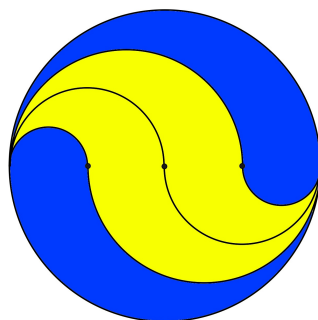# Mathematics Problem of the Week
Problem 286 Solution

Semicircles

Three curves divide a circle into 4 regions. What is the relationship amongst the areas of the four regions? Note that the three dots are equally spaced along a diameter of the circle and all curves are made up of semi-circles.

Solution: We colour the regions yellow and blue as shown in the figure. Note that the areas of the two blue regions are the same, and the areas of the two yellow regions are the same, because of symmetry. We will show that the area of a blue region is the same as the area of a yellow region, hence the area of all four regions are the same.

Let's split the regions into A, B, C, D as the following, and let R be the distance as indicated in the graph:
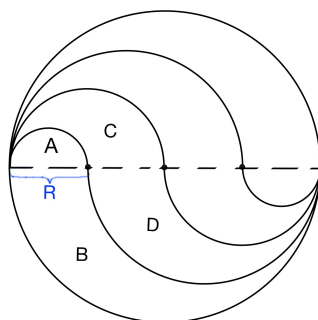
- The area of $A = \dfrac{1}{2}\pi\left(\dfrac{R}{2}\right)^2 = \dfrac{\pi R^2}{8}$

- The area of $B = \dfrac{1}{2}\pi(2R)^2 - \dfrac{1}{2}\pi\left(\dfrac{3R}{2}\right)^2 = 2\pi R^2 - \dfrac{9\pi R^2}{8}$

- The area of $C = \dfrac{1}{2}\pi(R)^2 - \dfrac{1}{2}\pi\left(\dfrac{R}{2}\right)^2 = \dfrac{\pi R^2}{2} - \dfrac{\pi R^2}{8}$

- The area of $D = \dfrac{1}{2}\pi(\dfrac{3R}{2})^2 - \dfrac{1}{2}\pi(R)^2 = \dfrac{9\pi R^2}{8} - \dfrac{\pi R^2}{2}$

Area of $A + B = \dfrac{\pi R^2}{8} + 2\pi R^2 - \dfrac{9\pi R^2}{8} = 2\pi R^2 - \pi R^2 = \pi R^2$

Area of $C + D = \dfrac{\pi R^2}{2} - \dfrac{\pi R^2}{8} + \dfrac{9\pi R^2}{8} - \dfrac{\pi R^2}{2} = \pi R^2 = $ Area of $A + B$

Correct submissions: Vojtech Pithart and Enguang Shen (Kirby) submitted correct answers.

Winner of the week: The winner of this week is Enguang Shen (Kirby)! Congratulations!