Chapter 3
Functions
3.1 Functions and Function Notation

Section Exercises
Verbal

1. What is the difference between a relation and a function?
   A relation is a set of ordered pairs. A function is a special kind of relation in which no two ordered pairs have the same first coordinate.

2. -

3. Why does the vertical line test tell us whether the graph of a relation represents a function?
   When a vertical line intersects the graph of a relation more than once, that indicates that for that input there is more than one output. At any particular input value, there can be only one output if the relation is to be a function.

4. -

5. Why does the horizontal line test tell us whether the graph of a function is one-to-one?
   When a horizontal line intersects the graph of a function more than once, that indicates that for that output there is more than one input. A function is one-to-one if each output corresponds to only one input.

Algebraic

For the following exercises, determine whether the relation represents a function.

6. -

7. $\{(a,b),(b,c),(c,c)\}$
   There are no input values with more than one output. This relation is a function.

For the following exercises, determine whether the relation represents y as a function of x.

8. -

9. $y = x^2$
   For each input x, there is only one value of y, so this is a function.

10. -

11. $3x^2 + y = 14$
    Solving for y gives $y = 14 - 3x^2$. For each input x, there is only one value of y, so this is a function.

12. -

13. $y = -2x^2 + 40x$

   For each input x, there is only one value of y, so this is a function.

14. -

15. $x = \dfrac{3y+5}{7y-1}$

   Solving for y gives $y = \dfrac{x+5}{7x-3}$. For each input x, there is only one value of y, so this is a function.

16. -

17. $y = \dfrac{3x+5}{7x-1}$

   For each input x, there is only one value of y, so this is a function.

18. -

19. $2xy = 1$

   Solving for y gives $y = \dfrac{1}{2x}$. For each input x, there is only one value of y, so this is a function.

20. -

21. $y = x^3$

   For each input x, there is only one value of y, so this is a function.

22. -

23. $x = \pm\sqrt{1-y}$

   Solving for y gives $y = 1 - x^2$. For each input x, there is only one value of y, so this is a function.

24. -

25. $y^2 = x^2$

   Solving for y gives $y = \pm x$. For each input x, for some values of x there is more than one value of y, so this is not a function.

26. -

For the following exercises, evaluate the function f at the indicated values.

a. $f(-3)$ b. $f(2)$ c. $f(-a)$ d. $-f(a)$ e. $f(a+h)$

27. $f(x) = 2x - 5$

   a. $f(-3) = 2(-3) - 5 = -11$ b. $f(2) = 2(2) - 5 = -1$

   c. $f(-a) = 2(-a) - 5 = -2a - 5$ d. $-f(a) = -(2a - 5) = -2a + 5$

   e. $f(a+h) = 2(a+h) - 5 = 2a + 2h - 5$

28. -

29. $f(x) = \sqrt{2-x} + 5$

  a. $f(-3) = \sqrt{2-(-3)} + 5 = \sqrt{5} + 5$

  b. $f(2) = \sqrt{2-2} + 5 = 5$

  c. $f(-a) = \sqrt{2-(-a)} + 5 = \sqrt{2+a} + 5$

  d. $-f(a) = -(\sqrt{2-a} + 5) = -\sqrt{2-a} - 5$

  e. $f(a+h) = \sqrt{2-(a+h)} + 5 = \sqrt{2-a-h} + 5$

30. -

31. $f(x) = |x-1| - |x+1|$

  a. $f(-3) = |-3-1| - |-3+1| = 4 - 2 = 2$

  b. $f(2) = |2-1| - |2+1| = 1 - 3 = -2$

  c. $f(-a) = |-a-1| - |-a+1|$

  d. $-f(a) = -(|a-1| - |a+1|) = -|a-1| + |a+1|$

  e. $f(a+h) = |a+h-1| - |a+h+1|$

32. -

33. Given the function $g(x) = x^2 + 2x$, evaluate $\dfrac{g(x) - g(a)}{x - a}$, $x \neq a$.

$$\frac{g(x) - g(a)}{x - a} = \frac{x^2 + 2x - (a^2 + 2a)}{x - a} = \frac{x^2 - a^2 + 2x - 2a}{x - a}$$
$$= \frac{(x+a)(x-a) + 2(x-a)}{x - a} = \frac{(x-a)(x+a+2)}{x - a} = x + a + 2, \ x \neq a$$

34. -

35. Given the function $f(x) = 8 - 3x$:

  a. Evaluate $f(-2)$.

  b. Solve $f(x) = -1$.

    a. $f(-2) = 8 - 3(-2) = 14$

    b. $8 - 3x = -1 \Rightarrow -3x = -9 \Rightarrow x = 3$

36. -

37. Given the function $f(x) = x^2 - 3x$:

    a. Evaluate $f(5)$.

    b. Solve $f(x) = 4$.

    a. $f(5) = (5)^2 - 3(5) = 10$

    b. $x^2 - 3x = 4 \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow (x - 4)(x + 1) = 0 \Rightarrow x = 4 \text{ or } x = -1$

38. -

39. Consider the relationship $3r + 2t = 18$.

    a. Write the relationship as a function $r = f(t)$.

    b. Evaluate $f(-3)$.

    c. Solve $f(t) = 2$.

    a. $3r + 2t = 18 \Rightarrow 3r = 18 - 2t \Rightarrow r = 6 - \dfrac{2}{3}t$

    b. $f(-3) = 6 - \dfrac{2}{3}(-3) = 6 + 2 = 8$

    c. $6 - \dfrac{2}{3}t = 2 \Rightarrow -\dfrac{2}{3}t = -4 \Rightarrow t = 6$

Graphical

For the following exercises, use the vertical line test to determine which graphs show relations that are functions.
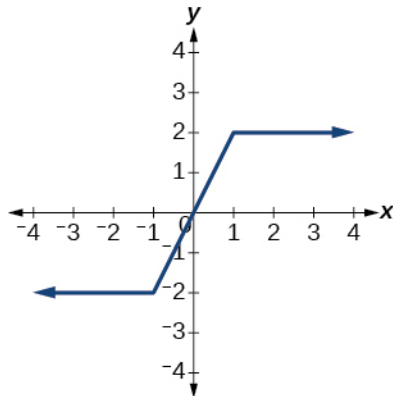
40. -

41.
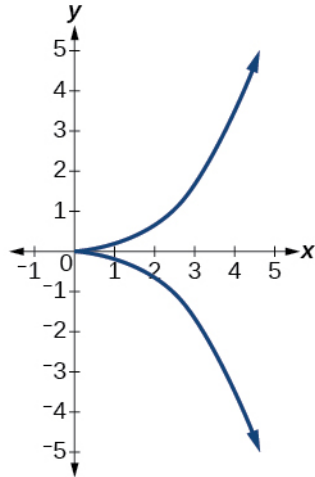
There are values of x for which a vertical line would intersect this graph more than once. This is not a function.

42. -
43.

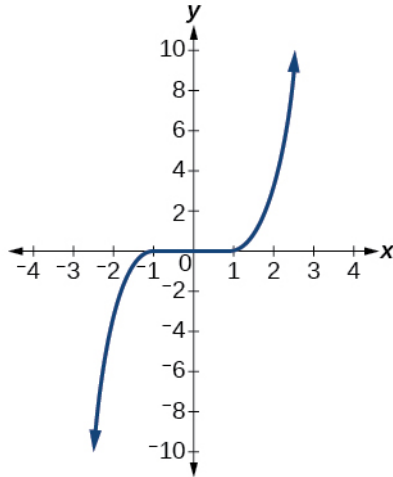There are no values of x for which a vertical line would intersect with this graph more than once. This is a function.
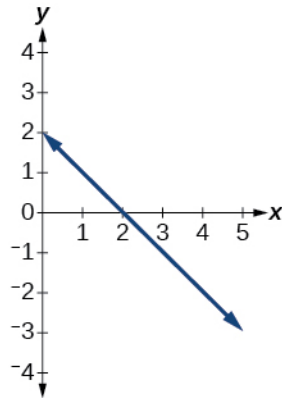
44. -

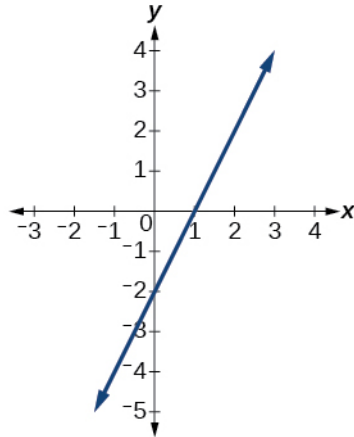45.

There are no values of x for which a vertical line would intersect with this graph more than once. This is a function.

46. -
47.

There are no values of x for which a vertical line would intersect with this graph more than once. This is a function.
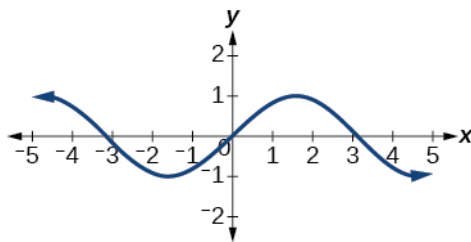
48. -
49.

There are no values of x for which a vertical line would intersect with this graph more than once. This is a function.
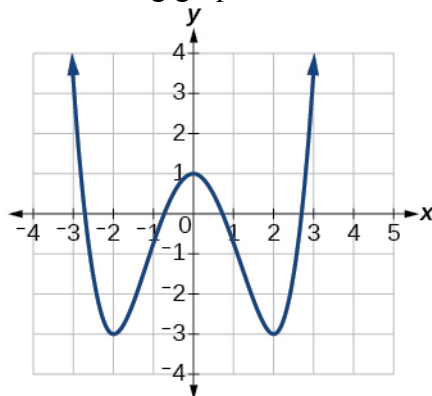
50. -
51.

There are no values of x for which a vertical line would intersect with this graph more than once. This is a function.

52. -
53. Given the following graph:
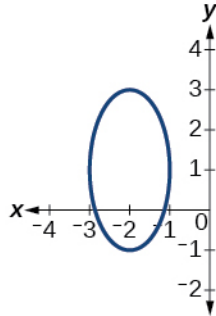

a. Evaluate $f(0)$.
b. Solve for $f(x) = -3$.
a. On the graph, when $x = 0$ the y-coordinate is 1. $f(0) = 1$
b. On the graph, when $y = -3$, the x-coordinate is –2 or 2. $f(-2) = -3$ and $f(2) = 3$
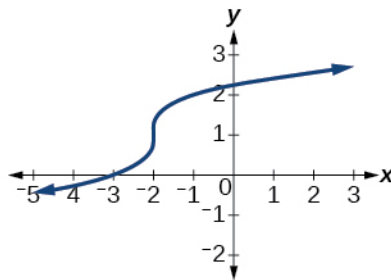
54. -

For the following exercises, determine if the given graph is a one-to-one function.
55.

There are values of x for which a vertical line would intersect this graph more than once. This is not a function, so it is not a one-to-one function.
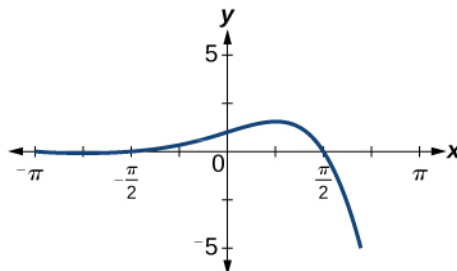
56. -

57.

There are no values of x for which a vertical line would intersect with this graph more than once. This is a function. There are no values for which a horizontal line would intersect with this graph more than once, so it is a one-to-one function.

58. -

59.

There are no values of x for which a vertical line would intersect with this graph more than once. This is a function. There are values for which a horizontal line would intersect with this graph more than once, so it is not a one-to-one function.

Numeric

For the following exercises, determine whether the relation represents a function.

60. -

61. $\{(3,4),\ (4,5),\ (5,6)\}$

There are no input values with more than one output. This relation is a function.

62. -

For the following exercises, determine if the relation represented in table form represents y as a function of x.

63.

x	5	10	15
y	3	8	14

There are no input values with more than one output. This relationship is a function.

64. -

65.

x	5	10	10
y	3	8	14

This relationship is not a function because the input 10 is associated with two different outputs.

For the following exercises, use the function f represented in table form below.

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	74	28	1	53	56	3	36	45	14	47

66. -

67. Solve $f(x) = 1$.

The table input value corresponding to $f(x) = 1$ is 2, so $f(2) = 1$.

For the following exercises, evaluate the function f at the values $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, and $f(2)$.

68. -

69. $f(x) = 8 - 3x$

$f(-2)$:	$8 - 3(-2) = 14$	$f(-2) = 14$
$f(-1)$:	$8 - 3(-1) = 11$	$f(-1) = 11$
$f(0)$:	$8 - 3(0) = 8$	$f(0) = 8$
$f(1)$:	$8 - 3(1) = 5$	$f(1) = 5$
$f(2)$:	$8 - 3(2) = 2$	$f(2) = 2$

70. -

71. $f(x) = 3 + \sqrt{x + 3}$

$f(-2)$:	$3 + \sqrt{-2 + 3} = 4$	$f(-2) = 4$
$f(-1)$:	$3 + \sqrt{-1 + 3} \approx 4.414$	$f(-1) \approx 4.414$
$f(0)$:	$3 + \sqrt{0 + 3} \approx 4.732$	$f(0) \approx 4.732$
$f(1)$:	$3 + \sqrt{1 + 3} = 5$	$f(1) = 5$
$f(2)$:	$3 + \sqrt{2 + 3} \approx 5.236$	$f(2) \approx 5.236$

$f(-2) = 4$; $f(-1) = 4.414$; $f(0) = 4.732$; $f(1) = 4.5$; $f(2) = 5.236$

72. -

73. $f(x) = 3^x$

$f(-2)$:	$3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$	$f(-2) = \dfrac{1}{9}$
$f(-1)$:	$3^{-1} = \dfrac{1}{3^1} = \dfrac{1}{3}$	$f(-1) = \dfrac{1}{3}$
$f(0)$:	$3^0 = 1$	$f(0) = 1$
$f(1)$:	$3^1 = 3$	$f(1) = 3$
$f(2)$:	$3^2 = 9$	$f(2) = 9$

For the following exercises, evaluate the expressions, given functions f, g, and h:

$$f(x) = 3x - 2$$
$$g(x) = 5 - x^2$$
$$h(x) = -2x^2 + 3x - 1$$

74. -

75. $f\left(\dfrac{7}{3}\right) - h(-2)$

$$f\left(\dfrac{7}{3}\right) - h(-2) = 3\left(\dfrac{7}{3}\right) - 2 - (-2(-2)^2 + 3(-2) - 1) = 7 - 2 - (-8 - 6 - 1) = 20$$
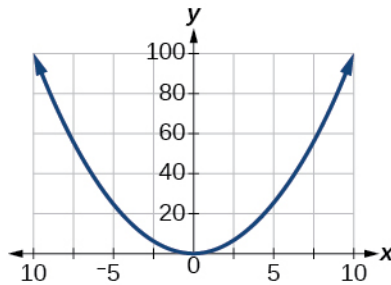
Technology

For the following exercises, graph $y = x^2$ on the given viewing windows. Determine the corresponding range for each viewing window. Show each graph.

76. -

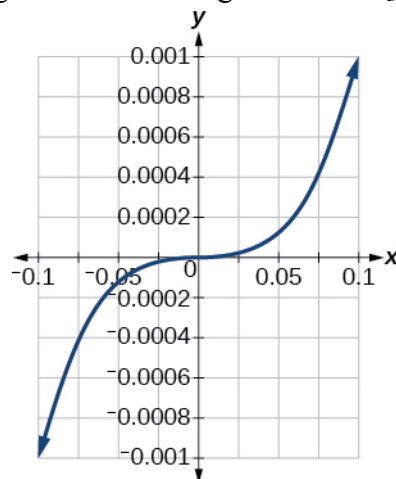77. $[-10,\ 10]$

The range for this viewing window is $[0,\ 100]$.

78. -

For the following exercises, graph $y = x^3$ on the given viewing window. Determine the corresponding range for each viewing window. Show each graph.

79. $[-0.1,\ 0.1]$
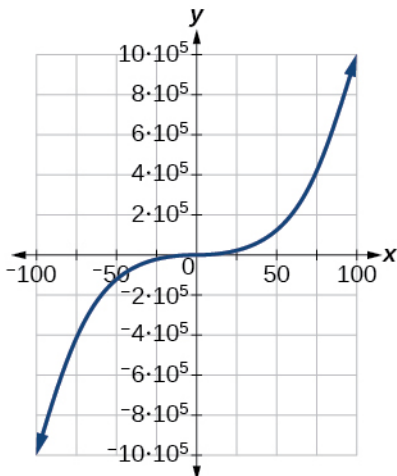
The range for this viewing window is $[-0.001,\ 0.001]$.

80. -
    The range for

81. $[-100,\ 100]$
    The range for this viewing window is $[-1{,}000{,}000,\ 1{,}000{,}000]$.
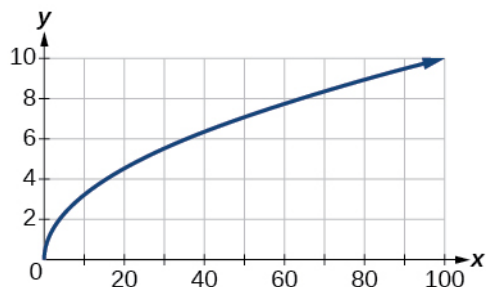

For the following exercises, graph $y = \sqrt{x}$ on the given viewing window. Determine the corresponding range for each viewing window. Show each graph.

82. -

83. $[0,\ 100]$
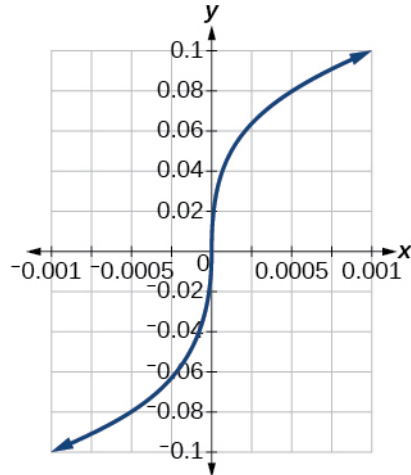   The range for this viewing window is $[0,\ 10]$.

84. -

For the following exercises, graph $y = \sqrt[3]{x}$ on the given viewing window. Determine the corresponding range for each viewing window. Show each graph.

85. $[-0.001,\ 0.001]$
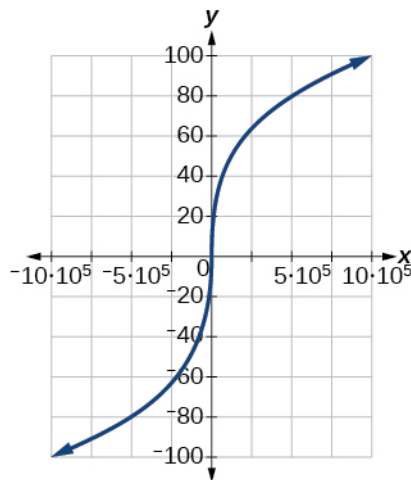   The range for this viewing window is $[-0.1,\ 0.1]$.

86. -
87. $[-1,000,000,\ 1,000,000]$

The range for this viewing window is $[-100,\ 100]$.

Real-World Applications

88. -
89. The number of cubic yards of dirt, $D,$ needed to cover a garden with area a square feet is given by $D = g(a).$

    a. A garden with area 5000 ft^2 requires 50 yd^3 of dirt. Express this information in terms of the function $g.$

    b. Explain the meaning of the statement $g(100) = 1.$

  a. $D = g(a)$, where $a = 5000$ and $D = 50,$ so $g(5000) = 50.$

b. $g(100) = 1$: where $a = 100$ and $D = 1$. This means that the number of cubic yards of dirt required for a garden of 100 square feet is 1.

90. -

91. Let $h(t)$ be the height above ground, in feet, of a rocket t seconds after launching. Explain the meaning of each statement:

    a. $h(1) = 200$.

    b. $h(2) = 350$.

    a. The height of the rocket above ground after 1 second is 200 ft.

    b. The height of the rocket above ground after 2 seconds is 350 ft.

92. –

Chapter 3
Functions
3.2 Domain and Range

SECTION EXERCISES
Verbal

1. Why does the domain differ for different functions?
   The domain of a function can be defined or can depend upon what values of the independent variable make the function undefined or imaginary.

2. -

3. Explain why the domain of $f(x) = \sqrt[3]{x}$ is different from the domain of $f(x) = \sqrt{x}$.

   There is no restriction on x for $f(x) = \sqrt[3]{x}$ because you can take the cube root of any real number. So the domain is all real numbers, $(-\infty, \infty)$. When dealing with the set of real numbers, you cannot take the square root of negative numbers. So x-values are restricted for $f(x) = \sqrt{x}$ to nonnegative numbers and the domain is $[0, \infty)$.

4. -

5. How do you graph a piecewise function?
   Graph each formula of the piecewise function over its corresponding domain. Use the same scale for the x-axis and y-axis for each graph. Indicate included endpoints with a solid circle and non-included endpoints with an open circle. Use an arrow to indicate $-\infty$ or ∞. Combine the graphs to find the graph of the piecewise function.

Algebraic

For the following exercises, find the domain of each function using interval notation.

6. -

7. $f(x) = 5 - 2x^2$

   There are no restrictions on the domain of this function. The domain is $(-\infty, \infty)$.

8. -

9. $f(x) = 3 - \sqrt{6 - 2x}$

   The radicand must be greater than or equal to zero. $6 - 2x \geq 0 \Rightarrow -2x \geq -6 \Rightarrow x \leq 3$ The domain is $(-\infty, 3]$.

10. -

11. $f(x) = \sqrt{x^2 + 4}$

    The radicand must be greater than or equal to zero. $x^2 + 4 \geq 0 \Rightarrow x^2 \geq -4$ This is always true, so the domain is $(-\infty, \infty)$.

12. -

13. $f(x) = \sqrt[3]{x-1}$

There are no restrictions on the domain of this function. The domain is $(-\infty, \infty)$.

14. -

15. $f(x) = \dfrac{3x+1}{4x+2}$

The denominator cannot be equal to zero. $4x+2 = 0 \Rightarrow 4x = -2 \Rightarrow x = -\dfrac{1}{2}$ So $x \neq -\dfrac{1}{2}$ and the domain is

$$\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right).$$

16. -

17. $f(x) = \dfrac{x-3}{x^2 + 9x - 22}$

The denominator cannot be equal to zero.
$x^2 + 9x - 22 = 0 \Rightarrow (x+11)(x-2) = 0 \Rightarrow x = 2$ or $x = -11$ So $x \neq 2$ and $x \neq -11$.
The domain is $(-\infty, -11) \cup (-11, 2) \cup (2, \infty)$.

18. -

19. $f(x) = \dfrac{2x^3 - 250}{x^2 - 2x - 15}$

The denominator cannot be equal to zero.
$x^2 - 2x - 15 = 0 \Rightarrow (x-5)(x+3) = 0 \Rightarrow x = 5$ or $x = -3$. So $x \neq 5$ and $x \neq -3$. The domain is $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$.

20. -

21. $\dfrac{2x+1}{\sqrt{5-x}}$

The radicand must be greater than zero (It cannot be equal to zero, as it is in the denominator).
$5 - x > 0 \Rightarrow 5 > x \Rightarrow x < 5$. The domain is $(-\infty, 5)$.

22. -

23. $f(x) = \dfrac{\sqrt{x-6}}{\sqrt{x-4}}$

The radicand in the denominator must be greater than zero (It cannot be equal to zero, as it is in the denominator) and the radicand in the numerator must also be greater than zero.
$x - 6 \geq 0 \Rightarrow x \geq 6$. Since this will also make the radicand in the denominator greater than zero, the domain is $[6, \infty)$.

24. -

25. $f(x) = \dfrac{x^2 - 9x}{x^2 - 81}$

The denominator cannot be equal to zero.

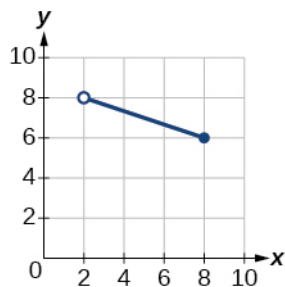$x^2 - 81 = 0 \Rightarrow (x+9)(x-9) = 0 \Rightarrow x = -9$ or $x = 9$. So $x \neq -9$ and $x \neq 9$. The domain is $\left(-\infty, -9\right) \cup \left(-9, 9\right) \cup \left(9, \infty\right)$.

26. -

Graphical

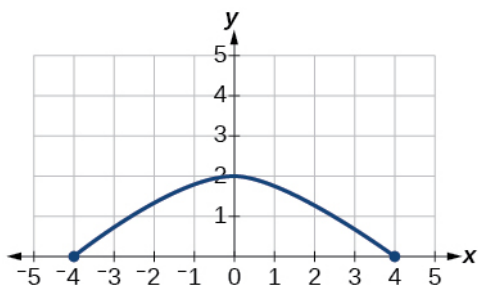For the following exercises, write the domain and range of each function using interval notation.

27.

The graph extends horizontally between 2 and 8, including the point when $x = 8$. The graph extends vertically from 6 to 8 including the point when $y = 6$. The domain is $\left(2, 8\right]$ and the range is $\left[6, 8\right)$.

28. -

29.
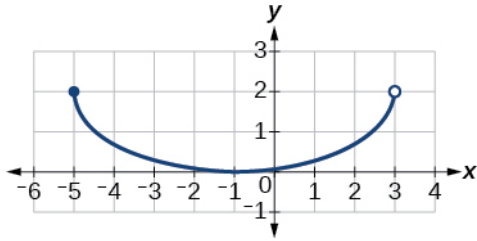

The graph extends horizontally between −4 and 4, including both endpoints. The graph extends vertically from 0 to 2, including the points when $y = 0$ and when $y = 2$. The domain is $\left[-4, 4\right]$ and the range is $\left[0, 2\right]$.
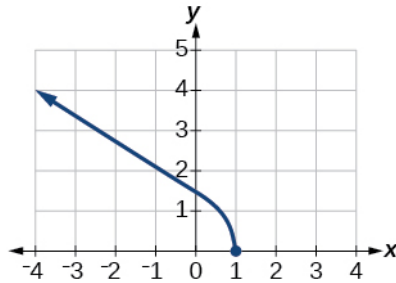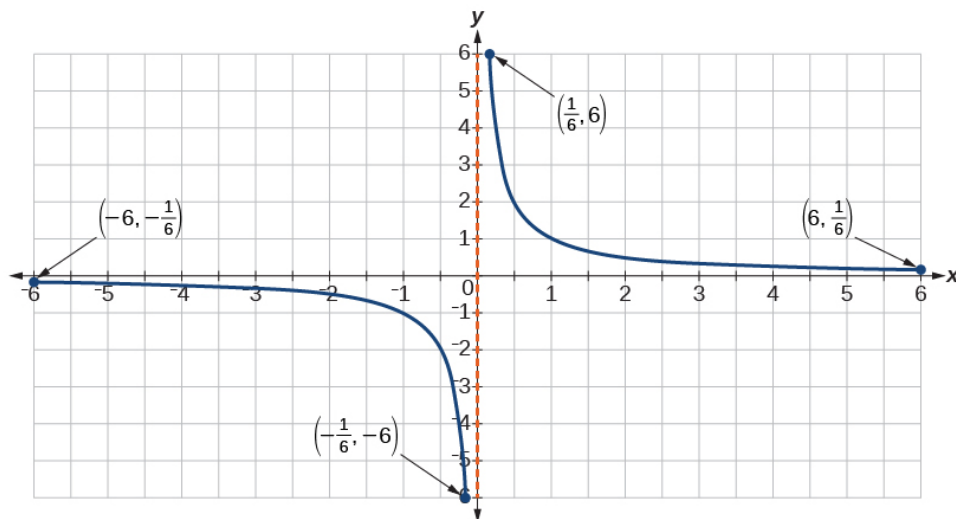
30. -

31.

The graph extends horizontally from −5 to 3, including the point when $x = -5$. The graph extends vertically from 0 to 2, including the points when $y = 0$ and when $y = 2$. The domain is $[-5, 3)$ and the range is $[0,2]$.

32. -

33.

The graph extends horizontally from 1 to the left without bound, including the point when $x = 1$. The graph extends vertically from 0 up without bound, including the point when $y = 0$. The domain is $(-\infty, 1]$ and the range is $[0,\infty)$.

34. -

35.

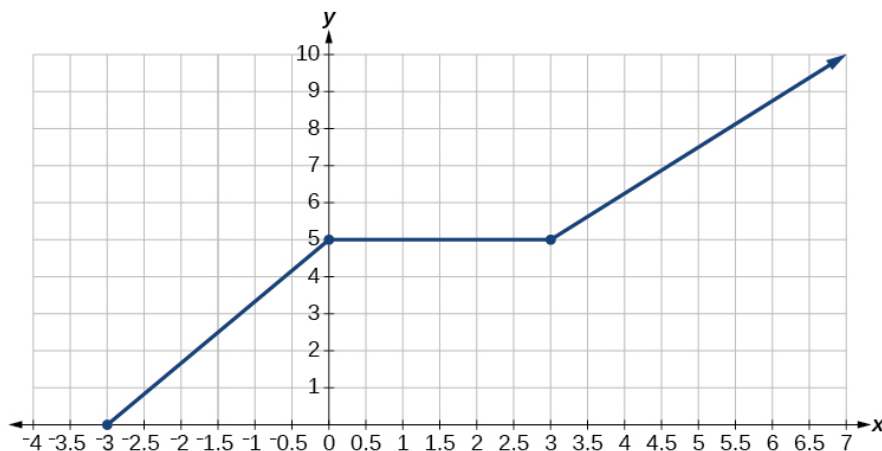Section 3.2

The graph extends horizontally from −6 to $-\frac{1}{6}$ including the endpoints and from $\frac{1}{6}$ to 6 including the endpoints. The graph extends vertically from −6 to $-\frac{1}{6}$ including the endpoints and from $\frac{1}{6}$ to 6 including the endpoints. The domain is $\left[-6,-\frac{1}{6}\right]\cup\left[\frac{1}{6},6\right]$ and the range is $\left[-6,-\frac{1}{6}\right]\cup\left[\frac{1}{6},6\right]$.

36. -
37.

The graph extends horizontally from −3 to the right without bound, including the point when x = −3. The graph extends vertically from 0 up without bound, including the point when y = 0. The domain is $[-3,\infty)$ and the range is $[0,\infty)$. [/answer]
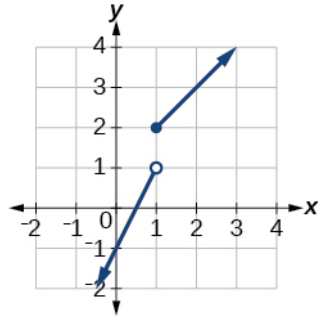
For the following exercises, sketch a graph of the piecewise function. Write the domain in interval notation.

38. -

39. $f(x)=\begin{cases} 2x-1 & \text{if} \quad x<1 \\ 1+x & \text{if} \quad x\geq 1 \end{cases}$

The function is defined for all values of x, so the domain is $(-\infty,\infty)$.
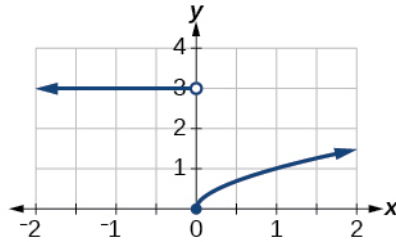
40. -

41. $f(x)=\begin{cases} 3 & \text{if} \quad x<0 \\ \sqrt{x} & \text{if} \quad x\geq 0 \end{cases}$

The function is defined for all values of x, so the domain is $(-\infty,\infty)$.

42. -

43. $f(x)=\begin{cases} x^2 & \text{if} \quad x<0 \\ x+2 & \text{if} \quad x\geq 0 \end{cases}$
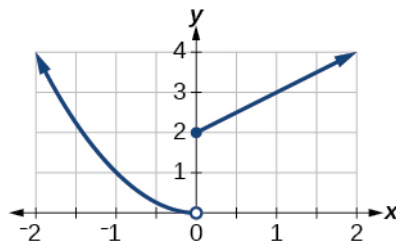
The function is defined for all values of x, so the domain is $(-\infty,\infty)$.

44. -

45. $f(x) = \begin{cases} |x| & \text{if} \quad x < 2 \\ 1 & \text{if} \quad x \geq 2 \end{cases}$

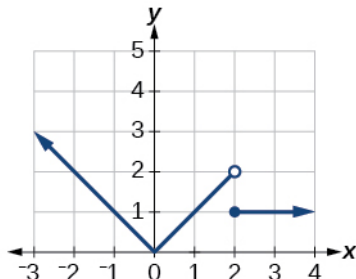   The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

Numeric

For the following exercises, given each function f, evaluate $f(-3)$, $f(-2)$, $f(-1)$, and $f(0)$.

46. -

47. $f(x) = \begin{cases} 1 & \text{if } x \leq -3 \\ 0 & \text{if } x > -3 \end{cases}$

   $\begin{array}{lll} f(-3): & 1 & f(-3) = 1 \\ f(-2): & 0 & f(-2) = 0 \\ f(-1): & 0 & f(-1) = 0 \\ f(0): & 0 & f(0) = 0 \end{array}$

48. -

For the following exercises, given each function f, evaluate $f(-1)$, $f(0)$, $f(2)$, and $f(4)$.

49. $f(x) = \begin{cases} 7x + 3 & \text{if} \quad x < 0 \\ 7x + 6 & \text{if} \quad x \geq 0 \end{cases}$

   $\begin{array}{lll} f(-1): & 7(-1) + 3 = -4 & f(-1) = -4 \\ f(0): & 7(0) + 6 = 6 & f(0) = 6 \\ f(2): & 7(2) + 6 = 20 & f(2) = 20 \\ f(4): & 7(4) + 6 = 34 & f(4) = 34 \end{array}$

50. -

51.
$$f(x) = \begin{cases} 5x & \text{if} & x < 0 \\ 3 & \text{if} & 0 \le x \le 3 \\ x^2 & \text{if} & x > 3 \end{cases}$$

$f(-1)$:	$5(-1) = -5$	$f(-1) = -5$
$f(0)$:	3	$f(0) = 3$
$f(2)$:	3	$f(2) = 3$
$f(4)$:	$(4)^2 = 16$	$f(4) = 16$

For the following exercises, write the domain for the piecewise function in interval notation.
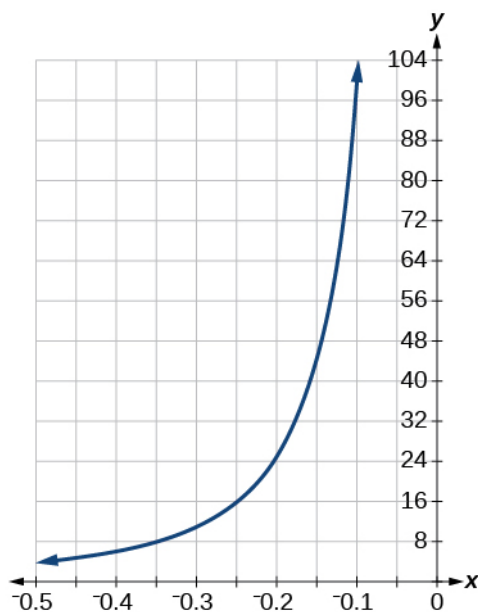
52. -

53. $f(x) = \begin{cases} x^2 - 2 & \text{if } x < 1 \\ -x^2 + 2 & \text{if } x > 1 \end{cases}$

The function is defined for all values of *x* except *x* = 1, so the domain is $(-\infty, 1) \cup (1, \infty)$.
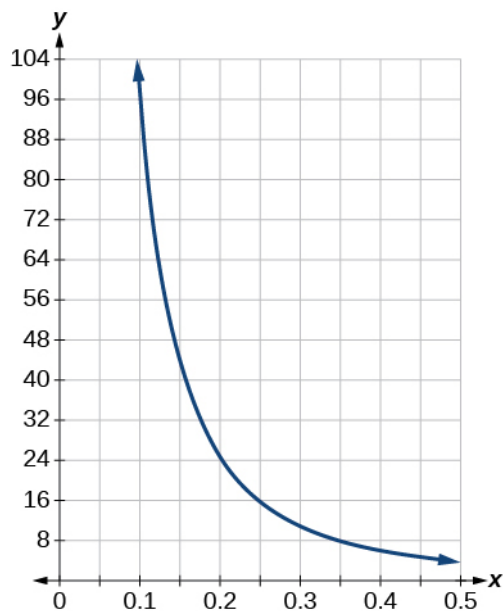
54. -

Technology

55. Graph $y = \dfrac{1}{x^2}$ on the viewing window $[-0.5, -0.1]$ and $[0.1, 0.5]$. Determine the corresponding range for the viewing window. Show the graphs.

The viewing window: $[-0.5, -0.1]$ has a range: $[4, 100]$.

The viewing window: $[0.1,\ 0.5]$ has a range: $[4,\ 100]$.

56. -

Extension

57. Suppose the range of a function f is $[-5,\ 8]$. What is the range of $|f(x)|$?

The absolute value will return outputs of greater than or equal to zero, so the range would be $[0,\ 8]$.

58. -

59. Create a function in which the domain is $x > 2$.

There are many answers. One such function is $f(x) = \dfrac{1}{\sqrt{x-2}}$.

Real-World Applications

60. -

61. The cost in dollars of making x items is given by the function $C(x) = 10x + 500$.

    a.   The fixed cost is determined when zero items are produced. Find the fixed cost for this item.

    b.   What is the cost of making 25 items?

    c.   Suppose the maximum cost allowed is $1500. What are the domain and range of the cost function, $C(x)$?

a. $C(0) = 10(0) + 500 = 500$ The fixed cost is $500.

b. $C(25) = 10(25) + 500 = 750$ The cost of making 25 items is $750.

c. $C(x) = 1500 \Rightarrow 10x + 500 = 1500 \Rightarrow 10x = 1000 \Rightarrow x = 100$ The domain is $[0,100]$ and the range is $[500,\ 1500]$.

Chapter 3
Functions

3.3 Rates of Change and Behavior Graphs

SECTION EXERCISES
Verbal

1. Can the average rate of change of a function be constant?
   Yes, the average rate of change of all linear functions is constant

2. -

3. How are the absolute maximum and minimum similar to and different from the local extrema?
   The absolute maximum and minimum relate to the entire graph, whereas the local extrema relate only to a specific region in an open interval.

4. -

Algebraic

For the following exercises, find the average rate of change of each function on the interval specified for real numbers b or h.

5. $f(x) = 4x^2 - 7$ on $[1,\ b]$

$$\frac{f(b) - f(1)}{b - 1} = \frac{4(b)^2 - 7 - (4(1)^2 - 7)}{b - 1} = \frac{4(b^2 - 1)}{b - 1} = \frac{4(b+1)(b-1)}{b - 1} = 4(b+1)$$

6. -

7. $p(x) = 3x + 4$ on $[2,\ 2 + h]$

$$\frac{p(2 + h) - p(2)}{2 + h - 2} = \frac{3(2 + h) + 4 - (3(2) + 4)}{h} = \frac{3h}{h} = 3; h \neq 0$$

8. -

9. $f(x) = 2x^2 + 1$ on $[x, x + h]$

$$\frac{f(x + h) - f(x)}{h} = \frac{2(x + h)^2 + 1 - (2x^2 + 1)}{h} = \frac{2(x^2 + 2xh + h^2) + 1 - 2x^2 - 1}{h}$$

$$= \frac{4xh + 2h^2}{h} = \frac{h(4x + 2h)}{h} = 4x + 2h; h \neq 0$$

10. -

11. $a(t) = \dfrac{1}{t + 4}$ on $[9, 9 + h]$

$$\frac{a(9 + h) - a(9)}{9 + h - 9} = \frac{\dfrac{1}{9 + h + 4} - \dfrac{1}{9 + 4}}{h} = \frac{13 - (13 + h)}{13h(13 + h)} = \frac{-h}{13h(13 + h)} = \frac{-1}{13(13 + h)}; h \neq 0$$

12. -

13. $j(x) = 3x^3$ on $[1, 1+h]$

$$\frac{j(1+h) - j(1)}{1+h-1} = \frac{3(1+h)^3 - 3(1)^3}{h} = \frac{3(1+3h+3h^2+h^3)-3}{h} = \frac{h(9+9h+3h^2)}{h}$$
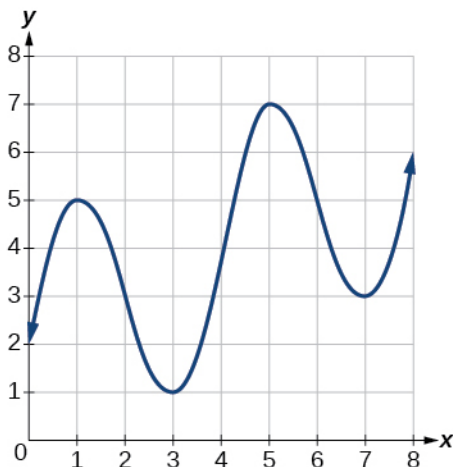$$= 3h^2 + 9h + 9; h \neq 0$$

14. -

15. Find $\frac{f(x+h) - f(x)}{h}$ given $f(x) = 2x^2 - 3x$ on $[x, x+h]$

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h} = \frac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h}$$
$$= \frac{h(4x + 2h - 3)}{h} = 4x + 2h - 3; h \neq 0$$

Graphical

For the following exercises, consider the graph of f shown in the figure.

16. -

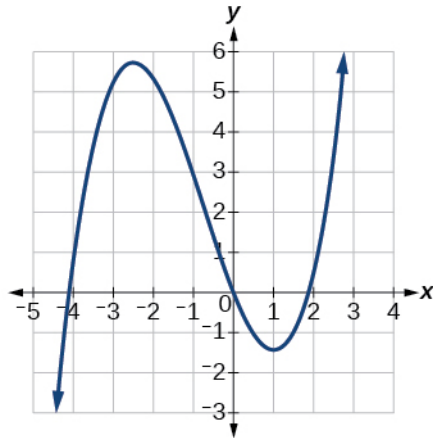17. Estimate the average rate of change from $x = 2$ to $x = 5$.

We have two points: $(2, 3)$ and $(5, 7)$. $\frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{5-2} = \frac{4}{3}$

For the following exercises, use the graph of each function to estimate the intervals on which the function is increasing or decreasing.
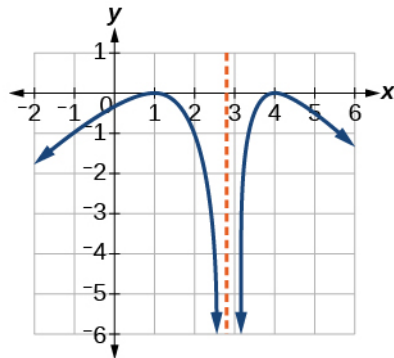
18. -

19.

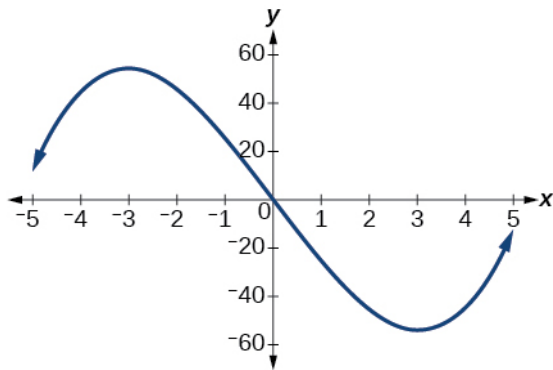The function is increasing on $\left(-\infty, -2.5\right) \cup \left(1, \infty\right)$ and decreasing on $(-2.5, \ 1)$.

20. -

21.

The function is increasing on $\left(-\infty, 1\right) \cup \left(3, 4\right)$ and decreasing on $\left(1, 3\right) \cup \left(4, \infty\right)$.

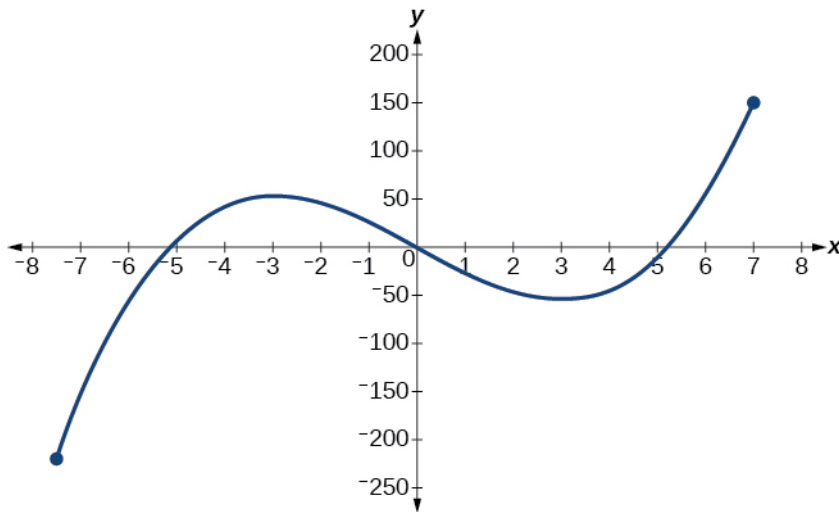For the following exercises, consider the graph shown in the figure.

22. -

23. Estimate the point(s) at which the graph of f has a local maximum or a local minimum.

   The function appears to have a local maximum: $(-3,\ 60)$ and a local minimum: $(3,-60)$.

For the following exercises, consider the graph in the figure.

24. -

25. If the complete graph of the function is shown, estimate the absolute maximum and absolute minimum.

   The function appears to have an absolute maximum of 150 at $x = 7$ and an absolute minimum of -220 at $x = -7.5$.

   The function appears to have an absolute maximum at approximately $(7,\ 150)$ and an absolute minimum at approximately $(-7.5,\ -220)$.

Numeric

26. -

27. The table below gives the population of a town (in thousands) from the year 2000 to the year 2008.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
Population (thousands)	87	84	83	80	77	76	78	81	85

What was the average rate of change of population:

a. between 2002 and 2004?

b. between 2002 and 2006?

a. $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{77 - 83}{2004 - 2002} = -\dfrac{6}{2} = -3.$ So the population has decreased by an average of 3000 per year.

b. $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{78 - 83}{2006 - 2002} = \dfrac{-5}{4} = -1.25.$ So the population has decreased by an average of 1250 per year.

For the following exercises, find the average rate of change of each function on the interval specified.

28. -

29. $h(x) = 5 - 2x^2$ on $[-2,\ 4]$

$$\frac{h(4) - h(-2)}{4 - (-2)} = \frac{5 - 2(4)^2 - (5 - 2(-2)^2)}{6} = \frac{-24}{6} = -4$$

30. -

31. $g(x) = 3x^3 - 1$ on $[-3,\ 3]$

$$\frac{g(3) - g(-3)}{3 - (-3)} = \frac{3(3)^3 - 1 - (3(-3)^3 - 1)}{6} = \frac{162}{6} = 27$$

32. -

33. $p(t) = \dfrac{\left(t^2 - 4\right)(t + 1)}{t^2 + 3}$ on $[-3,\ 1]$

$$p(1) = \frac{((1)^2 - 4)(1 + 1)}{(1)^2 + 3} = -\frac{3}{2} \qquad p(-3) = \frac{((-3)^2 - 4)(-3 + 1)}{(-3)^2 + 3} = -\frac{5}{6}$$

$$\frac{p(1) - p(-3)}{1 - (-3)} = \frac{-\dfrac{3}{2} - \left(-\dfrac{5}{6}\right)}{4} = -\frac{1}{6} \approx -0.167$$

34. -

Technology

For the following exercises, use a graphing utility to estimate the local extrema of each function and to estimate the intervals on which the function is increasing and decreasing.

35. $f(x) = x^4 - 4x^3 + 5$

There is a local minimum at $(3, -22)$. The function is decreasing on $(-\infty,\ 3)$ and increasing on $(3,\ \infty)$.

36. -

37. $g(t) = t\sqrt{t+3}$

There is a local minimum at $(-2, -2)$. The function is decreasing on $(-3, -2)$ and is increasing on $(-2,\ \infty)$.
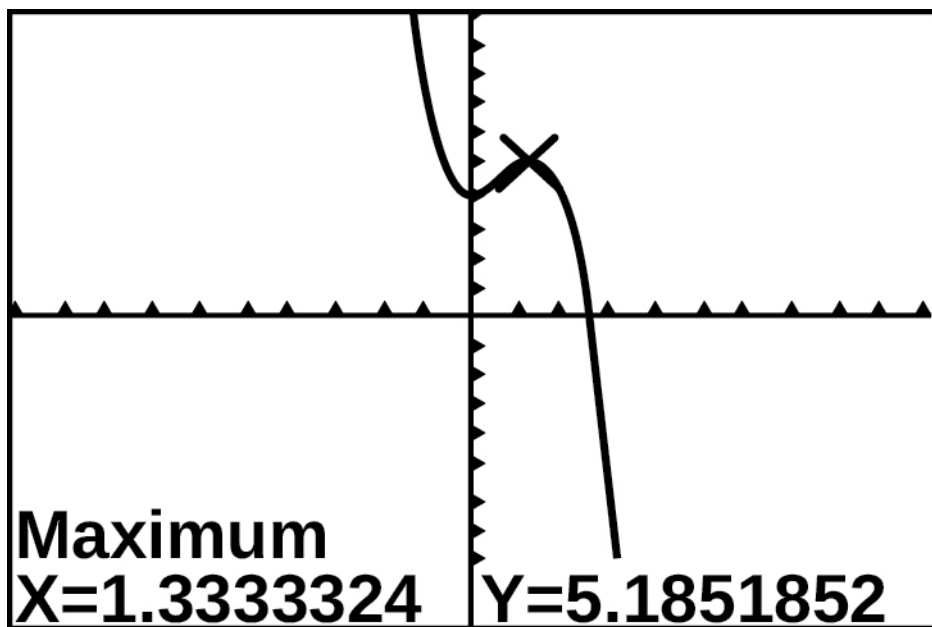
38. -

39. $m(x) = x^4 + 2x^3 - 12x^2 - 10x + 4$

There is a local maximum at $(-0.5,\ 6)$ and local minima at $(-3.25, -47)$ and $(2.1, -32)$. The function is decreasing on $(-\infty, -3.25)$ and $(-0.5,\ 2.1)$ and increasing on $(-3.25,\ -0.5)$ and $(2.1,\ \infty)$.

40. -

Extension

41. The graph of the function f is shown in the figure.


```
Maximum
X=1.3333324  Y=5.1851852
```

Based on the calculator screen shot, the point $(1.333,\ 5.185)$ is:

    A. a relative (local) maximum of the function
    B. the vertex of the function
    C. the absolute maximum of the function
    D. a zero of the function

  A. From the graph we can see that the point is a relative (local) maximum of the function.

42. -

43. Let $f(x) = \dfrac{1}{x}$. Find the number b such that the average rate of change of f on the interval

$(2,b)$ is $-\dfrac{1}{10}$.

$$\frac{f(b)-f(2)}{b-2} = -\frac{1}{10} \Rightarrow \frac{\frac{1}{b}-\frac{1}{2}}{b-2} = -\frac{1}{10} \Rightarrow \frac{2-b}{2b(b-2)} = -\frac{1}{10} \Rightarrow \frac{-1}{2b} = -\frac{1}{10} \Rightarrow b = 5$$

Real-World Applications

44. -

45. A driver of a car stopped at a gas station to fill up his gas tank. He looked at his watch, and the time read exactly 3:40 p.m. At this time, he started pumping gas into the tank. At exactly 3:44, the tank was full and he noticed that he had pumped 10.7 gallons. What is the average rate of flow of the gasoline into the gas tank?

We have two points: $(0,\ 0)$ and $(4,\ 10.7)$. $\dfrac{y_2 - y_1}{x_2 - x_2} = \dfrac{10.7-0}{4-0} \approx 2.7$. The gas was flowing at

approximately 2.7 gallons per minute.

46. -

47. The graph here illustrates the decay of a radioactive substance over t days.
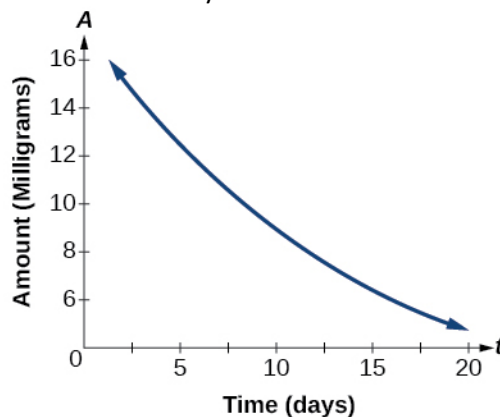
Use the graph to estimate the average decay rate from $t = 5$ to $t = 15$.

We have two points: $(5,\ 12)$ and $(15,\ 6)$. $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6-12}{15-5} = -0.6$. The radioactive

substance is decaying at approximately 0.6 milligrams per day.

Chapter 3
Functions
3.4 Composition of Functions

Verbal

1. How does one find the domain of the quotient of two functions, $\dfrac{f}{g}$?

   Find the numbers that make the function in the denominator g equal to zero, and check for any other domain restrictions on f and g, such as an even-indexed root or zeros in the denominator.

2. -

3. If the order is reversed when composing two functions, can the result ever be the same as the answer in the original order of the composition? If yes, give an example. If no, explain why not.

   Yes, although it is the exception and not the rule. Sample answer: Let $f(x) = x + 1$ and $g(x) = x - 1$. Then $f(g(x)) = f(x-1) = (x-1) + 1 = x$ and $g(f(x)) = g(x+1) = (x+1) - 1 = x$. So $f \circ g = g \circ f$.

4. -

Algebraic

For the following exercises, determine the domain for each function in interval notation.

5. Given $f(x) = x^2 + 2x$ and $g(x) = 6 - x^2$, find $f + g$, $f - g$, fg, and $\dfrac{f}{g}$.

   $f(x) + g(x) = (x^2 + 2x) + (6 - x^2) = 2x + 6$ domain: $(-\infty, \infty)$

   $f(x) - g(x) = (x^2 + 2x) - (6 - x^2) = 2x^2 + 2x - 6$ domain: $(-\infty, \infty)$

   $f(x) \cdot g(x) = (x^2 + 2x)(6 - x^2) = -x^4 - 2x^3 + 6x^2 + 12x$ domain: $(-\infty, \infty)$

   $\left(\dfrac{f}{g}\right)(x):$ $\dfrac{f(x)}{g(x)} = \dfrac{x^2 + 2x}{6 - x^2}$ domain: $(-\infty, -\sqrt{6}) \cup (-\sqrt{6}, \sqrt{6}) \cup (\sqrt{6}, \infty)$

6. -

7. Given $f(x) = 2x^2 + 4x$ and $g(x) = \dfrac{1}{2x}$, find $f + g$, $f - g$, fg, and $\dfrac{f}{g}$.

   $f(x) + g(x) = (2x^2 + 4x) + \left(\dfrac{1}{2x}\right) = \dfrac{4x^3 + 8x^2 + 1}{2x}$ domain: $(-\infty, 0) \cup (0, \infty)$

   $f(x) - g(x) = (2x^2 + 4x) - \left(\dfrac{1}{2x}\right) = \dfrac{4x^3 + 8x^2 - 1}{2x}$ domain: $(-\infty, 0) \cup (0, \infty)$

$$f(x) \cdot g(x) = (2x^2 + 4x)\left(\frac{1}{2x}\right) = \frac{2x(x+2)}{2x} = x+2 \qquad \text{domain: } (-\infty, 0) \cup (0, \infty)$$

$$\frac{f(x)}{g(x)} = \frac{2x^2 + 4x}{\frac{1}{2x}} = 2x(2x^2 + 4x) = 4x^2 + 8x^2 \qquad \text{domain: } (-\infty, 0) \cup (0, \infty)$$

8.   -

9.   Given and $f(x) = 3x^2$ and $g(x) = \sqrt{x-5}$, find $f + g, f - g, fg,$ and $\dfrac{f}{g}$.

$$f(x) + g(x) = 3x^2 + \sqrt{x-5} \qquad\qquad \text{domain: } [5, \infty)$$

$$f(x) - g(x) = 3x^2 - \sqrt{x-5} \qquad\qquad \text{domain: } [5, \infty)$$

$$f(x) \cdot g(x) = 3x^2\sqrt{x-5} \qquad\qquad \text{domain: } [5, \infty)$$

$$\frac{f(x)}{g(x)} = \frac{3x^2}{\sqrt{x-5}} \qquad\qquad\qquad \text{domain: } (5, \infty)$$

10. -

11. Given $f(x) = 2x^2 + 1$ and $g(x) = 3x - 5$, find the following:

    a. $f(g(2))$

    b. $f(g(x))$

    c. $g(f(x))$

    d. $(g \circ g)(x)$

    e. $(f \circ f)(-2)$

  a. $f(g(2)) = 2(3(2) - 5)^2 + 1 = 3$

  b. $f(g(x)) = 2(3x - 5)^2 + 1 = 2(9x^2 - 30x + 25) + 1 = 18x^2 - 60x + 51$

  c. $g(f(x)) = 3(2x^2 + 1) - 5 = 6x^2 - 2$

  d. $g(g(x)) = 3(3x - 5) - 5 = 9x - 20$

  e. $f(f(-2)) = 2(2(-2)^2 + 1)^2 + 1 = 2(9)^2 + 1 = 163$

For the following exercises, use each pair of functions to find $f(g(x))$ and $g(f(x))$. Simplify your answers.

12. -

13. $f(x) = \sqrt{x} + 2, \ g(x) = x^2 + 3$

$$f(g(x)) = \sqrt{x^2 + 3} + 2 \qquad\qquad g(f(x)) = (\sqrt{x} + 2)^2 + 3 = x + 4\sqrt{x} + 4 + 3 = x + 4\sqrt{x} + 7$$

14. -

15. $f(x) = \sqrt[3]{x}, \; g(x) = \dfrac{x+1}{x^3}$

$f(g(x)) = \sqrt[3]{\dfrac{x+1}{x^3}} = \dfrac{\sqrt[3]{x+1}}{x}$ $\qquad g(f(x)) = \dfrac{\sqrt[3]{x}+1}{\left(\sqrt[3]{x}\right)^3} = \dfrac{\sqrt[3]{x}+1}{x}$

16. -

17. $f(x) = \dfrac{1}{x-4}, \; g(x) = \dfrac{2}{x}+4$

$f(g(x)) = \dfrac{1}{\left(\dfrac{2}{x}+4\right)-4} = \dfrac{1}{\dfrac{2}{x}} = \dfrac{x}{2}, \;\; x \neq 0$

$g(f(x)) = \dfrac{2}{\left(\dfrac{1}{x-4}\right)}+4 = 2(x-4)+4 = 2x-4, \;\; x \neq 4$

For the following exercises, use each set of functions to find $f\big(g\big(h(x)\big)\big)$. Simplify your answers.

18. -

19. $f(x) = x^2 +1, \; g(x) = \dfrac{1}{x}, \;$ and $h(x) = x+3$

$f(g(h(x))) = \left(\dfrac{1}{(x+3)}\right)^2 +1 = \dfrac{1}{(x+3)^2}+1$

20. -

21. Given $f(x) = \sqrt{2-4x}$ and $g(x) = -\dfrac{3}{x}$, find the following:

    a. $(g \circ f)(x)$

    b. the domain of $(g \circ f)(x)$ in interval notation

    a. $g(f(x)) = -\dfrac{3}{\sqrt{2-4x}}$

    b. The domain of $g(f(x))$ is $\left(-\infty, \dfrac{1}{2}\right)$.

22. -

23. Given functions $p(x) = \dfrac{1}{\sqrt{x}}$ and $m(x) = x^2 - 4$, state the domain of each of the following functions using interval notation:

    a. $\dfrac{p(x)}{m(x)}$

    b. $p(m(x))$

    c. $m(p(x))$

a. $\dfrac{p(x)}{m(x)} = \dfrac{\frac{1}{\sqrt{x}}}{x^2 - 4} = \dfrac{1}{\sqrt{x}(x^2 - 4)}$ The domain is $(0, 2) \cup (2, \infty)$.

b. $p(m(x)) = \dfrac{1}{\sqrt{x^2 - 4}}$ The domain is $(-\infty,\ 2) \cup (2,\ \infty)$.

c. $m(p(x)) = \left(\dfrac{1}{\sqrt{x}}\right)^2 - 4 = \dfrac{1}{x} - 4$ The domain is $(0,\ \infty)$.

24. -

25. For $f(x) = \dfrac{1}{x}$ and $g(x) = \sqrt{x-1}$, write the domain of $(f \circ g)(x)$ in interval notation.

    $f(g(x)) = \dfrac{1}{\sqrt{x-1}}$ The domain is $(1, \infty)$.

For the following exercises, find functions $f(x)$ and $g(x)$ so the given function can be expressed as $h(x) = f\big(g(x)\big)$.

26. -

27. $h(x) = (x-5)^3$

    There are many possible solutions. Here is one possible answer: $\begin{aligned} f(x) &= x^3 \\ g(x) &= x - 5 \end{aligned}$

28. -

29. $h(x) = \dfrac{4}{(x+2)^2}$

    There are many possible solutions. Here is one possible answer: $\begin{aligned} f(x) &= \dfrac{4}{x} \\ g(x) &= (x+2)^2 \end{aligned}$

30. -

31. $h(x) = \sqrt[3]{\dfrac{1}{2x-3}}$

    There are many possible solutions. Here is one possible answer: $\begin{aligned} f(x) &= \sqrt[3]{x} \\ g(x) &= \dfrac{1}{2x-3} \end{aligned}$

32. -

33. $h(x) = \sqrt[4]{\dfrac{3x-2}{x+5}}$

There are many possible solutions. Here is one possible answer:
$$f(x) = \sqrt[4]{x}$$
$$g(x) = \frac{3x - 2}{x + 5}$$

34. -

35. $h(x) = \sqrt{2x + 6}$

There are many possible solutions. Here is one possible answer:
$$f(x) = \sqrt{x}$$
$$g(x) = 2x + 6$$

36. -

37. $h(x) = \sqrt[3]{x - 1}$

There are many possible solutions. Here is one possible answer:
$$f(x) = \sqrt[3]{x}$$
$$g(x) = x - 1$$

38. -

39. $h(x) = \dfrac{1}{(x - 2)^3}$

There are many possible solutions. Here is one possible answer:
$$f(x) = x^3$$
$$g(x) = \frac{1}{x - 2}$$

40. -

41. $h(x) = \sqrt{\dfrac{2x - 1}{3x + 4}}$

There are many possible solutions. Here is one possible answer:
$$f(x) = \sqrt{x}$$
$$g(x) = \frac{2x - 1}{3x + 4}$$

Graphical

For the following exercises, use the graphs of f and g to evaluate the expressions.
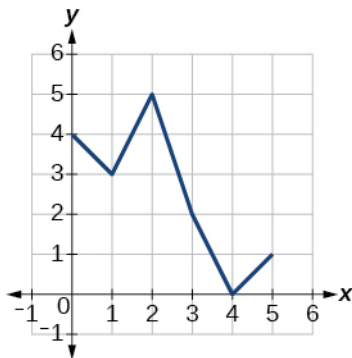
Section 3.4
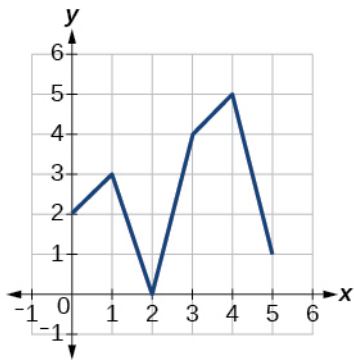
42. -

43. $f\left(g(1)\right)$

From the graph of g we have $g(1)=3$ and from the graph of f we have $f(3)=2$. So $f(g(1))=2$.

44. -

45. $g\left(f(0)\right)$

From the graph of f we have $f(0)=4$ and from the graph of g we have $g(4)=5$. So $g(f(0))=5$.

46. -

47. $f\left(f(4)\right)$

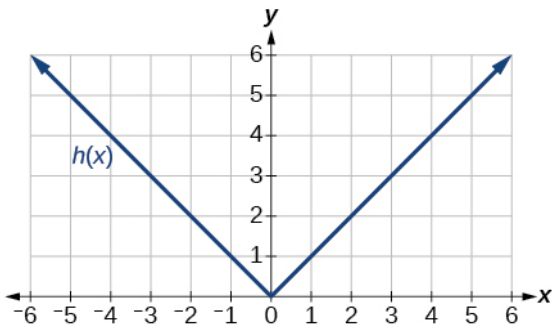From the graph of f we have $f(4)=0$ and $f(0)=4$. So $f(f(4))=4$.

48. -

49. $g\left(g(0)\right)$

From the graph of g we have $g(0)=2$ and $g(2)=0$. So $g(g(0))=0$.

For the following exercises, use graphs of $f(x)$, $g(x)$, and $h(x)$, to evaluate the expressions.
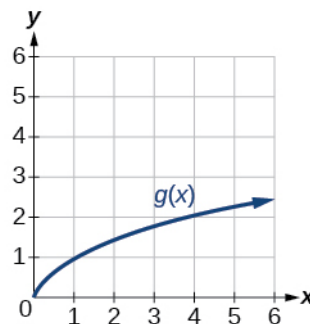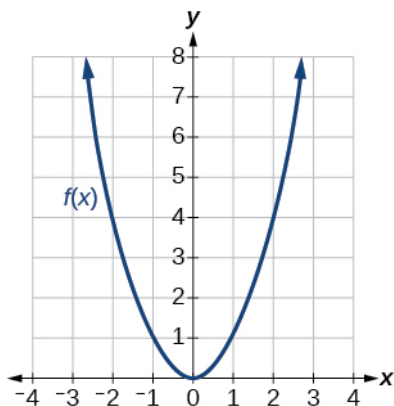
50. -

51. $g\left(f(2)\right)$

From the graph of f we have $f(2)=4$ and from the graph of g we have $g(4)=2$. This gives us $g(f(2))=2$.

52. -

53. $f\left(g(1)\right)$

From the graph of g we have $g(1)=1$ and from the graph of f we have $f(1)=1$. This gives us $f(g(1))=1$.

54. -

55. $h\left(f(2)\right)$

From the graph of f we have $f(2)=4$ and from the graph of h we have $h(4)=4$. This gives us $h(f(2))=4$.

56. -

57. $f\left(g\left(f(-2)\right)\right)$

From the graph of f we have $f(-2)=4$, from the graph of g we have $g(4)=2$ and from the graph of f we have $f(2)=4$. This gives us $f(g(f(-2)))=4$.

Numeric

For the following exercises, use the function values for f and g shown in the table to evaluate each expression.

x	$f(x)$	$g(x)$
0	7	9
1	6	5
2	5	6
3	8	2
4	4	1

5	0	8
6	2	7
7	1	3
8	9	4
9	3	0

58. -

59. $f\big(g(5)\big)$

From the table we have $g(5)=8$ and $f(8)=9$. This gives us $f(g(5))=9$.

60. -

61. $g\big(f(3)\big)$

From the table we have $f(3)=8$ and $g(8)=4$. This gives us $g(f(3))=4$.

62. -

63. $f\big(f(1)\big)$

From the table we have $f(1)=6$ and $f(6)=2$. This gives us $f(f(1))=2$.

64. -

65. $g\big(g(6)\big)$

From the table we have $g(6)=7$ and $g(7)=3$. This gives us $g(g(6))=3$.

For the following exercises, use the function values for f and g shown in the table to evaluate the expressions.

x	−3	−2	−1	0	1	2	3
$f(x)$	11	9	7	5	3	1	−1
$g(x)$	−8	−3	0	1	0	−3	−8

66. -

67. $(f \circ g)(2)$

From the table we have $g(2)=-3$ and $f(-3)=11$. This gives us $f(g(2))=11$.

68. -

69. $(g \circ f)(3)$

From the table we have $f(3)=-1$ and $g(-1)=0$. This gives us $g(f(3))=0$.

70. -

71. $(f \circ f)(3)$

From the table we have $f(3) = -1$ and $f(-1) = 7$. This gives us $f(f(-3)) = 7$.

For the following exercises, use each pair of functions to find $f\big(g(0)\big)$ and $g\big(f(0)\big)$.

72. -

73. $f(x) = 5x + 7, \ g(x) = 4 - 2x^2$

$f(g(0)) = 5(4 - 2(0)^2) + 7 = 27$ $\qquad$ $g(f(0)) = 4 - 2(5(0) + 7)^2 = -94$

74. -

75. $f(x) = \dfrac{1}{x+2}, \ g(x) = 4x + 3$

$f(g(0)) = \dfrac{1}{4(0) + 3 + 2} = \dfrac{1}{5}$ $\qquad$ $g(f(0)) = 4\left(\dfrac{1}{0+2}\right) + 3 = 5$

For the following exercises, use the functions $f(x) = 2x^2 + 1$ and $g(x) = 3x + 5$ to evaluate or find the composite function as indicated.

76. -

77. $f\big(g(x)\big)$

$f(g(x)) = 2(3x + 5)^2 + 1 = 2(9x^2 + 30x + 25) + 1 = 18x^2 + 60x + 51$

78. -

79. $(g \circ g)(x)$

$g(g(x)) = 3(3x + 5) + 5 = 9x + 20 \quad g \circ g(x) = 9x + 20$

Extensions

For the following exercises, use $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$.

80. -

81. Find $(f \circ g)(2)$ and $(g \circ f)(2)$.

$f(g(2)) = (\sqrt[3]{2 - 1})^3 + 1 = 2 - 1 + 1 = 2$

$g(f(2)) = \sqrt[3]{(2^3 + 1) - 1} = \sqrt[3]{2^3} = 2$

The answers are both 2.

82. -

83. What is the domain of $(f \circ g)(x)$?

There are no restrictions on x in $g(f(x))$ so the domain is $(-\infty, \infty)$.

84. -

For the following exercises, let $F(x) = (x+1)^5$, $f(x) = x^5$, and $g(x) = x+1$.

85. True or False: $(g \circ f)(x) = F(x)$.

False: $g(f(x)) = x^5 + 1 \neq (x+1)^5$

86. -

For the following exercises, find the composition when $f(x) = x^2 + 2$ for all $x \geq 0$ and $g(x) = \sqrt{x-2}$.

87. $(f \circ g)(6)$; $(g \circ f)(6)$

$f(g(6)) = (\sqrt{6-2})^2 + 2 = 6 - 2 + 2 = 6$ $g(f(6)) = \sqrt{(6^2+2)-2} = \sqrt{36} = 6$

88. -

89. $(f \circ g)(11)$; $(g \circ f)(11)$

$f(g(11)) = (\sqrt{11-2})^2 + 2 = 11 - 2 + 2 = 11$ $g(f(11)) = \sqrt{(11^2+2)-2} = \sqrt{11^2} = 11$

Real-World Applications

90. -

91. The function $A(d)$ gives the pain level on a scale of 0 to 10 experienced by a patient with d milligrams of a pain-reducing drug in their system. The milligrams of the drug in the patient's system after t minutes is modeled by $m(t)$. To determine when the patient will be at a pain level of 4, you would need to:

a. Evaluate $A(m(4))$.

b. Evaluate $m(A(4))$.

c. Solve $A(m(t)) = 4$.

d. Solve $m(A(d)) = 4$.

After t minutes, we can determine the amount of drug with $m(t)$ and the pain level based on this many milligrams of the drug can be found with $A(m(t))$. The answer is c.

92. -

93. A rain drop hitting a lake makes a circular ripple. If the radius, in inches, grows as a function of time in minutes according to $r(t) = 25\sqrt{t+2}$, find the area of the ripple as a function of time. Find the area of the ripple at $t = 2$.

The area of the ripple is given by the function $A(t) = \pi(r(t))^2 = \pi(25\sqrt{t+2})^2$. At $t = 2$ this is

$A(2) = \pi(25\sqrt{2+2})^2 = 2500\pi$ square inches.

94. -

95. Use the function you found to find the total area burned after 5 minutes.

After 5 minutes, the burned area is $A(5) = \pi(2(5))^2 = 121\pi$ square units.

96. -

97. The number of bacteria in a refrigerated food product is given by $N(T) = 23T^2 - 56T + 1$, $3 < T < 33$, where T is the temperature of the food. When the food is removed from the refrigerator, the temperature is given by $T(t) = 5t + 1.5$, where t is the time in hours.

    a. Find the composite function $N(T(t))$.

    b. Find the time (round to two decimal places) when the bacteria count reaches 6752

a.
$N(T(t)) = 23(5t + 1.5)^2 - 56(5t + 1.5) + 1 = 23(25t^2 + 15t + 2.25) - 280t - 84 + 1$
$= 575t^2 + 65t - 31.25$

b. $N(T(t)) = 6752 \Rightarrow 575t^2 + 65t - 31.25 = 6752 \Rightarrow 575t^2 + 65t - 6783.25 = 0$ Using the quadratic formula with $a = 575$, $b = 65$ and $c = -6783.25$ gives us approximately 3.38 hours.

Chapter 3
Functions
3.5 Transformation of Functions

Verbal

1. When examining the formula of a function that is the result of multiple transformations, how can you tell a horizontal shift from a vertical shift?
   A horizontal shift results when a constant is added to or subtracted from the input. A vertical shift results when a constant is added to or subtracted from the output.

2. -

3. When examining the formula of a function that is the result of multiple transformations, how can you tell a horizontal compression from a vertical compression?
   A horizontal compression results when a constant greater than 1 multiplies the input. A vertical compression results when a constant between 0 and 1 multiplies the output.

4. -

5. How can you determine whether a function is odd or even from the formula of the function?
   For a function f, substitute $(-x)$ for (x) in $f(x)$ and simplify. If the resulting function is the same as the original function, $f(-x) = f(x)$, then the function is even. If the resulting function is the opposite of the original function, $f(-x) = -f(x)$, then the original function is odd. If the function is not the same or the opposite, then the function is neither odd nor even.

Algebraic

6. -

7. Write a formula for the function obtained when the graph of $f(x) = |x|$ is shifted down 3 units and to the right 1 unit.
   A shift down 3 units and to the right 1 unit will result in $g(x) = f(x-1) - 3$, which gives us $g(x) = |x-1| - 3$.

8. -

9. Write a formula for the function obtained when the graph of $f(x) = \dfrac{1}{x^2}$ is shifted up 2 units and to the left 4 units.
   A shift up 2 units and to the left 4 units will result in $g(x) = f(x+4) + 2$, which gives us $g(x) = \dfrac{1}{(x+4)^2} + 2.$

For the following exercises, describe how the graph of the function is a transformation of the graph of the original function f.

10. -

11. $y = f(x + 43)$

The graph of $f(x + 43)$ is a horizontal shift to the left 43 units of the graph of f.

12. -

13. $y = f(x - 4)$

The graph of $f(x - 4)$ is a horizontal shift to the right 4 units of the graph of f.

14. -

15. $y = f(x) + 8$

The graph of $f(x) + 8$ is a vertical shift up 8 units of the graph of f.

16. -

17. $y = f(x) - 7$

The graph of $f(x) - 7$ is a vertical shift down 7 units of the graph of f.

18. -

19. $y = f(x + 4) - 1$

The graph of $f(x + 4) - 1$ is a horizontal shift to the left 4 units and a vertical shift down 1 unit of the graph of f.

For the following exercises, determine the interval(s) on which the function is increasing and decreasing.

20. -

21. $g(x) = 5(x + 3)^2 - 2$

This is a parabola with its vertex shifted from (0, 0) to (–3, –2), so the graph is decreasing on $(-\infty, -3)$ and increasing on $(-3, \infty)$.

22. -

23. $k(x) = -3\sqrt{x} - 1$

This is a square root function reflected across the x-axis, stretched by a factor of 3 and shifted down 1 unit, so it is decreasing on $(0, \infty)$.

Graphical

For the following exercises, use the graph of $f(x) = 2^x$ shown to sketch a graph of each transformation of $f(x)$.
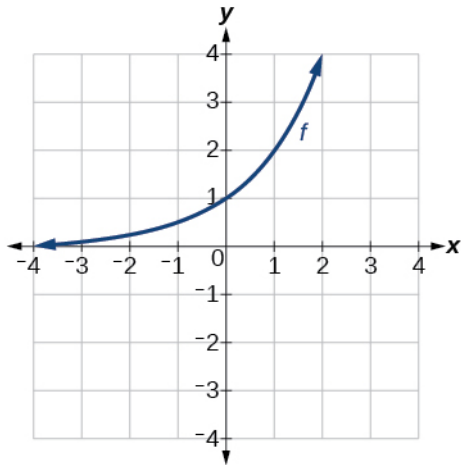
24. -

25. $h(x) = 2^x - 3$

The original graph has been shifted down 3 units.

26. -
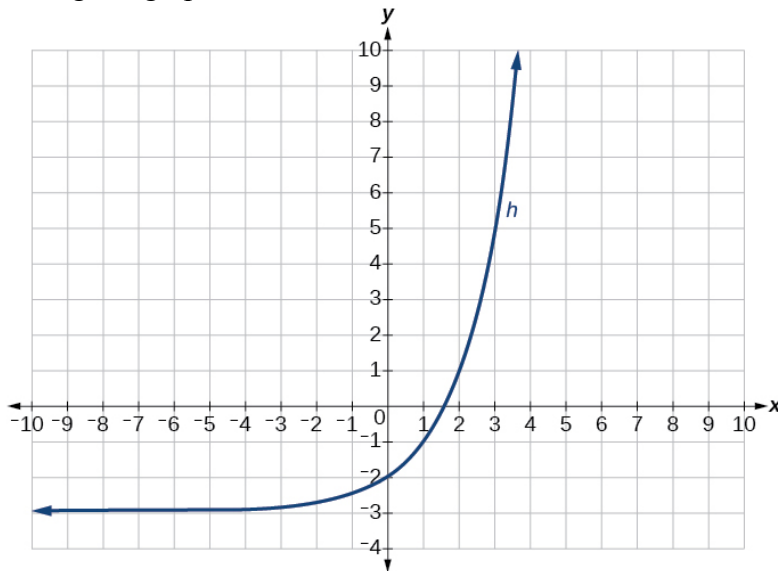
For the following exercises, sketch a graph of the function as a transformation of the graph of one of the toolkit functions.

27. $f(t) = (t+1)^2 - 3$

This is a parabola that has been shifted down 3 units and to the left 1 unit.

28. –
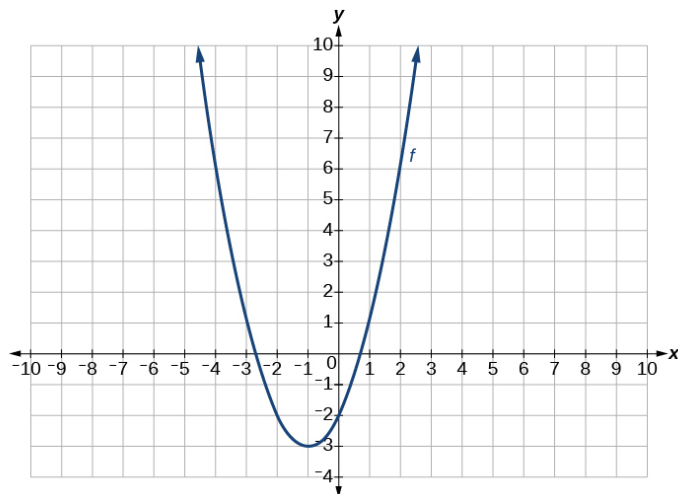
29. $k(x) = (x-2)^3 - 1$

This is a cubic function that has been shifted down 1 unit and to the right 2 units.

30. -
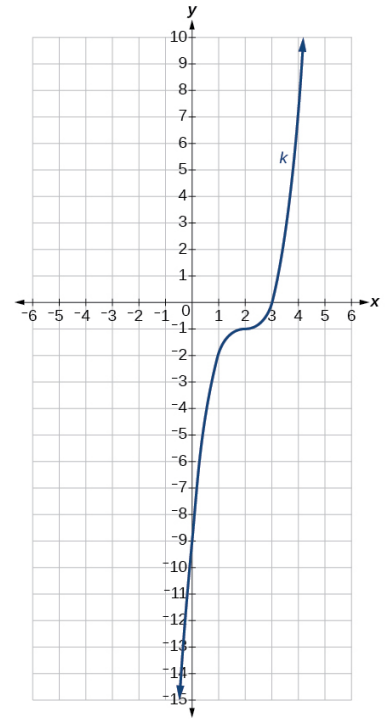
Numeric

31. Tabular representations for the functions f, g, and h are given below. Write $g(x)$ and $h(x)$ as transformations of $f(x)$.

x	−2	−1	0	1	2
$f(x)$	−2	−1	−3	1	2

x	−1	0	1	2	3
$g(x)$	−2	−1	−3	1	2

x	−2	−1	0	1	2
$h(x)$	−1	0	−2	2	3

The output values for $g(x)$ have been shifted 1 unit to the right in relation to the input values, so $g(x) = f(x-1)$. The output values for $h(x)$ have all been increased by 1, so $h(x) = f(x)+1$.

32. -

For the following exercises, write an equation for each graphed function by using transformations of the graphs of one of the toolkit functions.

33.

This is a transformation of the absolute value function 2 units down and 3 units to the right, so $f(x) = |x-3|-2$.
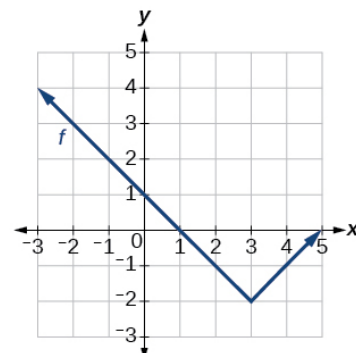
34. -

35.

This is a transformation of the square
root function 1 unit down and 3 units
to the left, so $f(x) = \sqrt{x+3} - 1$.
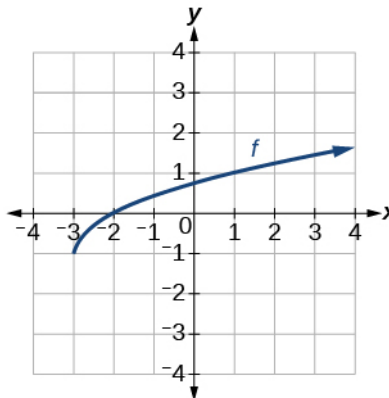
36. -

37.
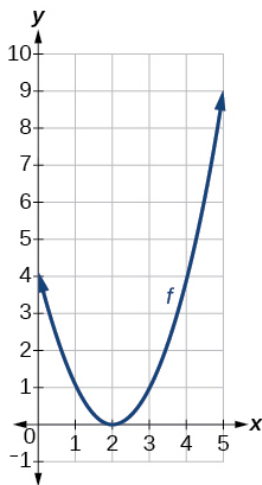
This is a transformation of the parabola
function to the right 2 units, so
$f(x) = (x-2)^2$.

38. -

39.

This is a transformation of the absolute value
function 2 units down and 3 units to the left, so
$f(x) = |x+3| - 2$.
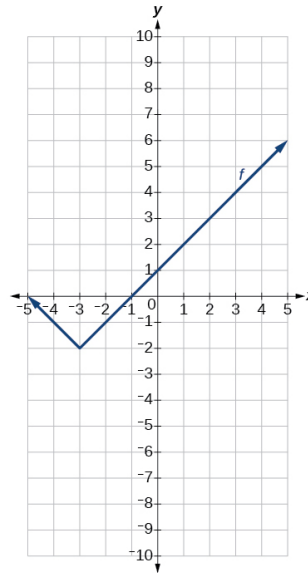
40. –

Section 3.5

For the following exercises, use the graphs of transformations of the square root function to find a formula for each of the functions.

41.
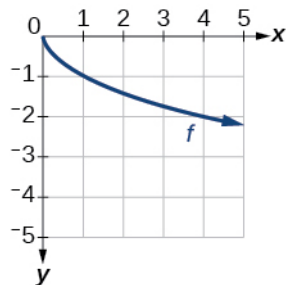
This is a reflection of the square root function across the *x*-axis, so $f(x) = -\sqrt{x}$.

42. -

For the following exercises, use the graphs of the transformed toolkit functions to write a formula for each of the resulting functions.

43.
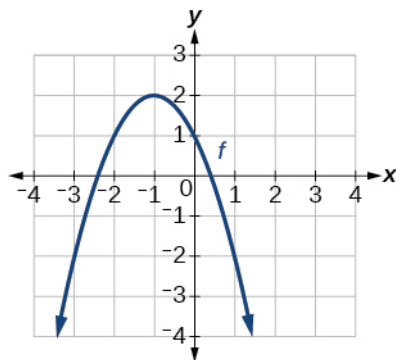
This is a transformation of the parabola function. It is reflected across the *x*-axis, shifted up 2 units and to the left 1 unit, so $f(x) = -(x+1)^2 + 2$.

44. -
45.
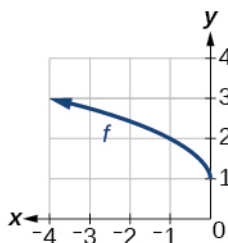
This is a transformation of the square root function. It is reflected across the *y*-axis and shifted up 1 unit, so $f(x) = \sqrt{-x} + 1$.

46. -

For the following exercises, determine whether the function is odd, even, or neither.

47. $f(x) = 3x^4$

   Evaluating $f(-x) = 3(-x)^4 = 3x^4 = f(x)$, so this function is even.

48. -

49. $h(x) = \dfrac{1}{x} + 3x$

   Evaluating $g(-x) = \dfrac{1}{-x} + 3(-x) = -\dfrac{1}{x} - 3x = -\left(\dfrac{1}{x} + 3\right) = -g(x)$, so this function is odd.

50. -

51. $g(x) = 2x^4$

   Evaluating $g(-x) = 2(-x)^4 = 2x^4 = g(x)$, so this function is even.

52. -

For the following exercises, describe how the graph of each function is a transformation of the graph of the original function f.

53. $g(x) = -f(x)$

   The graph of g is a vertical reflection (across the x-axis) of the graph of f.

54. -

55. $g(x) = 4f(x)$

   The graph of g is a vertical stretch by a factor of 4 of the graph of f.

56. -

57. $g(x) = f(5x)$

   The graph of g is a horizontal compression by a factor of $\dfrac{1}{5}$ of the graph of f.

58. -

59. $g(x) = f\left(\dfrac{1}{3}x\right)$

   The graph of g is a horizontal stretch by a factor of 3 of the graph of f.

60. -

61. $g(x) = 3f(-x)$

   The graph of g is a horizontal reflection across the y-axis and a vertical stretch by a factor of 3 of the graph of f.

62. -

For the following exercises, write a formula for the function g that results when the graph of a given toolkit function is transformed as described.

Section 3.5

63. The graph of $f(x) = |x|$ is reflected over the y-axis and horizontally compressed by a factor of $\frac{1}{4}$.

This will result in $g(x) = f(-4)$, so $g(x) = |-4x|$.

64. -

65. The graph of $f(x) = \frac{1}{x^2}$ is vertically compressed by a factor of $\frac{1}{3}$, then shifted to the left 2 units and down 3 units.

This will result in $g(x) = f(3(x+2)) - 3$, so $g(x) = \frac{1}{3(x+2)^2} - 3$.

66. -

67. The graph of $f(x) = x^2$ is vertically compressed by a factor of $\frac{1}{2}$, then shifted to the right 5 units and up 1 unit.

This will result in $g(x) = \frac{1}{2}f(x-5) + 1$, so $g(x) = \frac{1}{2}(x-5)^2 + 1$.

68. -

For the following exercises, describe how the formula is a transformation of a toolkit function. Then sketch a graph of the transformation.

69. $g(x) = 4(x+1)^2 - 5$

This is a parabola shifted to the left 1 unit, stretched vertically by a factor of 4, and shifted down 5 units.

70. -
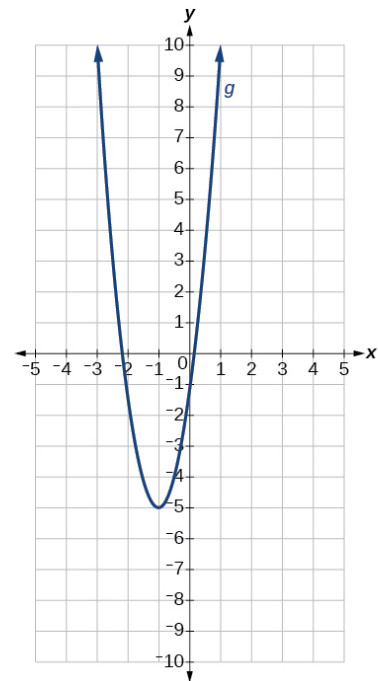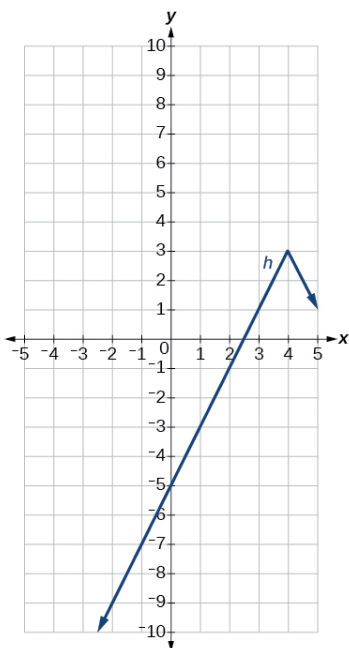
71. $h(x) = -2|x-4| + 3$

This is an absolute value function stretched vertically by a factor of 2, shifted 4 units to the right, reflected across the horizontal axis, and then shifted 3 units up.

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Section 3.5

72. -

73. $m(x) = \dfrac{1}{2}x^3$

This is a cubic function compressed vertically by a factor of $\dfrac{1}{2}$.
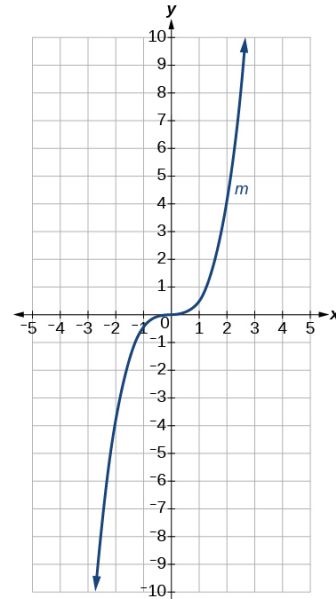
74. -

75. $p(x) = \left(\dfrac{1}{3}x\right)^3 - 3$

The graph of the function is stretched horizontally by a factor of 3 and then shifted downward by 3 units.
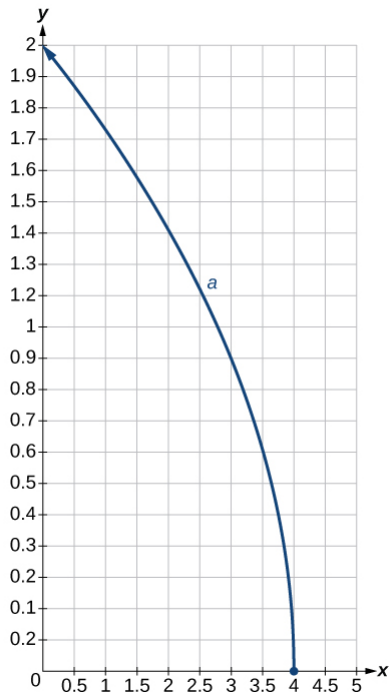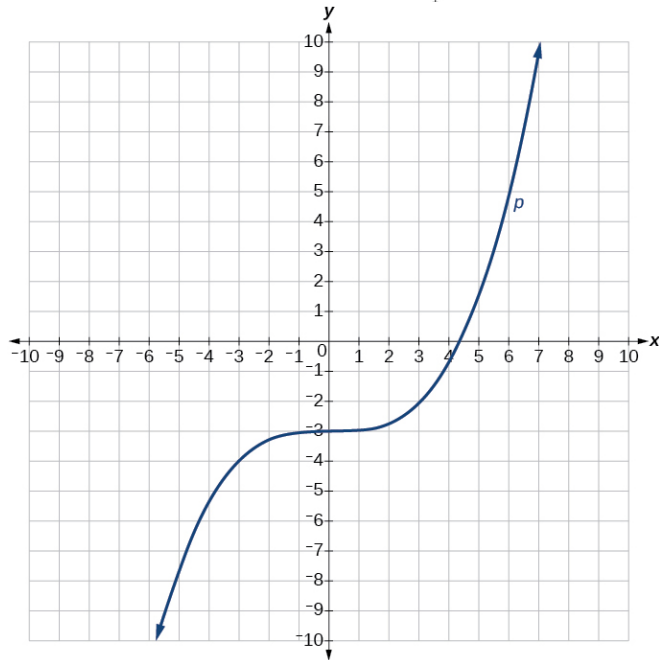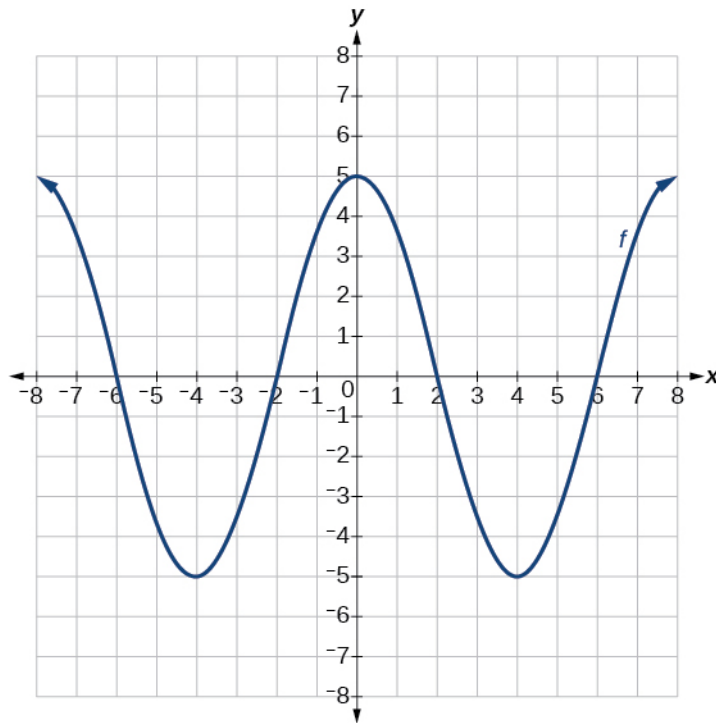
76. -

77. $a(x) = \sqrt{-x+4}$

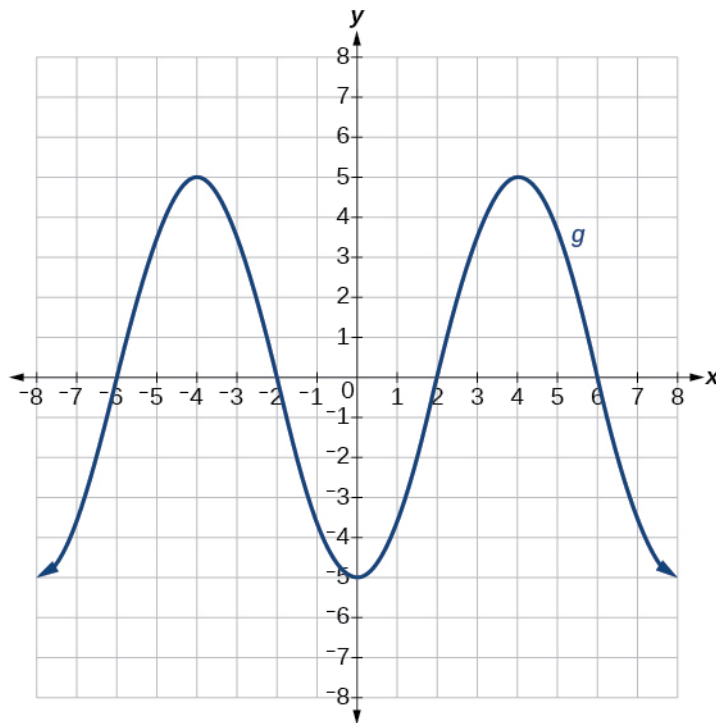The graph of $f(x) = \sqrt{x}$ is shifted 4 units to the right and then reflected across the y-axis.

224

For the following exercises, use the graph to sketch the given transformations.

78. -

79. $g(x) = -f(x)$

The graph of $g(x)$ will be the same as the graph of $f(x)$, but reflected across the x-axis.

80. -

81. $g(x) = f(x-2)$

The graph of $g(x)$ will be the same as the graph of $f(x)$, but shifted 2 units to the right.
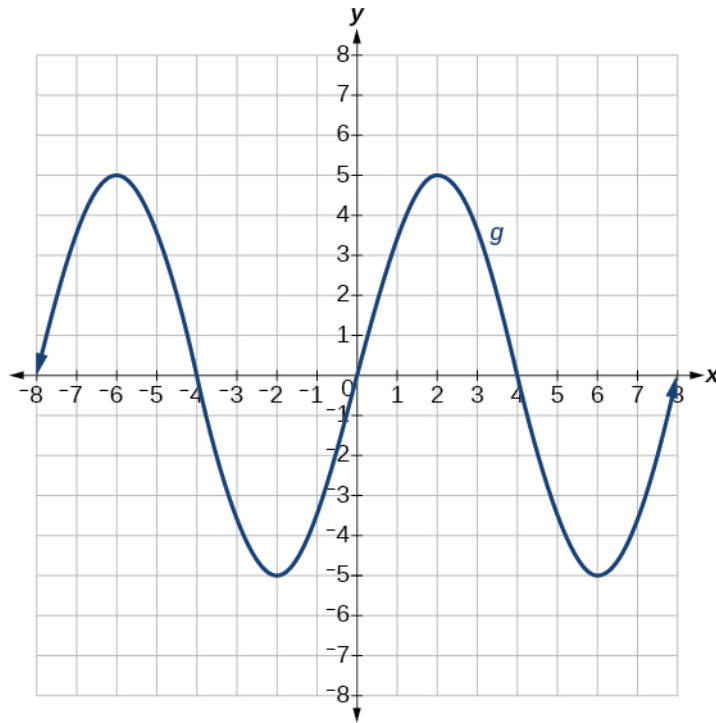
Chapter 3
Functions
3.6 Absolute Value Functions

Verbal

1. How do you solve an absolute value equation?
   Isolate the absolute value term so that the equation is of the form $|A| = B$. Form one equation by setting the expression inside the absolute value symbol, A, equal to the expression on the other side of the equation, B. Form a second equation by setting A equal to the opposite of the expression on the other side of the equation, $-B$. Solve each equation for the variable.

2. -

3. When solving an absolute value function, the isolated absolute value term is equal to a negative number. What does that tell you about the graph of the absolute value function?
   The graph of the absolute value function does not cross the x-axis, so the graph is either completely above or completely below the x-axis.

4. -

Algebraic

5. Describe all numbers x that are at a distance of 4 from the number 8. Express this using absolute value notation.
   The distance from x to 8 can be represented using the absolute value statement:
   $|x - 8| = 4$.

6. -

7. Describe the situation in which the distance that point x is from 10 is at least 15 units. Express this using absolute value notation.
   The distance from x to 10 can be represented using the absolute value statement:
   $|x - 10| \geq 15$.

8. -

For the following exercises, find the x- and y-intercepts of the graphs of each function.

9. $f(x) = 4|x - 3| + 4$

   Let $x = 0$: $f(0) = 4|0 - 3| + 4 = 16$, so the y-intercept is $(0, 16)$.

   Let $f(x) = 0$: $4|x - 3| + 4 = 0 \Rightarrow 4|x - 3| = -4 \Rightarrow |x - 3| = -1$. It is not possible for an absolute value to be negative, so there are no x-intercepts.

10. -

11. $f(x) = -2|x + 1| + 6$

   Let $x = 0$: $-2|0 + 1| + 6 = 4$, so the y-intercept is $(0, 4)$.

   Let $f(x) = 0$: $-2|x + 1| + 6 = 0 \Rightarrow -2|x + 1| = -6 \Rightarrow |x + 1| = 3$. This gives two different equations: $x + 1 = 3 \Rightarrow x = 2$ and $x + 1 = -3 \Rightarrow x = -4$, so the x-intercepts are $(-4, 0)$ and $(2, 0)$.

12. -

13. $f(x) = 2|x-1| - 6$

Let $x = 0$: $2|0-1| - 6 = -4$ so the y-intercept is $(0, -4)$.

Let $f(x) = 0$: $2|x-1| - 6 = 0 \Rightarrow 2|x-1| = 6$. This gives two different equations: $x - 1 = 3$

and $x - 1 = -3$, so the x-intercepts are $(-2, 0)$ and $(4, 0)$.

14. -

15. $f(x) = -|x-9| + 16$

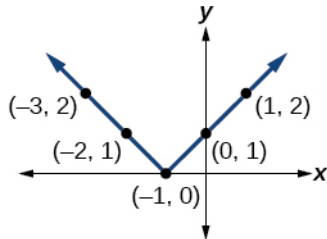Let $x = 0$: $-|0-9| + 16 = 7$ so the y-intercept is $(0, 7)$.

Let $f(x) = 0$: $-|x-9| + 16 = 0 \Rightarrow |x-9| = 16$. This gives two different equations:

$x - 9 = 16$ and $x - 9 = -16$, so the x-intercepts are $(25, 0)$ and $(-7, 0)$

Graphical

For the following exercises, graph the absolute value function. Plot at least five points by hand for each graph.

16. -

17. $y = |x+1|$

18. -

For the following exercises, graph the given functions by hand.

19. $y = |x| - 2$

This is a transformation of the absolute value function, shifted down 2 units.

20. -
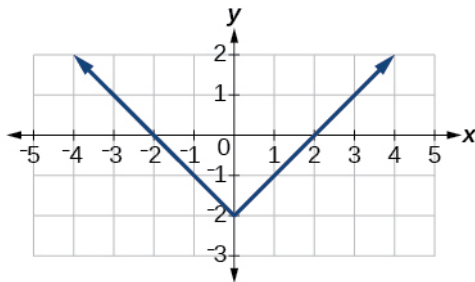
21. $y = -|x| - 2$

This is a transformation of the absolute value function, reflected across the *x*-axis and shifted down 2 units.
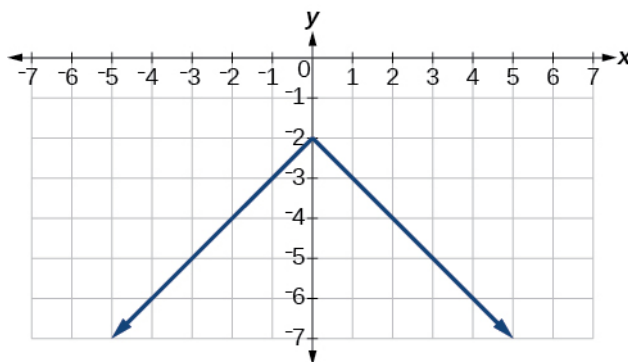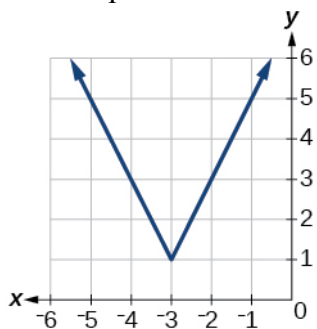
22. -

23. $f(x) = -|x-1| - 2$

This is a transformation of the absolute value function. It is reflected across the *x*-axis, shifted down 2 units and to the right 1 unit.
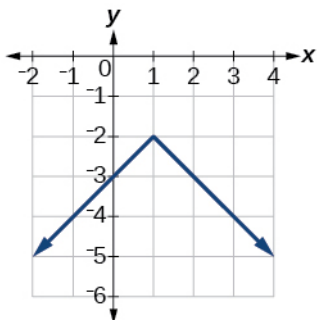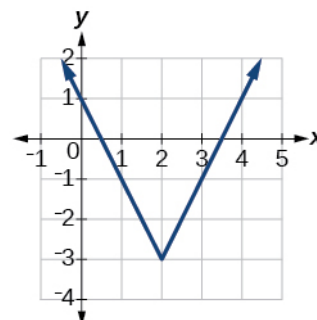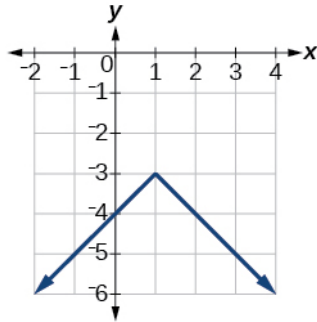
24. -

25. $f(x) = 2|x+3| + 1$

This is a transformation of the absolute value function. It is stretched vertically by a factor of 2, shifted up 1 unit and shifted to the left 3 units.

26. -

27. $f(x) = |2x-4| - 3$

This is a transformation of the absolute value function. It is compressed horizontally by a factor of $\frac{1}{2}$, shifted to the right 2 units and shifted down 3 units.
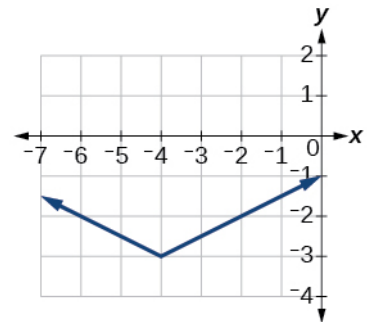
28. -

29. $f(x) = -|x-1| - 3$

This is a transformation of the absolute value function. It is reflected across the x-axis, shifted down 3 units and shifted to the right 1 unit.

30. -

31. $f(x) = \frac{1}{2}|x+4| - 3$

This is a transformation of the absolute value function. It is compressed vertically by a factor of $\frac{1}{2}$, shifted down 3 units and shifted to the left 4 units.
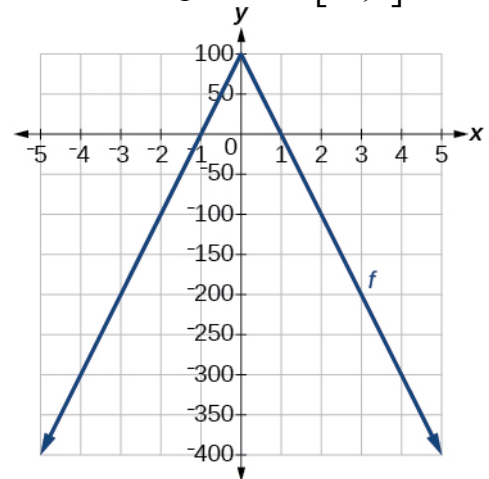
Technology

32. -

33. Use a graphing utility to graph $f(x) = -100|x| + 100$ on the viewing window $[-5, 5]$. Identify the corresponding range. Show the graph.
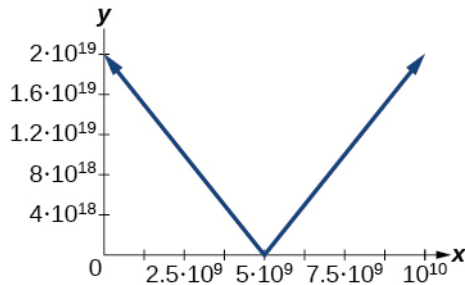
The range of the function is $[-400, 100]$.

For the following exercises, graph each function using a graphing utility. Specify the viewing window.

34. -

35. $f(x) = 4 \times 10^9 |x - (5 \times 10^9)| + 2 \times 10^9$

Extensions

For the following exercises, solve the inequality.

36. -

37. If possible, find all values of a such that there are no x-intercepts for $f(x) = 2|x+1| + a.$

   $a > 0$ If possible, find all values of a such that there are no y-intercepts for $f(x) = 2|x+1| + a.$

   $f(0)$: $2|0+1| + a = a + 2$ This is a real number for all values of a, so there is no value for a that will keep the function from having a y-intercept.

   $f(x) = 0$: $2|x+1| + a = 0 \Rightarrow 2|x+1| = -a \Rightarrow |x+1| = -\dfrac{a}{2}$ This will have a solution for x as long as a is not positive. The function will have no x-intercept for $a > 0$.

Real-World Applications

38. -

39. The true proportion, p, of people who give a favorable rating to Congress is 8% with a margin of error of 1.5%. Describe this statement using an absolute value equation.
    The difference between p and the estimated value of 8% is less than or equal to 0.015.
    This can be written with the inequality: $|p - 0.08| \leq 0.015.$

40. -

41. A machinist must produce a bearing that is within 0.01 inches of the correct diameter of 5.0 inches. Using x as the diameter of the bearing, write this statement using absolute value notation.
    The difference between the diameter of the bearing and 5.0 inches must be less than 0.01.
    This can be written with the inequality: $|x - 5.0| \leq 0.01.$

42. -

Chapter 3
Functions
3.7 Inverse Functions

Verbal

1. Describe why the horizontal line test is an effective way to determine whether a function is one-to-one?
   Each output of a function must have exactly one input for the function to be one-to-one. If any horizontal line crosses the graph of a function more than once, that means that y-values repeat and the function is not one-to-one. If no horizontal line crosses the graph of the function more than once, then no y-values repeat and the function is one-to-one.

2. -

3.
   Yes. For example, $f(x) = \dfrac{1}{x}$ is its own inverse.

4. -

5. How do you find the inverse of a function algebraically?
   Given a function $y = f(x)$, interchange the x and y. Solve the new equation for y. The expression for y is the inverse, $y = f^{-1}(x)$.

Algebraic

6. –

   For the following exercises, find $f^{-1}(x)$ for each function.

7. $f(x) = x + 3$
   Let $y = x + 3$. Interchange x and y: $x = y + 3$. Solve for y: $x = y + 3 \Rightarrow y = x - 3$. So the inverse is: $f^{-1}(x) = x - 3$.

8. -

9. $f(x) = 2 - x$
   Let $y = 2 - x$. Interchange x and y: $x = 2 - y$. Solve for y:
   $x = 2 - y \Rightarrow x + y = 2 \Rightarrow y = 2 - x$. So the inverse is: $f^{-1}(x) = 2 - x$.

10. -

11. $f(x) = 11x + 7$
    Let $y = 11x + 7$. Interchange x and y: $x = 11y + 7$. Solve for y:
    $x = 11y + 7 \Rightarrow 11y = x - 7 \Rightarrow y = \dfrac{x-7}{11}$. So the inverse is: $f^{-1}(x) = \dfrac{x-7}{11}$.

12. -

For the following exercises, find a domain on which each function f is one-to-one and non-decreasing. Write the domain in interval notation. Then find the inverse of f restricted to that domain.

13. $f(x) = (x+7)^2$

This is a parabola shifted 7 units to the left. It is one-to-one and increasing on $[-7, \infty)$. Let $y = (x+7)^2$. Interchange x and y: $x = (y+7)^2$. Solve for y:

$x = (y+7)^2 \Rightarrow \pm\sqrt{x} = y+7 \Rightarrow y = \sqrt{x} - 7$. So the inverse is: $f^{-1}(x) = \sqrt{x} - 7$

14. -

15. $f(x) = x^2 - 5$

This is a parabola shifted 5 units down. It is one-to-one and increasing on $[0, \infty)$. Let $y = x^2 - 5$. Interchange x and y: $x = y^2 - 5$. Solve for y:

$x = y^2 - 5 \Rightarrow x+5 = y^2 \Rightarrow y = \pm\sqrt{x+5} \Rightarrow y = \sqrt{x+5}$. So the inverse is $f^{-1}(x) = \sqrt{x+5}$.

16. -

For the following exercises, use function composition to verify that $f(x)$ and $g(x)$ are inverse functions.

17. $f(x) = \sqrt[3]{x-1}$ and $g(x) = x^3 + 1$

$f(g(x)) = \sqrt[3]{x^3 + 1 - 1} = \sqrt[3]{x^3} = x$ and $g(f(x)) = \left(\sqrt[3]{x-1}\right)^3 + 1 = x - 1 + 1 = x$. So $f(x)$ and $g(x)$ are inverses of each other.

18. -

Graphical

For the following exercises, use a graphing utility to determine whether each function is one-to-one.

19. $f(x) = \sqrt{x}$

This function is one-to-one.

20. -

21. $f(x) = -5x + 1$

This function is one-to-one.

22. -

For the following exercises, determine whether the graph represents a one-to-one function.

23.
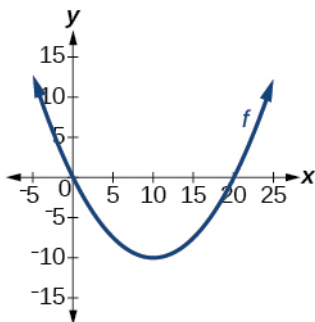
There are places where a horizontal line would intersect the graph in more than one point, so it is not one-to-one.

24. -

For the following exercises, use the graph of f shown.

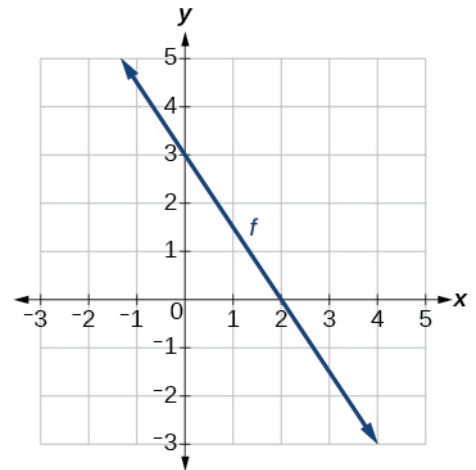For the following exercises, evaluate or solve.

25. Find $f(0)$.

The y-coordinate when $x = 0$ is 3, so $f(0) = 3$.

26. -

27. Find $f^{-1}(0)$.

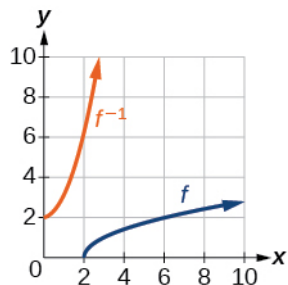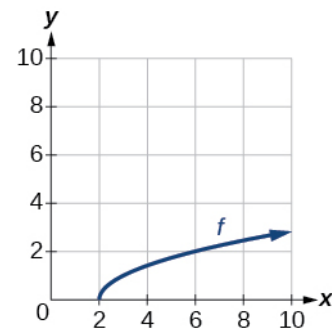The graph of $f(x)$ contains the point $(2, 0)$ so $f^{-1}(0) = 2$.

28. -

For the following exercises, use the graph of the one-to-one function shown.

29. Sketch the graph of f^{-1}.

The graph of f^{-1} is the reflection of the graph of f across the line $y = x$.

30. -

31. If the complete graph of f is shown, find the domain of f.

The graph extends horizontally from 2 to 10, including the endpoints, so the domain is $[2,10]$.

32. -

Numeric

For the following exercises, evaluate or solve, assuming that the function f is one-to-one.

33. If $f(6) = 7$, find $f^{-1}(7)$.

The inverse function reverses the input and output quantities, so $f^{-1}(7) = 6$.

34. -

234

35. If $f^{-1}(-4) = -8$, find $f(-8)$.

The inverse function reverses the input and output quantities, so $f(-8) = -4$.

36. -

For the following exercises, use the values listed in the table to evaluate or solve.

x	0	1	2	3	4	5	6	7	8	9
f(x)	8	0	7	4	2	6	5	3	9	1

37. Find $f(1)$.

From the table we can see that $f(1) = 0$.

38. -

39. Find $f^{-1}(0)$.

From the table we can see that $f(1) = 0$, so $f^{-1}(0) = 1$.

40. -

41. Use the tabular representation of f to create a table for $f^{-1}(x)$.

x	3	6	9	13	14
f(x)	1	4	7	12	16

The inverse function reverses the input and output quantities.

x	1	4	7	12	16
f⁻¹(x)	3	6	9	13	14

Technology

For the following exercises, find the inverse function.
Then, graph the function and its inverse.

42. -

43. $f(x) = x^3 - 1$

Let $y = x^3 - 1$. Interchange x and y: $x = y^3 - 1$.

Solve for y: $x = y^3 - 1 \Rightarrow y^3 = x + 1 \Rightarrow y = \sqrt[3]{x+1}$,

so the inverse function is $f^{-1}(x) = (1+x)^{1/3}$.

44. -

235
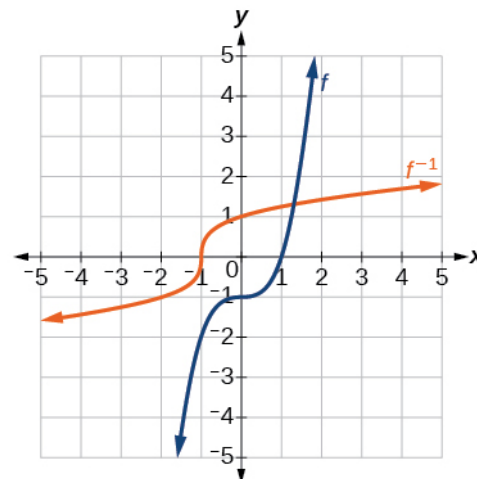
Real-World Applications

45. To convert from x degrees Celsius to y degrees Fahrenheit, we use the formula

$f(x) = \dfrac{9}{5}x + 32$. Find the inverse function, if it exists, and explain its meaning.

Let $y = \dfrac{9}{5}x + 32$. Interchange x and y: $x = \dfrac{9}{5}y + 32$. Solve for y:

$x = \dfrac{9}{5}y + 32 \Rightarrow \dfrac{9}{5}y = x - 32 \Rightarrow y = \dfrac{5}{9}(x - 32)$. so the inverse function is

$f^{-1}(x) = \dfrac{5}{9}(x - 32)$. This formula allows you to convert from x degrees Celsius to y

degrees Fahrenheit.

46. -

47. A car travels at a constant speed of 50 miles per hour. The distance the car travels in miles is a function of time, t, in hours given by $d(t) = 50t$. Find the inverse function by expressing the time of travel in terms of the distance traveled. Call this function $t(d)$. Find $t(180)$ and interpret its meaning.

Let $d = 50t$. Solve for t: $d = 50t \Rightarrow t = \dfrac{d}{50}$. So the inverse function is $t(d) = \dfrac{d}{50}$;

$t(180) = \dfrac{180}{50}$. The time it takes for the car to travel 180 miles is 3.6 hours.

Chapter 3 Review Exercises

3.1 Functions and Function Notation
For the following exercises, determine whether the relation is a function.

1. $\{(a,b),(c,d),(e,d)\}$

   There are no input values with more than one output. This relation is a function.

2. -

3. $y^2 + 4 = x$, for x the independent variable and y the dependent variable.

   Solving for y gives $y = \pm\sqrt{x-4}$; for all values of x in the domain there is more than one value of y, so this is not a function.

4. -

For the following exercises, evaluate the function at the indicated values:

a. $f(-3)$; b. $f(2)$; c. $f(-a)$; d. $-f(a)$; e. $f(a+h)$.

5. $f(x) = -2x^2 + 3x$

   a. $f(-3) = -2(-3)^2 + 3(-3) = -27$ b. $f(2) = -2(2)^2 + 3(2) = -2$

   c. $f(-a) = -2(-a)^2 + 3(-a) = -2a^2 - 3a$ d. $-f(a) = -(-2a^2 + 3a) = 2a^2 - 3a$

   e. $f(a+h) = -2(a+h)^2 + 3(a+h) = -2a^2 - 4ah - 2h^2 + 3a + 3h$

6. -

For the following exercises, determine whether the functions are one-to-one.

7. $f(x) = -3x + 5$

   This is a linear function, so it is one-to-one.

8. -

For the following exercises, use the vertical line test to determine if the relation whose graph is provided is a function.

9.

There are no values of x for which a vertical line would intersect with this graph more than once. This is a function.
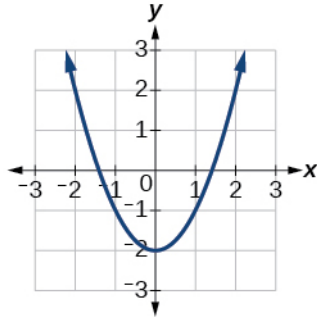
10.   -

11.

There are no values of x for which a vertical line would intersect with this graph more than once. This is a function.

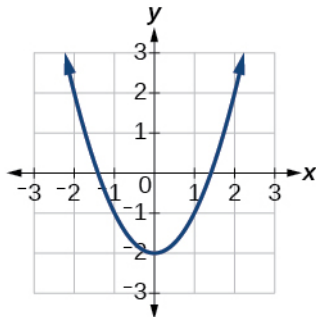For the following exercises, graph the functions.

12.　-

13.　$f(x) = x^2 - 2$

This is a parabola shifted 2 units down.

For the following exercises, use the figure to approximate the values.

14.　-

15.　$f(-2)$

From the graph we can see that the y-coordinate is 2 when the x-coordinate is –2, so $f(-2) = 2$.

16.　-

17.　If $f(x) = 1$, then solve for x.

From the graph we can see that the x-coordinates are –1.8 and 1.8 when the y-coordinate is 1, so the solutions are $x = -1.8$ and $x = 1.8$.

For the following exercises, use the function $h(t) = -16t^2 + 80t$ to find the values.

18. -

19. $\dfrac{h(a) - h(1)}{a - 1}$

$\dfrac{h(a) - h(1)}{a - 1} = \dfrac{-16a^2 + 80a - (-16(1)^2 + 80(1))}{a - 1} = \dfrac{-16a^2 + 80a - 64}{a - 1}$

$= \dfrac{(-16a + 64)(a - 1)}{a - 1} = -16a + 64; a \neq 1$

3.2 Domain and Range

For the following exercises, find the domain of each function, expressing answers using interval notation.
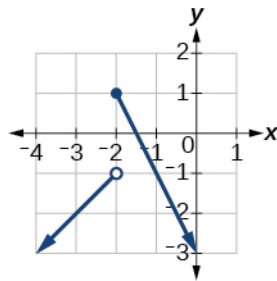
20.   -

21. $f(x) = \dfrac{x - 3}{x^2 - 4x - 12}$

The denominator cannot be equal to zero.

$x^2 - 4x - 12 = 0 \Rightarrow (x - 6)(x + 2) = 0 \Rightarrow x = 6$ or $x = -2$ so $x \neq 6$ and $x \neq -2$, and the domain is $(-\infty, -2) \cup (-2, 6) \cup (6, \infty)$.

22.   -

23.   Graph this piecewise function: $f(x) = \begin{cases} x + 1 & x < -2 \\ -2x - 3 & x \geq -2 \end{cases}$

3.3 Rates of Change and Behavior of Graphs

For the following exercises, find the average rate of change of the functions from $x = 1$ to $x = 2$.
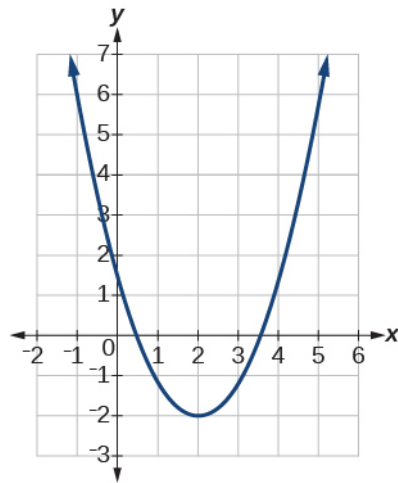
24.   -

25. $f(x) = 10x^2 + x$

$\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{10(2)^2 + 2 - (10(1)^2 + 1)}{1} = 31$

26.   -

For the following exercises, use the graphs to determine the intervals on which the functions are increasing, decreasing, or constant.
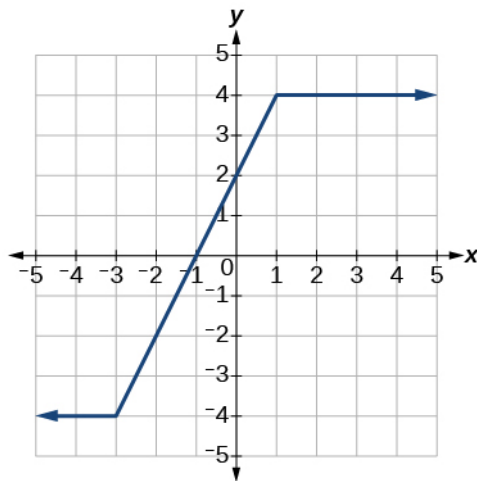
27.

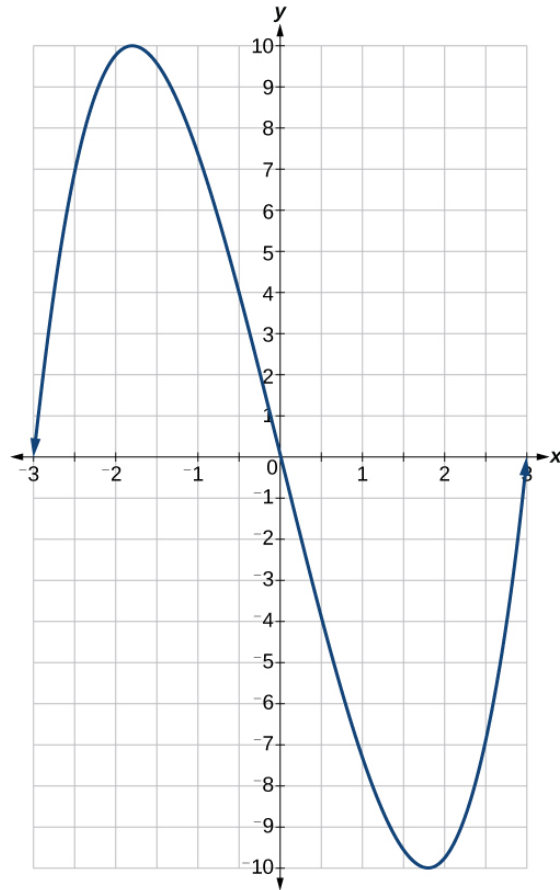The function is increasing on $(2,\infty)$ and decreasing on $(-\infty,2)$.

28. -

29.

The function is increasing on $(-3,1)$ and constant on $(-\infty,-3)$ and $(1, \infty)$.

30. -

31. Find the local extrema for the function graphed in Exercise 495.

The function has a local minimum $(-2,-3)$ and a local maximum at $(1,3)$.

32.   -

33.   Find the absolute maximum of the function graphed in Figure 01_07_219.
       The absolute maximum of the function on this interval is 10.

3.4 Composition of Functions

For the following exercises, find $(f \circ g)(x)$ and $(f \circ g)(x)$ for each pair of functions.

34.   -

35. $f(x) = 3x + 2, \ g(x) = 5 - 6x$

       $f(g(x)) = 3(5 - 6x) + 2 = 17 - 18x$ $g(f(x)) = 5 - 6(3x + 2) = -7 - 18x$

36.   -

37. $f(x) = \sqrt{x + 2}, \ g(x) = \dfrac{1}{x}$

       $f(g(x)) = \sqrt{\dfrac{1}{x} + 2}$ $g(f(x)) = \dfrac{1}{\sqrt{x + 2}}$

38.   -

For the following exercises, find $(f \circ g)$ and the domain for $(f \circ g)(x)$ for each pair of functions.

39. $f(x) = \dfrac{x+1}{x+4}$, $g(x) = \dfrac{1}{x}$

$f(g(x)) = \dfrac{\dfrac{1}{x}+1}{\dfrac{1}{x}+4} = \dfrac{1+x}{1+4x}$. From the un-simplified version we can see that $x \neq 0$ and

from the simplified version we have $x \neq -\dfrac{1}{4}$, so the domain is

$\left(-\infty, -\dfrac{1}{4}\right) \cup \left(-\dfrac{1}{4}, 0\right) \cup (0, \infty)$

40.   -

41. $f(x) = \dfrac{1}{x}$, $g(x) = \sqrt{x}$

$f(g(x)) = \dfrac{1}{\sqrt{x}}$. The radicand must be greater than or equal to zero, so the domain is

$(0, \infty)$

42.   -

For the following exercises, express each function H as a composition of two functions f and g where $H(x) = (f \circ g)(x)$.

43. $H(x) = \sqrt{\dfrac{2x-1}{3x+4}}$

There are many possible solutions. Here is one possible answer:

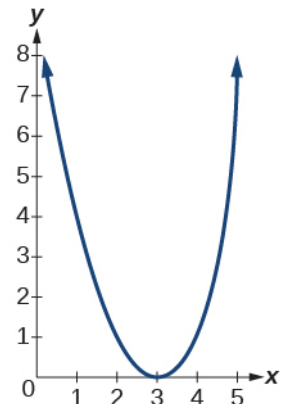$g(x) = \dfrac{2x-1}{3x+4}$ and $f(x) = \sqrt{x}$.

44.   -

3.5 Transformation of Functions
For the following exercises, sketch a graph of the given function.

45. $f(x) = (x-3)^2$
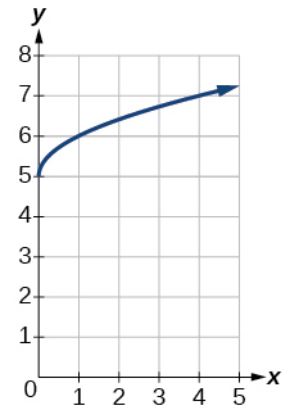   This is a parabola shifted 3 units to the right.
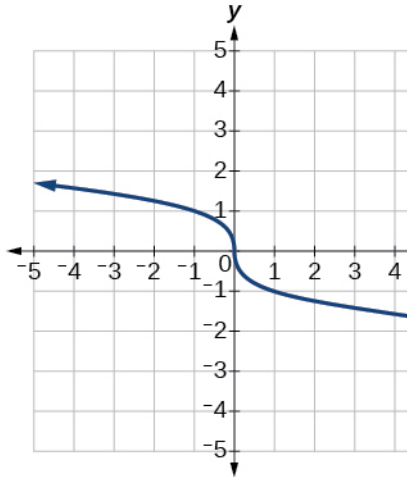
46. -

47. $f(x) = \sqrt{x} + 5$

This is a square root function shifted 5 units up.

48. -

49. $f(x) = \sqrt[3]{-x}$

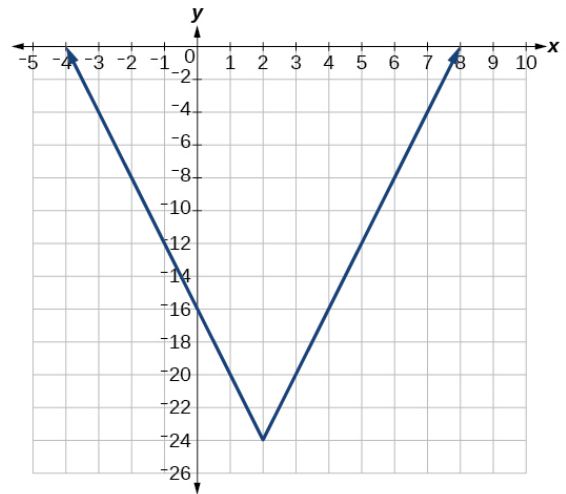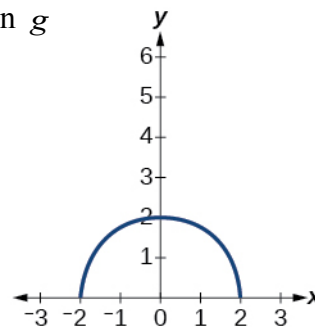This is a cube root function reflected across the y-axis.

50. -

51. $f(x) = 4\left[\left|x-2\right| - 6\right]$

This is an absolute value function shifted to the right 2 units, stretched vertically by a factor of 4 and shifted down 24 units.

52. -

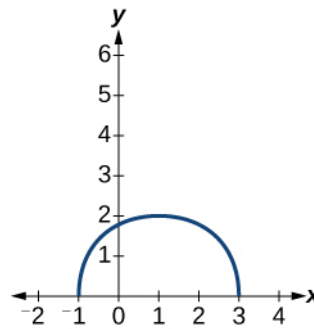For the following exercises, sketch the graph of the function g if the graph of the function f is shown in the graph.

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53. $g(x) = f(x - 1)$

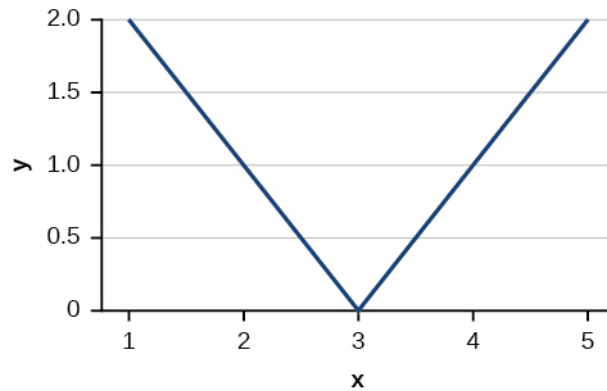This is the graph of $f(x)$ shifted
unit to the right.

1

54. -

For the following exercises, write the equation for the standard function represented by each of the graphs below.

55.

This is an absolute value function
shifted 3 units to the right.
$f(x) = |x - 3|$.

56. -

For the following exercises, determine whether each function below is even, odd, or neither.

57. $f(x) = 3x^4$

Evaluating $f(-x) = 3(-x)^4 = 3x^4 = f(x)$, so this function is even.
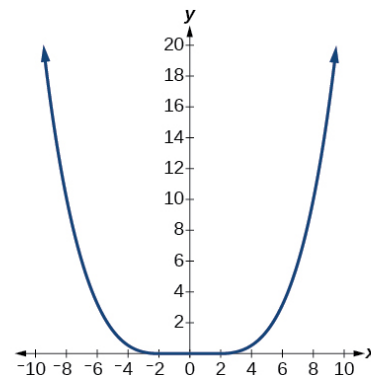
58. -

59. $h(x) = \dfrac{1}{x} + 3x$

Evaluating $g(-x) = \dfrac{1}{-x} + 3(-x) = -\left(\dfrac{1}{x} + 3\right) = -g(x)$, so this function is odd.

For the following exercises, analyze the graph and determine whether the graphed function is even, odd, or neither.

60. -

61.

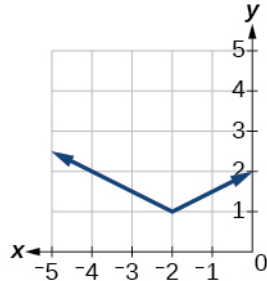This graph is symmetric with respect to the y-axis, so it is even.

62. -

244

3.6 Absolute Value Functions

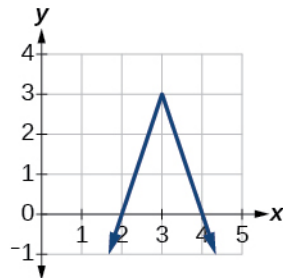For the following exercises, write an equation for the transformation of $f(x) = |x|$.

63.

This is an absolute value function stretched vertically by a factor of $\frac{1}{2}$, shifted to the left 2 units and shifted up 1 unit. $f(x) = \frac{1}{2}|x+2|+1$.
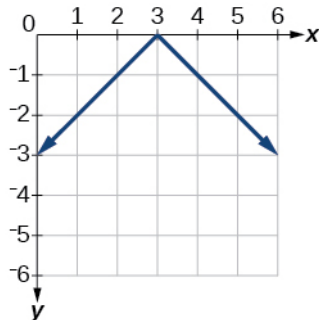
64. -

65.

This is an absolute value function reflected across the x-axis, stretched vertically by a factor of 3, shifted to the right 3 and up 3. $f(x) = -3|x-3|+3$.

For the following exercises, graph the absolute value function.

66. -

67. $f(x) = -|x-3|$

This is an absolute value function reflected across the x-axis and shifted to the right 3 units.

68. -

3.7 Inverse Functions

For the following exercises, find $f^{-1}(x)$ for each function.

    69. $f(x) = 9 + 10x$

        Let $y = 9 + 10x$. Interchange x and y: $x = 9 + 10y$. Solve for y:

$$x = 9 + 10y \Rightarrow 10y = x - 9 \Rightarrow y = \frac{x-9}{10}, \text{ so the inverse is } f^{-1}(x) = \frac{x-9}{10}.$$

    70. -

For the following exercise, find a domain on which the function f is one-to-one and non-decreasing. Write the domain in interval notation. Then find the inverse of f restricted to that domain.

    71. $f(x) = x^2 + 1$

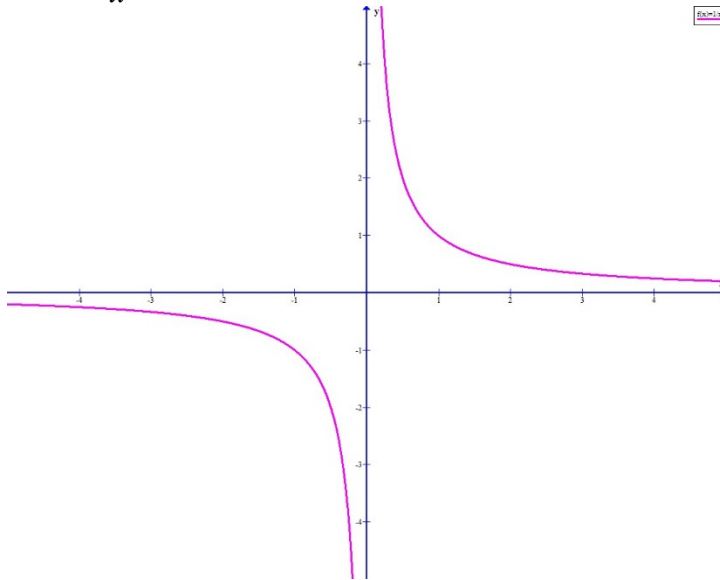   The function is one-to-one and increasing on a domain of $[0, \infty)$.

        Let $y = x^2 + 1$. Interchange x and y: $x = y^2 + 1$. Solve for y:

        $x = y^2 + 1 \Rightarrow y^2 = x - 1 \Rightarrow y = \pm\sqrt{x-1}$, so $f^{-1}(x) = \sqrt{x-1}$.

    72. -

For the following exercises, use a graphing utility to determine whether each function is one-to-one.

    73. $f(x) = \dfrac{1}{x}$

        The function is one-to-one.

    74. -

    75. If $f(5) = 2$, find $f^{-1}(2)$.

        Because the inverse function reverses the input and output values, $f^{-1}(2) = 5$.

    76. -

Chapter 3 Practice Test

For the following exercises, determine whether each of the following relations is a function.

1. $y = 2x + 8$

   This is a linear relation, so it is a function.

2. -

For the following exercises, evaluate the function $f(x) = -3x^2 + 2x$ at the given input.

3. $f(-2)$

   $f(-2) = -3(-2)^2 + 2(-2) = -16$

4. -

5. Show that the function $f(x) = -2(x-1)^2 + 3$ is not one-to-one.

6. -

7. Given $f(x) = 2x^2 - 5x$, find $f(a+1) - f(1)$.

   $f(a+1) - f(1) = 2(a+1)^2 - 5(a+1) - (2(1)^2 - 5(1)) = 2(a^2 + 2a + 1) - 5a - 5 + 3$

   $= 2a^2 + 4a + 2 - 5a - 2 = 2a^2 - a$

8. -

9. Find the average rate of change of the function $f(x) = 3 - 2x^2 + x$ by finding $\dfrac{f(b) - f(a)}{b - a}$.

   $\dfrac{f(b) - f(a)}{b - a} = \dfrac{3 - 2b^2 + b - (3 - 2a^2 + a)}{b - a} = \dfrac{2a^2 - 2b^2 + b - a}{b - a} = \dfrac{2(a+b)(a-b) + b - a}{b - a}$

   $= -2(a + b) + 1; b \neq a$

For the following exercises, use the functions $f(x) = 3 - 2x^2 + x$ and $g(x) = \sqrt{x}$ to find the composite functions.

10. -

11. $(g \circ f)(1)$

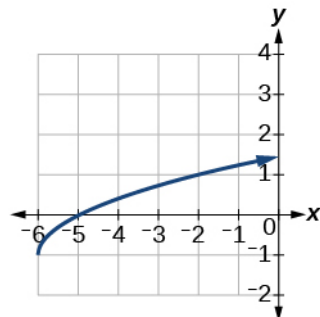   Using $g(f(x)) = \sqrt{3 - 2x^2 + x}$ from the previous problem we get

   $g(f(2)) = \sqrt{3 - 2(1)^2 + 1} = \sqrt{2}$.

12. -

For the following exercises, graph the functions by translating, stretching, and/or compressing a toolkit function.

13. $f(x) = \sqrt{x + 6} - 1$

   This is a square root function shifted 6 units to the left and 1 unit down.

14. -

For the following exercises, determine whether the functions are even, odd, or neither.

15. $f(x) = -\dfrac{5}{x^2} + 9x^6$

$f(-x) = -\dfrac{5}{(-x)^2} + 9(-x)^6 = -\dfrac{5}{x^2} + 9x^6 = f(x),$ so this function is even.

16. -

17. $f(x) = \dfrac{1}{x}$

$f(-x) = \dfrac{1}{-x} = -\dfrac{1}{x} = -f(x),$ so this function is odd.

18. -

For the following exercises, find the inverse of the function.

19. $f(x) = 3x - 5$

Let $y = 3x - 5$. Interchange x and y: $x = 3y - 5$. Solve for y:

$x = 3y - 5 \Rightarrow 3y = x + 5 \Rightarrow y = \dfrac{x+5}{3},$ so the inverse is $f^{-1}(x) = \dfrac{x+5}{3}.$

20. -

For the following exercises, use the graph of g shown.
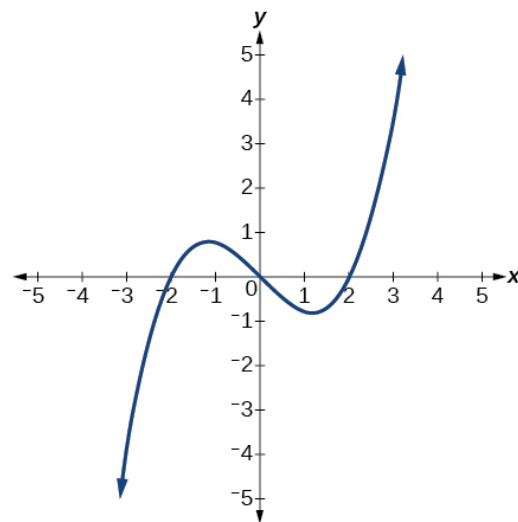
21. On what intervals is the function increasing?
The function is increasing on approximately $(-\infty, -1.1)$ and $(1.1, \infty)$.

22. -

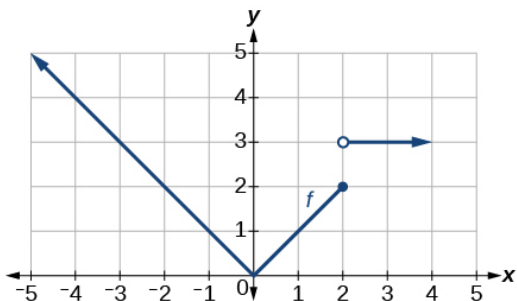23. Approximate the local minimum of the function. Express the answer as an ordered pair.
The local minimum is at $(1.1, -0.9)$.

24. –

For the following exercises, use the graph of the piecewise function shown.

25. Find $f(2)$.

When the x-coordinate is 2, the y-coordinate is 2, so $f(2) = 2$.

26. -

27. Write an equation for the piecewise function.
When $x \le 2$ we have the absolute value function, and when $x > 2$ we have the constant function, so

$f(x) = \begin{cases} |x| & \text{if } x \le 2 \\ 3 & \text{if } x > 2 \end{cases}$

For the following exercises, use the values listed in the table.

x	0	1	2	3	4	5	6	7	8
$F(x)$	1	3	5	7	9	11	13	15	17

28. -

29. Solve the equation $F(x) = 5$.

From the table we can see that the output is 5 when the input is 2, so the solution to $F(x) = 5$ is $x = 2$.

30. -

31. Is the function represented by the graph one-to-one?

Yes – for each input value there is only one output value, so the function is one-to-one.

32. -

33. Given $f(x) = -2x + 11$, find $f^{-1}(x)$.

Let $y = -2x + 11$. Interchange x and y: $x = -2y + 11$. Solve for y:

$$x = -2y + 11 \Rightarrow -2y = x - 11 \Rightarrow y = -\frac{x-11}{2} = \frac{11-x}{2}, \text{ so } f^{-1}(x) = -\frac{x-11}{2} \text{ or } \frac{11-x}{2}.$$