KWANTLEN POLYTECHNIC UNIVERSITY

Mathematics Problem of the Week

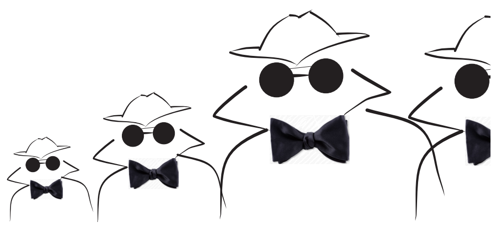
Problem 284 Solution

Four Robbers

Four bank robbers of different ages have a treasure of 200 gold bars. On their get apart vehicle, they decided to split the bars using this scheme:

- The oldest robber proposes how to share the bars, and ALL robbers (including the oldest) vote for or against it.
- If 50% or more of the robbers vote for it, then the bars will be shard that way. Otherwise, the proposing robber will have to leave the group for good, and the process is repeated with the remaining robbers.
- As robbers tend to be ruthless, if a robber would get the same number of bars if he voted for or against a proposal, he will vote against so that the robber who proposed the plan will be removed from the group.

Assuming that all four robbers are intelligent, rational, greedy, and do not wish to leave the group, (and are rather good at math for robbers) what should the oldest robber propose to get as much gold bars as possible without having to leave the group?



Solution: The robbers get 199, 0, 1, 0 bars in order from the oldest to the youngest.

Let A, B, C and D be the robbers from oldest to the youngest. The oldest robber A gets 199 bars and C gets 1 bar and the other two get nothing. The point is that C takes any offers because he does not want the game be reduced to three robbers. Here is why:

- If C and D are the only two left in the game, then C can propose 200 bars for himself and 0 bars for D. Then C can vote for himself get at least 50% vote for the proposal. In this case, D will get nothing. Therefore D will try to avoid this situation as long as he can get at least 1 bar.
- If B, C, and D are left in the game, as long as B gives one bar to D, D will vote for B to avoid being left with C alone as in the previous case. In this case, B's best proposal is to keep 199 bars to himself, giving 0 to C, and 1 to D. In this case C gets nothing, and he will try to avoid being left with B, C, and D as long as C gets at least 1 coin.
- When A propose, he knows that as long as he gives C one coin, C will vote for A to avoid being left with B, C and D. Therefore A can propose: 199 coins for himself, 0 coin for B, 1 coin for C, and 0 coin for D. With this proposal, both A and C will vote for A, and B and D will vote against A. Therefore A got 50% of votes. Hence his proposal will be accepted.

Unfortunately we had no winners for this puzzle.