

# Mathematics Problem of the Week

## Problem Number 288 Solution

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### Hypnotherapist

Joseph, a hypnotherapist, goes to Bank of Canada to cash a bank draft. He hypnotizes the bank teller, Ava, so that she confuses the dollars and cents in the case that she gives him dollars instead of cents, and cents instead of dollars. For example, if the bank draft is \$2.10, Ava gives him \$10.02.

After leaving the bank, Joseph went home put the cash on table where there was 30 cents. Then he noticed that the sum of this 30 cents and the money Ava gave him, is exactly 4 times the amount of the original bank draft. What was the amount on the bank draft?

**Solution:** Assuming Joseph has  $x$  dollars and  $y$  cents on the original bank draft. Note that both  $x$  and  $y$  are in the interval  $[0, 99]$ . Otherwise the confusing dollar with cents situation won't really make sense.

With this set up, we know that: The original bank draft has a total of:  $100x + y$  cents. After Joseph went home and put the money Ava gave him together with the 30 cents on the table he has:  $100y + x + 30$  cents in total. Therefore

$$100y + x + 30 = 4(100x + y)$$

$$100y + x + 30 = 400x + 4y$$

$$399x - 96y = 30$$

$$133x - 32y = 10$$

Note that the greatest common divisor of 133 and 32 is 1. Therefore use Euclidean Algorithm, we can get:

$$133 = 32 * 4 + 5 \tag{1}$$

$$32 = 5 * 6 + 2 \tag{2}$$

$$5 = 2 * 2 + 1 \tag{3}$$

$$(3) \Rightarrow 1 = 5 - 2 * 2$$

$$(2) \Rightarrow 2 = 32 - 5 * 6$$

$$(1) \Rightarrow 5 = 133 - 32 * 4$$

Therefore:

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$$1 = 5 - (32 - 5 * 6) * 2 = 133 - 32 * 4 - (32 - (133 - 32 * 4) * 6) * 2$$

$$1 = 133 * 13 - 32 * 54$$

Therefore

$$133 * 130 - 32 * 540 = 10$$

However,  $x, y \in [0, 99]$ . Note that for any value of  $k \in \mathbb{Z}$ ,

$$133 * (130 - 32k) - 32 * (540 - 133k) = 133 * 130 - 32 * 540 + 32 * 133k - 133 * 32k = 133 * 130 - 32 * 540 = 10$$

To make sure  $x, y \in [0, 99]$ ,  $k = 4$

$$x = 130 - 2 * 4 = 2$$

$$y = 540 - 133 * 4 = 8$$

The original bank draft has \$2.08.

**Correct answers:** Navdeep Singh and Enguang Shen (Kirby) submitted correct answers!

**Winner:** The winner of the week is Navdeep Singh! Congratulations Navdeep!