use the distributive property

$$\frac{1}{y - 4} \left(\frac{(3y - 1)(y - 3)}{(3y - 1)(y + 2)} \cdot \frac{(2y + 5)(y - 4)}{(2y + 5)(y - 3)} \right)$$

Simplify

$$\frac{1}{y + 2}$$

56. -

$$57. \frac{x^2 + 7x + 12}{x^2 + x - 6} \div \frac{3x^2 + 19x + 28}{8x^2 - 4x - 24} \div \frac{2x^2 + x - 3}{3x^2 + 4x - 7}$$

Section 1.6

$$\frac{x^2 + 7x + 12}{x^2 + x - 6} \div \frac{3x^2 + 19x + 28}{8x^2 - 4x - 24} \div \frac{2x^2 + x - 3}{3x^2 + 4x - 7}$$

Rewrite the expressions; divide

$$\frac{x^2 + 4x + 3x + 12}{x^2 + 3x - 2x - 6} \cdot \frac{8x^2 - 16x + 12x - 24}{3x^2 + 12x + 7x + 28} \cdot \frac{3x^2 + 7x - 3x - 7}{2x^2 + 3x - 2x - 3}$$

Factor out the GCF of each expression

$$\frac{x(x+4) + 3(x+4)}{x(x+3) - 2(x+3)} \cdot \frac{8x(x-2) + 12(x-2)}{3x(x+4) + 7(x+4)} \cdot \frac{x(3x+7) - (3x+7)}{x(2x+3) - (2x+3)}$$

Use the distributive property

$$\frac{(x+3)(x+4)}{(x-2)(x+3)} \cdot \frac{(8x+12)(x-2)}{(3x+7)(x+4)} \cdot \frac{(x-1)(3x+7)}{(x-1)(2x+3)}$$

Simplify

$$\frac{4(2x+3)}{2x+3} = 4$$

Chapter 1 Review Exercises**Section 1.1**

For the following exercises, perform the given operations.

1. $(5 - 3 \cdot 2)^2 - 6$
 $(5 - 3 \cdot 2)^2 - 6$ Multiply
 $(5 - 6)^2 - 6$ Subtract inside of parenthesis
 $(-1)^2 - 6$ Square the -1
 $1 - 6$ Subtract
 -5
2. -
3. $2 \cdot 5^2 + 6 \div 2$
 $2 \cdot 5^2 + 6 \div 2$ Square the 5
 $2 \cdot 25 + 6 \div 2$ Multiply
 $50 + 6 \div 2$ Divide
 $50 + 3$ Add
 53

For the following exercises, solve the equation.

4. -
5. $2y + 4^2 = 64$
 $2y + 4^2 = 64$ Square the 4
 $2y + 16 = 64$ Subtract 16 from both sides
 $2y = 48$ Divide 2 from both sides
 $y = 24$

For the following exercises, simplify the expression.

6. -
7. $3m(4 + 7) - m$
 $3m(4 + 7) - m$ Add inside of the parenthesis
 $3m(11) - m$ Multiply
 $33m - m$ Subtract
 $32m$

For the following exercises, identify the number as rational, irrational, whole, or natural. Choose the most descriptive answer.

Chapter 1 Review Exercises

8. -
9. 0
whole
10. -
11. $\sqrt{11}$
irrational

Section 1.2

For the following exercises, simplify the expression.

12. -

$$13. \frac{4^5}{4^3}$$
$$\frac{4^5}{4^3} = 4^{5-3}$$
$$4^2 = 16$$

14. -

$$15. \frac{6a^2 \cdot a^0}{2a^{-4}}$$
$$\frac{6a^2 \cdot 1}{2a^{-4}}$$
$$\frac{6a^2 a^4}{2}$$
$$3a^{2+4}$$
$$3a^6$$

16. -

$$17. \frac{4^{-2} x^3 y^{-3}}{2x^0}$$
$$\frac{x^3}{(4^2 y^3)(2 \cdot 1)}$$
$$\frac{x^3}{2(16y^3)}$$
$$\frac{x^3}{32y^3}$$

18. -

$$19. \left(\frac{16a^3}{b^2}\right)(4ab^{-1})^{-2}$$

$$\left(\frac{16a^3}{b^2}\right)\left(\frac{1}{4ab^{-1}}\right)^2$$

$$\left(\frac{16a^3}{b^2}\right)\left(\frac{b}{4a}\right)^2$$

$$\left(\frac{16a^3}{b^2}\right)\left(\frac{b^2}{16a^2}\right)$$

a

20. -

21. Write the number in scientific notation: 16,340,000

$$1.634 \times 10^7$$

Section 1.3

For the following exercises, find the principal square root.

22. -

$$23. \sqrt{196}$$

14

24. -

$$25. \sqrt{75}$$

$$\sqrt{75} \quad 75 \text{ has factors } 25 \text{ and } 3$$

$$\sqrt{25 \cdot 3} \quad \text{Multiplication property of radicals}$$

$$\sqrt{25} \cdot \sqrt{3} \quad 25 \text{ is a perfect square}$$

$$5\sqrt{3}$$

26. -

Chapter 1 Review Exercises

$$27. \sqrt{\frac{32}{25}}$$

$$\sqrt{\frac{32}{25}} \quad \text{Division property of radicals}$$

$$\frac{\sqrt{32}}{\sqrt{25}} \quad \text{32 has factors 16 and 2; 25 is a perfect square}$$

$$\frac{\sqrt{16 \cdot 2}}{5} \quad \text{Multiplication property of radicals}$$

$$\frac{\sqrt{16}\sqrt{2}}{5} \quad \text{16 is a perfect square}$$

$$\frac{4\sqrt{2}}{5}$$

28. -

$$29. \sqrt{\frac{49}{1250}}$$

$$\sqrt{\frac{49}{1250}} \quad \text{Division property of radicals}$$

$$\frac{\sqrt{49}}{\sqrt{1250}} \quad \text{49 is a perfect square; 1250 has factors 625 and 2}$$

$$\frac{7}{\sqrt{625 \cdot 2}} \quad \text{Multiplication property of radicals}$$

$$\frac{7}{\sqrt{625}\sqrt{2}} \quad \text{625 is a perfect square}$$

$$\frac{7}{25\sqrt{2}} \quad \text{Rationalize the denominator}$$

$$\frac{7}{25\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{Simplify}$$

$$\frac{7\sqrt{2}}{50}$$

30. -

$$31. 4\sqrt{3} + 6\sqrt{3}$$

$$10\sqrt{3}$$

32. -

$$33. \sqrt[5]{-243}$$

$$\sqrt[5]{-243} \quad \text{Note that } -243 = -3^5$$

$$\sqrt[5]{-3^5} \quad \text{Evaluate}$$

$$-3$$

34. -

Section 1.4

For the following exercises, perform the given operations and simplify.

$$35. (3x^3 + 2x - 1) + (4x^2 - 2x + 7)$$

$$(3x^3 + 2x - 1) + (4x^2 - 2x + 7)$$

$$3x^3 + 2x - 1 + 4x^2 - 2x + 7$$

$$3x^3 + 4x^2 + 6$$

36. -

$$37. (2x^2 + 3x - 6) + (3x^2 - 4x + 9)$$

$$(2x^2 + 3x - 6) + (3x^2 - 4x + 9)$$

$$5x^2 - x + 3$$

38. -

$$39. (k + 3)(k - 6)$$

$$(k + 3)(k - 6)$$

$$k^2 - 6k + 3k - 18$$

$$k^2 - 3k - 18$$

40. -

$$41. (x + 1)(x^2 + 1)$$

$$(x + 1)(x^2 + 1)$$

$$x^3 + x^2 + x + 1$$

42. -

$$43. (a + 2b)(3a - b)$$

$$(a + 2b)(3a - b)$$

$$3a^2 - ab + 6ab - 2b^2$$

$$3a^2 + 5ab - 2b^2$$

44. -

Section 1.5

For the following exercises, find the greatest common factor.

Chapter 1 Review Exercises

45. $81p + 9pq - 27p^2q^2$
 $9p$

46. -

47. $88a^3b + 4a^2b - 144a^2$
 $4a^2$

For the following exercises, factor the polynomial.

48. -

49. $8a^2 + 30a - 27$
 $8a^2 + 30a - 27$ Rewrite $30a$ as $36a - 6a$
 $(8a^2 + 36a) + (-6a - 27)$ Factor out the GCF of each expression
 $4a(2a + 9) - 3(2a + 9)$ Use the distributive property
 $(4a - 3)(2a + 9)$

50. -

51. $x^2 + 10x + 25$
 $x^2 + 10x + 25$ Rewrite $10x$ as $5x + 5x$
 $(x^2 + 5x) + (5x + 25)$ Factor out the GCF of each expression
 $x(x + 5) + 5(x + 5)$ Use the distributive property
 $(x + 5)(x + 5)$ Rewrite
 $(x + 5)^2$

52. -

53. $4h^2 - 12hk + 9k^2$
 $4h^2 - 12hk + 9k^2$ Rewrite $-12hk$ as $-6hk - 6hk$
 $4h^2 - 6hk - 6hk + 9k^2$ Factor out the GCF of each expression
 $2h(2h - 3k) - 3k(2h - 3k)$ Use the distributive property
 $(2h - 3k)(2h - 3k)$ Rewrite
 $(2h - 3k)^2$

54. -

55. $p^3 + 216$
 $p^3 + 216$ Sum of cubes
 $(p + 6)(p^2 - 6p + 36)$

56. -

57. $64q^3 - 27p^3$

$64q^3 - 27p^3$

Difference of cubes

$(4q - 3p)(16q^2 + 12pq + 9p^2)$

58. -

59. $3p(p+3)^{\frac{1}{3}} - 8(p+3)^{\frac{4}{3}}$

$3p(p+3)^{\frac{1}{3}} - 8(p+3)^{\frac{4}{3}}$

Factor out the GCF of the expression

$(p+3)^{\frac{1}{3}}[3p - 8(p+3)]$

Distribute the -8

$(p+3)^{\frac{1}{3}}(3p - 8p - 24)$

Combine like terms

$(p+3)^{\frac{1}{3}}(-5p - 24)$

60.

Section 1.6

For the following exercises, simplify the expression.

61. $\frac{x^2 - x - 12}{x^2 - 8x + 16}$

$\frac{x^2 - x - 12}{x^2 - 8x + 16}$

Rewrite $-x$ as $-4x + 3x$; Rewrite $-8x$ as $-4x - 4x$

$\frac{(x^2 - 4x) + (3x - 12)}{(x^2 - 4x) + (-4x + 16)}$

Factor out the GCF of each expression

$\frac{x(x - 4) + 3(x - 4)}{x(x - 4) - 4(x - 4)}$

Use the distributive property

$\frac{(x + 3)(x - 4)}{(x - 4)(x - 4)}$

Simplify

$\frac{x + 3}{x - 4}$

62. -

$\frac{2a^2 - a - 3}{2a^2 - 6a - 8} \cdot \frac{5a^2 - 19a - 4}{10a^2 - 13a - 3}$

Rewrite the expressions

63.

$\frac{(2a^2 + 2a) + (-3a - 3)}{(2a^2 + 2a) + (-8a - 8)} \cdot \frac{(5a^2 - 20a) + (a - 4)}{(10a^2 - 15a) + (2a - 3)}$

Factor out the GCF of each expression

64. -

$\frac{2a(a + 1) - 3(a + 1)}{2a(a + 1) - 8(a + 1)} \cdot \frac{5a(a - 4) + (a - 4)}{5a(2a - 3) + (2a - 3)}$

Use the distributive property

$\frac{(2a - 3)(a + 1)}{(2a - 8)(a + 1)} \cdot \frac{(5a + 1)(a - 4)}{(5a + 1)(2a - 3)}$

Simplify

$\frac{a - 4}{2(a - 4)} = \frac{1}{2}$

Chapter 1 Review Exercises

65. $\frac{m^2 + 5m + 6}{2m^2 - 5m - 3} \div \frac{2m^2 + 3m - 9}{4m^2 - 4m - 3}$ Rewrite the expressions; divide

66. - $\frac{(m^2 + 3m) + (2m + 6)}{(2m^2 - 6m) + (m - 3)} \cdot \frac{(4m^2 - 6m) + (2m - 3)}{(2m^2 + 6m) + (-3m - 9)}$ Factor out the GCF of each expression

$\frac{m(m + 3) + 2(m + 3)}{2m(m - 3) + (m - 3)} \cdot \frac{2m(2m - 3) + (2m - 3)}{2m(m + 3) - 3(m + 3)}$ Use the distributive property

$\frac{(m + 2)(m + 3)}{(2m + 1)(m - 3)} \cdot \frac{(2m + 1)(2m - 3)}{(2m - 3)(m + 3)}$ Simplify

$\frac{m + 2}{m - 3}$

67. $\frac{10}{x} + \frac{6}{y}$

$\frac{10}{x} + \frac{6}{y}$ The LCD is xy

$\frac{10}{x} \cdot \frac{y}{y} + \frac{6}{y} \cdot \frac{x}{x}$ Multiply

$\frac{10y}{xy} + \frac{6x}{xy}$ Add

$\frac{6x + 10y}{xy}$

68. -

$$69. \frac{\frac{1}{d} + \frac{2}{c}}{6c + 12d}$$

$$dc$$

$$\frac{\frac{1}{d} + \frac{2}{c}}{6c + 12d}$$

Rewrite the expression

$$dc$$

$$\frac{dc}{6c + 12d} \left(\frac{1}{d} + \frac{2}{c} \right)$$

The LCD is dc

$$\frac{dc}{6c + 12d} \left(\frac{1}{d} \cdot \frac{c}{c} + \frac{2}{c} \cdot \frac{d}{d} \right)$$

Multiply

$$\frac{dc}{6c + 12d} \left(\frac{c}{dc} + \frac{2d}{dc} \right)$$

Multiply

$$\frac{dc(c + 2d)}{dc(6c + 12d)}$$

Simplify

$$\frac{c + 2d}{6(c + 2d)} = \frac{1}{6}$$

-

Chapter 1 Practice Test

For the following exercises, identify the number as rational, irrational, whole, or natural. Choose the most descriptive answer.

1. -13
rational
2. -

For the following exercises, evaluate the equations.

3. $2(x+3)-12=18$
 $2(x+3)-12=18$ Distribute the 2
 $2x+6-12=18$ Combine like terms on the right-hand side
 $2x-6=18$ Add 6 to both sides
 $2x=24$ Divide both sides by 2
 $x=12$
4. -
5. Write the number in standard notation: 3.1415×10^6 .
3,141,500
6. -

For the following exercises, simplify the expression.

7. $-2 \cdot (2+3 \cdot 2)^2 + 144$
 $-2 \cdot (2+3 \cdot 2)^2 + 144$ Multiply
 $-2 \cdot (2+6)^2 + 144$ Add
 $-2 \cdot (8)^2 + 144$ Square the 8
 $-2 \cdot 64 + 144$ Multiply
 $-128 + 144$ Add
16
8. -
9. $3^5 \cdot 3^{-3}$
 $3^5 \cdot 3^{-3} = 3^{5-3}$
 $3^2 = 9$
10. -

Chapter 1 Practice Test

$$11. \frac{8x^3}{(2x)^2}$$

$$\frac{8x^3}{4x^2}$$

$$2x^{3-2}$$

$$2x$$

12. -

$$13. \frac{\sqrt{441}}{21}$$

14. -

$$15. \sqrt{\frac{9x}{16}}$$

$$\frac{\sqrt{9x}}{\sqrt{16}}$$

$$\frac{\sqrt{9x}}{\sqrt{16}}$$

$$\frac{\sqrt{9}\sqrt{x}}{4}$$

$$\frac{3\sqrt{x}}{4}$$

Division property of radicals

Multiplication property of radicals; 16 is a perfect square

9 is a perfect square

16. -

$$17. 6\sqrt{24} + 7\sqrt{54} - 12\sqrt{6}$$

$$6\sqrt{24} + 7\sqrt{54} - 12\sqrt{6}$$

24 has factors 4 and 6; 54 has factors 9 and 6

$$6\sqrt{4 \cdot 6} + 7\sqrt{9 \cdot 6} - 12\sqrt{6}$$

Multiplication property of radicals

$$6\sqrt{4}\sqrt{6} + 7\sqrt{9}\sqrt{6} - 12\sqrt{6}$$

4 and 9 are perfect squares

$$6(2)\sqrt{6} + 7(3)\sqrt{6} - 12\sqrt{6}$$

Multiply

$$12\sqrt{6} + 21\sqrt{6} - 12\sqrt{6}$$

Combine like terms

$$21\sqrt{6}$$

18. -

Chapter 1 Practice Test

$$19. (13q^3 + 2q^2 - 3) - (6q^2 + 5q - 3)$$

$$(13q^3 + 2q^2 - 3) - (6q^2 + 5q - 3) \quad \text{Distribute the minus symbol}$$

$$13q^3 + 2q^2 - 3 - 6q^2 - 5q + 3 \quad \text{Combine like terms}$$

$$13q^3 - 4q^2 - 5q$$

20. -

$$21. (n - 2)(n^2 - 4n + 4)$$

$$(n - 2)(n^2 - 4n + 4) \quad \text{Use the distributive property}$$

$$n^3 - 4n^2 + 4n - 2n^2 + 8n - 8 \quad \text{Combine like terms}$$

$$n^3 - 6n^2 + 12n - 8$$

22. -

For the following exercises, factor the polynomial.

$$23. 16x^2 - 81$$

$$16x^2 - 81 \quad \text{Difference of squares}$$

$$(4x - 9)(4x + 9)$$

24. -

$$25. 27c^3 - 1331$$

$$27c^3 - 1331 \quad \text{Difference of cubes}$$

$$(3c - 11)(9c^2 + 33c + 121)$$

26. -

For the following exercises, simplify the expression.

Chapter 1 Practice Test

27. $\frac{2z^2 + 7z + 3}{z^2 - 9} \cdot \frac{4z^2 - 15z + 9}{4z^2 - 1}$ Rewrite the expressions; difference of squares

$\frac{2z^2 + 7z + 3}{z^2 - 9} \cdot \frac{4z^2 - 15z + 9}{4z^2 - 1}$

$\frac{(2z^2 + 6z) + (z + 3)}{(z - 3)(z + 3)} \cdot \frac{(4z^2 - 12z) + (-3z + 9)}{(2z - 1)(2z + 1)}$ Factor out the GCF of each expression

$\frac{2z(z + 3) + (z + 3)}{(z - 3)(z + 3)} \cdot \frac{4z(z - 3) - 3(z - 3)}{(2z - 1)(2z + 1)}$ Use the distributive property

$\frac{(2z + 1)(z + 3)}{(z - 3)(z + 3)} \cdot \frac{(4z - 3)(z - 3)}{(2z - 1)(2z + 1)}$ Simplify

$\frac{4z - 3}{2z - 1}$

28. -

29. $\frac{\frac{a}{2b} - \frac{2b}{9a}}{\frac{3a - 2b}{6a}}$ Rewrite the expression

$\frac{\frac{a}{2b} - \frac{2b}{9a}}{\frac{3a - 2b}{6a}}$

$\frac{6a}{3a - 2b} \left(\frac{a}{2b} - \frac{2b}{9a} \right)$ The LCD is $18ab$

$\frac{6a}{3a - 2b} \left(\frac{a}{2b} \cdot \frac{9a}{9a} - \frac{2b}{9a} \cdot \frac{2b}{2b} \right)$ Multiply

$\frac{6a}{3a - 2b} \left(\frac{9a^2}{18ab} - \frac{4b^2}{18ab} \right)$ Multiply

$\frac{6a(9a^2 - 4b^2)}{18ab(3a - 2b)}$ Difference of squares

$\frac{6a(3a - 2b)(3a + 2b)}{18ab(3a - 2b)}$ Simplify

$\frac{3a + 2b}{3b}$