

Mathematics Problem of the Week

Problem Number: 313

Posted on Monday, October 3, 2022

Deadline: Tuesday, October 11, 2022 at noon

Sequences Wonders

Three numbers are in arithmetic progression, three other numbers in geometric progression. Adding the corresponding terms of these two progressions successively, we obtain

75, 67, and 74,

respectively, and adding all three terms of the arithmetic progression, we obtain 111. Find the terms in both progressions.

What You Need and Some Hints:

Geometric progression: we start with a number, say 2, and to get the following term, we multiply the current term by a constant number, say 3. In this example, we get the following sequence:

2, 6, 18, ...

Note that the product of the first and third terms is 6^2 (middle term)!



Arithmetic progression: we start with a value, say 3, and to get the following term, we add the current term by a constant number, say 5. Thus, we get the following sequence in this example:

3, 8, 13, ...

If we call the middle term d , then the first term is $d - 5$ and the third term is $d + 5$. Also, the average of the three terms is the middle term (here 8)!



Now, according to the problem above, we are supposed to add the two terms of the same colour from the two sequences.