1. What is the net force on charge A in each configuration shown below? The distances are $r_1 = 12.0$ cm and $r_2 = 20.0$ cm.

(a) 

Charge A is the target and charges B and C are sources. Charge B and A have the same sign, so they repel. That is, charge A feels a force $F_{BA}$ directed in the $+i$ direction. Charge C and A have the opposite sign, so they attract. That is, charge A feels a force $F_{CA}$ directed in the $-i$ direction. The situation is shown in the diagram below.

Since we know the exact direction of these forces we need only calculate the magnitude of these forces:

$$F_{BA} = k\left| \frac{Q_B Q_A}{r_1^2} \right| = \left(8.99 \times 10^9 \text{ N} \cdot \text{m} / \text{C}^2\right) \left| \frac{(-3 \times 10^{-6} \text{ C})(-5 \times 10^{-6} \text{ C})}{(0.12 \text{ m})^2} \right| = 9.36458 \text{ N}.$$

and

$$F_{CA} = k\left| \frac{Q_C Q_A}{r_2^2} \right| = \left(8.99 \times 10^9 \text{ N} \cdot \text{m} / \text{C}^2\right) \left| \frac{(3 \times 10^{-6} \text{ C})(-5 \times 10^{-6} \text{ C})}{(0.20 \text{ m})^2} \right| = 3.37125 \text{ N}.$$

In component vector form our two forces are $F_{BA} = +i[9.36458 \text{ N}]$ and $F_{CA} = +i[3.37125 \text{ N}]$. Since the forces are along the same axis we find

$$F_{\text{net}} = F_{BA} + F_{CA} = i 5.99 \text{ N}.$$

The net force acting on charge A due to the other forces is 5.99 N along the positive x axis.
The magnitudes of $F_{BA}$ and $F_{CA}$ are the same as in part (a), however, the direction of the force of charge C on charge A is different. That force is now $F_{CA} = +i \ 3.37125 \text{ N}$ directed to the right as shown in the diagram below.

Since the forces are along the same axis we find 

$$F_{\text{net}} = F_{BA} + F_{CA} = i \ 9.36458 \text{ N} + i \ 3.37125 \text{ N} = i \ 12.74 \text{ N}.$$ 

The net force acting on charge A due to the other forces is 12.74 N along the positive x axis.

2. We are asked to calculate the force on an electric charge due to other electric charges. To do this we follow the following steps:

- determine the vector distance to the charge of interest,
- determine the vector force between the two charges using Coulomb’s Law,
- check the direction of the force using the fact that opposite attract and like repel,
- sum all the $i$ and $j$ components separately,
- find the magnitude and direction of the net force.

Since $Q_A$ is the charge of interest, the one we wish to find the force acting on, our vector distances are

<table>
<thead>
<tr>
<th>Vector Distance</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>$3 \mu C$</td>
</tr>
<tr>
<td>$r_2$</td>
<td>$5 \mu C$</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\mathbf{r}_1 &= +i \, 0.12 \quad \mathbf{r}_1 = 0.12 \\
\mathbf{r}_2 &= +j \, 0.20 \quad \mathbf{r}_2 = 0.20 
\end{align*}
\]

Charge A feels a force from charge B whose magnitude is

\[
F_{BA} = k \frac{Q_B Q_A}{r_1^3} \mathbf{r}_1 = (8.99 \times 10^9 \, N \cdot m / C^2) \frac{(-3 \times 10^{-6} \, C)(-5 \times 10^{-6} \, C)}{(0.12 \, m)^3} (i \, 0.12 \, m) = 9.36458 \, N.
\]

The charges have the same sign so they repel. Charge A feels force \( F_{BA} \) directed to the right (+i direction) as we found.

Charge A feels a force from charge C whose magnitude is

\[
F_{CA} = k \frac{Q_C Q_A}{r_2^3} \mathbf{r}_2 = (8.99 \times 10^9 \, N \cdot m / C^2) \frac{(+3 \times 10^{-6} \, C)(-5 \times 10^{-6} \, C)}{(0.20 \, m)^3} (j \, 0.20 \, m) = -j \, 3.37125 \, N.
\]

The charges have the opposite sign so they attract. Charge A feels force \( F_{AC} \) directed straight down (–j direction).

We sketch the forces

\[
F_{\text{net}} = F_{BA} + F_{CA}
\]

The net force is

\[
F_{\text{net}} = 9.34568 \, N - 3.37125 \, N.
\]

We can use the Pythagorean Theorem to find magnitude of the net force,

\[
F_{\text{net}} = \left[ (F_{BA})^2 + (F_{CA})^2 \right]^{1/2} = \left[ (9.36458)^2 + (3.37125)^2 \right]^{1/2} = 9.95 \, N.
\]

We use trigonometry to find the angle \( \theta \),

\[
\theta = \arctan\left( \frac{|F_{CA}|}{|F_{BA}|} \right) = \arctan\left( \frac{3.37125}{9.36458} \right) = 19.8^\circ.
\]
3. Where would you put a positive charge of +1 μC in the diagram below so that the net electrostatic force on it is zero?

We would have to put the 1 μC where the force from the right 2.0 μC charge is cancelled by the force from the left 5.0 μC charge. Since forces are vectors, we would have to put the charge somewhere on the line joining the two charges as shown below. We will assume that the 1 μC charge is some distance x from the 2.0 μC.

We are asked to calculate the force on an electric charge due to other electric charges. To do this we follow the following steps:

- determine the magnitude of the force between the two charges using Coulomb’s Law,
- determine the direction of the force using the fact that opposite attract, like repel,
- recall that forces are vectors, and that a net force is a vector addition.

The 1 μC charge feels a force from the 5.0 μC charge whose magnitude is \( F_{51} = k \frac{Q_5 Q_1}{(3 - x)^2} \). The charges have the same sign so they repel. The 1 μC charge feels force \( F_{51} \) directed to the right. The 1 μC charge feels a force from the 2.0 μC charge whose magnitude is \( F_{21} = k \frac{Q_2 Q_1}{x^2} \). The charges have the same sign so they repel. The 1 μC charge feels force \( F_{21} \) directed to the left.

We sketch the forces as shown in the diagram below.

For the net force to be zero, \( F_{21} \) must have the same magnitude as \( F_{51} \), \( F_{21} = F_{51} \). Thus we have

\[
k \frac{Q_2 Q_1}{x^2} = k \frac{Q_5 Q_1}{(3 - x)^2}.
\]

We eliminate the common factor \( kQ_1 \) and we get

\[
\frac{Q_2}{x^2} = \frac{Q_5}{(3 - x)^2}.
\]

Next we
take the square root of each side, \( \frac{\sqrt{Q_2}}{x} = \frac{\sqrt{Q_5}}{3-x} \). We cross–multiply, collect terms, and find

\[
x = 3 \frac{\sqrt{Q_2}}{\sqrt{Q_2} + \sqrt{Q_5}} = 3 \frac{\sqrt{2}}{\sqrt{2} + \sqrt{3}} = 1.162 \text{ m}.
\]

We would have to place the 1 μC charge 1.16 m from the 2 μC charge for it to feel no net force.

4. The distances on the graph below are in metres. The charges are \( Q_1 = -3 \mu \text{C} \), \( Q_2 = 2 \mu \text{C} \), and \( Q_3 = 5 \mu \text{C} \).

(a) Find the net force on charge \( Q_1 \) due to charges \( Q_2 \) and \( Q_3 \)
(b) Find the net force on charge \( Q_2 \) due to charges \( Q_1 \) and \( Q_3 \).
(c) Find the net force on charge \( Q_3 \) due to charges \( Q_1 \) and \( Q_2 \).

(a) To find the net force (magnitude and direction) on charge \( Q_3 \) due to charges \( Q_1 \), \( Q_2 \), \( Q_4 \), and \( Q_5 \), we must first find the net electric field at the current location of \( Q_3 \). That require the vector distance \( \mathbf{r} \) for each case.
Coulomb's Law says \( \mathbf{E} = kQ \frac{\mathbf{r}}{r^3} \), where \( \mathbf{r} \) is the vector distance from one charge to the charge on which you want to know the force. The constant is \( k = 8.99 \times 10^9 \text{ N-m}^2/\text{C}^2 \). The vector distances, which point from the other charges to \( Q_3 \), are read off the graph and give:

<table>
<thead>
<tr>
<th>Vector Distance</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{13} = i 12 - j 1 )</td>
<td>( r_{13} =</td>
</tr>
<tr>
<td>( r_{23} = i 10 + j 8 )</td>
<td>( r_{23} =</td>
</tr>
<tr>
<td>( r_{43} = -i 5 + j 2 )</td>
<td>( r_{43} =</td>
</tr>
<tr>
<td>( r_{53} = -i 2 + j 6 )</td>
<td>( r_{53} =</td>
</tr>
</tbody>
</table>

The electric fields due to each charge, therefore, are:

\[
\mathbf{E}_{13} = (8.99 \times 10^9)(-3 \times 10^{-6})(i\ 12 - j\ 1)/(12.04159)^3 \\
= i\ (-185.358) + j\ (15.446) \text{ N/C}
\]

\[
\mathbf{E}_{23} = (8.99 \times 10^9)(2 \times 10^{-6})(i\ 10 + j\ 8)/(12.80625)^3 \\
= i\ (85.610) + j\ (68.488) \text{ N/C}
\]

\[
\mathbf{E}_{43} = (8.99 \times 10^9)(-1 \times 10^{-6})(-i\ 5 + j\ 2)/(5.38516)^3 \\
= i\ (287.828) + j\ (-115.131) \text{ N/C}
\]

\[
\mathbf{E}_{53} = (8.99 \times 10^9)(4 \times 10^{-6})(-i\ 2 + j\ 6)/(6.32456)^3 \\
= i\ (-284.289) + j\ (852.866) \text{ N/C}
\]

We get the net electric field by adding up the \( i \) and \( j \) terms separately.
\[ \mathbf{E}_{\text{net}} = \mathbf{E}_{13} + \mathbf{E}_{23} + \mathbf{E}_{43} + \mathbf{E}_{53} = i (-96.209) + j (821.670) \text{ N/C} \]

Using the Pythagorean Theorem, \( \mathbf{E}_{\text{net}} = 827.283 \text{ N/C} \) at \( \theta = 96.68^\circ \) above horizontal.

In component form, the force on \( Q_3 \) is then given by

\[ \mathbf{F}_{\text{net}} = Q_3 \mathbf{E}_{\text{net}} = (5 \times 10^{-6})(-i 96.209 + j 821.670) = -i (4.8104 \times 10^{-4}) + j (4.1083 \times 10^{-3}) \text{ N}. \]

The magnitude and direction can also be found

\[ \mathbf{F}_{\text{net}} = Q_3 \mathbf{E}_{\text{net}} = (5 \times 10^{-6})(827.283) = 4.1364 \times 10^{-3} \text{ N}. \]

Since \( Q_3 \) is positive, \( \mathbf{F}_{\text{net}} \) and \( \mathbf{E}_{\text{net}} \) are parallel, so \( \mathbf{F}_{\text{net}} \) also points along \( \theta = 96.68^\circ \) above horizontal.

(b) To find the net force (magnitude and direction) on charge \( Q_2 \) due to charges \( Q_3 \) and \( Q_5 \), we must first find the vector distance \( \mathbf{r} \) for each case.

Coulomb's Law says \( \mathbf{E} = kQ \mathbf{r}/r^3 \), where \( \mathbf{r} \) is the vector distance from one charge to the charge on which you want to know the force. The constant is \( k = 8.99 \times 10^9 \text{ N-m}^2/\text{C}^2 \). The vector distances, which point from the other charges to \( Q_2 \), are read off the graph and give

**Vector Distance**  
\[ r_{32} = -i 10 - j 8 \quad r_{32} = \mid r_{32} \mid = \sqrt{(-10)^2 + (-8)^2} = 12.80625 \]
\[ r_{52} = -i 12 - j 2 \quad r_{52} = \mid r_{52} \mid = \sqrt{(-12)^2 + (-2)^2} = 12.16552 \]

The electric fields, therefore, are
\[ \mathbf{E}_{32} = (8.99 \times 10^9)(5 \times 10^{-6})(-i 10 - j 8)/(12.80625)^3 \\
= i (-214.025) + j (-171.220) \text{ N/C} \]
\[ \mathbf{E}_{52} = (8.99 \times 10^9)(4 \times 10^{-6})(-i 12 - j 2)/(12.16552)^3 \\
= i (-239.667) + j (-39.945) \text{ N/C} \]

We get the net force by adding up the \(i\) and \(j\) terms separately.

\[ \mathbf{E}_{\text{net}} = \mathbf{E}_{32} + \mathbf{E}_{52} \\
= i (-453.692) + j (-211.164) \]

Using the Pythagorean Theorem, \(\mathbf{E}_{\text{net}} = 500.426 \text{ N/C} \) at \(\theta = 204.96^\circ \) above horizontal.

In component form, the force on \(Q_3\) is then given by

\[ \mathbf{F}_{\text{net}} = Q_2 \mathbf{E}_{\text{net}} = (2 \times 10^{-6})(-i 453.692 - j 211.164) = -i (9.0738 \times 10^{-4}) - j (4.2233 \times 10^{-3}) \text{ N}. \]

The magnitude and direction can also be found

\[ \mathbf{F}_{\text{net}} = Q_2 \mathbf{E}_{\text{net}} = (2 \times 10^{-6})(500.426) = 1.0009 \times 10^{-3} \text{ N}. \]

Since \(Q_2\) is positive, \(\mathbf{F}_{\text{net}}\) and \(\mathbf{E}_{\text{net}}\) are parallel, so \(\mathbf{F}_{\text{net}}\) also points along \(\theta = 204.96^\circ \) above horizontal.

(c) To find the net force (magnitude and direction) charge \(Q_5\) due to charges \(Q_1, Q_2, \) and \(Q_4\), we must first find the vector distance \(\mathbf{r}\) for each case.

Coulomb's Law says \(\mathbf{F} = k q_1 q_2 \mathbf{r}/r^3\), where \(\mathbf{r}\) is the vector distance from one charge to the charge on which you want to know the force. The constant is \(k = 8.99 \times 10^9 \text{ N-m}^2/\text{C}^2\). The vector distances, which point from the
other charges to $Q_5$, are read off the graph and give

$$
\begin{align*}
\textbf{Vector Distance} & \quad \textbf{Magnitude} \\
r_{15} = +i 14 - j 7 & \quad r_{15} = |r_{15}| = [(14)^2 + (-7)^2]^{\frac{1}{2}} = 15.65248 \\
r_{25} = +i 12 + j 2 & \quad r_{25} = |r_{25}| = [(12)^2 + (2)^2]^{\frac{1}{2}} = 12.16552 \\
r_{45} = -i 3 - j 4 & \quad r_{45} = |r_{45}| = [(-3)^2 + (-4)^2]^{\frac{1}{2}} = 5
\end{align*}
$$

The electric fields, therefore, are

$$
\begin{align*}
E_{15} &= (8.99 \times 10^9)(-3 \times 10^{-6})(i 14 - j 7)/(15.65248)^3 \\
&= i (-98.460) + j (49.230) \text{ N/C} \\
E_{25} &= (8.99 \times 10^9)(2 \times 10^{-6})(i 12 + j 2)/(12.16552)^3 \\
&= i (119.834) + j (19.972) \text{ N/C} \\
E_{52} &= (8.99 \times 10^9)(-1 \times 10^{-6})(-i 3 - j 4)/(5)^3 \\
&= i (215.760) + j (287.680) \text{ N/C}
\end{align*}
$$

We get the net electric field by adding up the $i$ and $j$ terms separately.

$$
E_{\text{net}} = E_{15} + E_{25} + E_{45} = i (237.134) + j (356.882) \text{ N/C}
$$

Using the Pythagorean Theorem, $E_{\text{net}} = 237.134 \text{ N/C}$ at $\theta = 56.40^\circ$ above horizontal.

In component form, the force on $Q_5$ is then given by

$$
F_{\text{net}} = Q_5 E_{\text{net}} = (4 \times 10^{-6})(i 237.134 + j 356.882) = i (9.48534 \times 10^{-4}) + j (1.42753 \times 10^{-3}) \text{ N}.
$$

The magnitude and direction can also be found

$$
F_{\text{net}} = Q_5 E_{\text{net}} = (4 \times 10^{-6})(237.134) = 1.7139 \times 10^{-3} \text{ N}.
$$

Since $Q_5$ is positive, $F_{\text{net}}$ and $E_{\text{net}}$ are parallel, so $F_{\text{net}}$ also points along $\theta = 56.40^\circ$ above horizontal.

---

5. A charged ball of mass $m = 0.265 \text{ kg}$ and unknown charge $q$ is hanging by a light thread from a ceiling. A fixed charge $Q = +5.00 \mu\text{C}$ on an insulated stand is brought close to the unknown charge. As a result, the unknown charge hangs at an angle $\theta = 38.0^\circ$ to the vertical as shown in the diagram below. The distance between the two charges is $r = 22.0 \text{ cm}$. 

---

[Top]
(a) What is the sign of the unknown charge? Explain how you know this.

(b) What is the magnitude of the unknown charge?

(a) The unknown charge \( q \) must be negative since it is attracted to the positive charge \( Q \).

(b) Notice that the unknown charge is not moving, so all the forces acting on it must balance. In dealing with forces, we draw a free body diagram and apply Newton’s Second Law. Since it is not moving, the acceleration is zero. The forces acting on the unknown charge are its weight, \( mg \), the tension in the string \( T \), and the Coulomb force \( F \). The magnitude of \( F \) is given by \( F = \frac{|kqQ|}{r^2} \).

\[
F_x = ma_x \\
F_y = ma_y \\
k|qQ|/r^2 - T\sin(\theta) = 0 \\
T\cos(\theta) - mg = 0
\]

The second column gives us an expression for \( T \), \( T = \frac{mg}{\cos(\theta)} \). If we substitute this into the first equation, we find an equation, \( k|qQ|/r^2 = mg\sin(\theta)/\cos(\theta) \), or

\[
k|qQ|/r^2 = mg\tan(\theta) \]

Solving for \( q \), we get \( |q| = \frac{r^2mg\tan(\theta)}{k|Q|} \). Using the given values we find

\[
|q| = \frac{(0.22)^2(0.265)(9.81)\tan(38^\circ)}{(8.99\times 10^{-9})(5\times 10^{-6})} = 2.19 \times 10^{-6} \text{ C}.
\]

So the unknown charge is \( q = -2.19 \mu\text{C} \).
6. What is the net electric field at point A in each configuration shown below? The distances are \(r_1 = 12.0\, \text{cm}\) and \(r_2 = 20.0\, \text{cm}\). In each case, determine the magnitude and direction of the electrostatic force on a charge put at point A if the charge is (i) \(-5\, \mu\text{C}\), (ii) \(2\, \mu\text{C}\), or (iii) \(-7\, \mu\text{C}\).

We are asked to calculate the net electric field at a point due to electric charges. To do this we follow the following steps:

- determine the vector distance to the point of interest,
- determine the vector electric field due to a charge using \(E = kQ / r^3\),
- check the direction of the field using the fact that field of a positive charge points outwards while the field due to a negative charge points in,
- sum all the \(i\) and \(j\) components separately,
- find the magnitude and direction of the net electric field.

Finally, the find the force \(F\) on a charge \(q\) in an electric field \(E\) we use the vector equation \(F = qE\).

(a)

Since A is the point of interest, the point where we wish to find the field, our vector distances are

<table>
<thead>
<tr>
<th>Vector Distance</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_1 = +i 0.12)</td>
<td>(r_1 = 0.12)</td>
</tr>
<tr>
<td>(r_2 = +j 0.20)</td>
<td>(r_2 = 0.20)</td>
</tr>
</tbody>
</table>

At point A there is an electric field from charge B whose magnitude is

\[
E_B = k \frac{Q_B}{r_1^3} \cdot r_1 = (8.99 \times 10^9 \, \text{N} \cdot \text{m}/\text{C}^2) \frac{-3 \times 10^{-6} \, \text{C}}{(0.12 \, \text{m})^3} (i \, 0.12 \, \text{m}) = -i \, 1.8729 \times 10^6 \, \text{N}/\text{C}.
\]

The charge B is negative so \(E_B\) is directed to the left \((-i\) direction) into charge B.

At point A there is an electric field from charge C whose magnitude is

\[
E_C = k \frac{Q_C}{r_2^3} \cdot r_2 = (8.99 \times 10^9 \, \text{N} \cdot \text{m}/\text{C}^2) \frac{3 \times 10^{-6} \, \text{C}}{(0.20 \, \text{m})^3} (j \, 0.20 \, \text{m}) = j \, 6.7425 \times 10^5 \, \text{N}/\text{C}.
\]

The charge C is positive so \(E_C\) is directed upwards \((+j\) direction) away from charge C.

The net field is
\[ \mathbf{E}_{\text{net}} = \mathbf{E}_B + \mathbf{E}_C = -i \times 1.87291 \times 10^6 \text{ N/C} + j0.67425 \times 10^6 \text{ N/C}. \]

We sketch the electric field

We use the Pythagorean Theorem to find \( E_{\text{net}} \),

\[ E_{\text{net}} = \sqrt{(E_B^2 + E_C^2)} = \sqrt{(-1.87291 \times 10^6)^2 + (0.67425 \times 10^6)^2} = 1.99058 \times 10^6 \text{ N/C}. \]

We use trigonometry to find the angle \( \theta \),

\[ \theta = \arctan\left(\frac{|E_C|}{E_B}\right) = \arctan\left(\frac{0.67425}{-1.87291}\right) = 19.8^\circ. \]

The net electric field acting at point A due to the charges is \( 1.99 \times 10^6 \text{ N/C} \) at \( 19.8^\circ \) above the negative x axis or \( 160.2^\circ \) a.h.

If we place a positive charge at point A, it will feel a force in the same direction as the net electric field. A negative charge would experience a force in the opposite direction, \( 19.8^\circ \) below the positive x axis.

Using \( F = qE \) to find the magnitude of the force, we get:

<table>
<thead>
<tr>
<th>( q )</th>
<th>( F ) (N)</th>
<th>( \text{direction} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) (-5 \mu\text{C})</td>
<td>9.95</td>
<td>19.8^\circ \text{ b.h.}</td>
</tr>
<tr>
<td>(ii) (+2 \mu\text{C})</td>
<td>3.98</td>
<td>160.2^\circ \text{ a.h.}</td>
</tr>
<tr>
<td>(iii) (-7 \mu\text{C})</td>
<td>13.93</td>
<td>19.8^\circ \text{ b.h.}</td>
</tr>
</tbody>
</table>

(b)

The magnitudes of \( E_B \) and \( E_C \) are the same as in part (a), however, the direction of the electric field of charge C is now directed away from charge C to the right at point A, \( E_C = +i \times 6.7425 \times 10^5 \text{ N/C} \).
Since the forces are along the same axis we find

\[ \mathbf{E}_{\text{net}} = \mathbf{E}_B + \mathbf{E}_C = -i \ 1.87291 \times 10^6 \text{ N/C} + i \ 0.67425 \times 10^6 \text{ N/C} = -i \ 1.19866 \times 10^6 \text{ N/C} . \]

The net electric field acting at point A due to the charge is \( 1.199 \times 10^6 \text{ N/C} \) to the left.

If we place a positive charge at point A, it will feel a force in the same direction as the net electric field. A negative charge would experience a force in the opposite direction, to the right.

Using \( F = qE \) to find the magnitude of the force, we get:

<table>
<thead>
<tr>
<th>( q )</th>
<th>( F ) (N)</th>
<th>direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) (-5 \mu \text{C})</td>
<td>5.99</td>
<td>right</td>
</tr>
<tr>
<td>(ii) (+2 \mu \text{C})</td>
<td>2.40</td>
<td>left</td>
</tr>
<tr>
<td>(iii) (-7 \mu \text{C})</td>
<td>8.39</td>
<td>right</td>
</tr>
</tbody>
</table>

(c)

The magnitudes of \( \mathbf{E}_B \) and \( \mathbf{E}_C \) are the same as in part (a), however, the direction of the electric field of charge C is now directed away from charge C to the left at point A, \( \mathbf{E}_C = -i \ 6.7425 \times 10^5 \text{ N/C} \).

Since the forces are along the same axis we find

\[ \mathbf{E}_{\text{net}} = \mathbf{E}_B + \mathbf{E}_C = -i \ 1.87291 \times 10^6 \text{ N/C} - i \ 0.67425 \times 10^6 \text{ N/C} = -i \ 2.54726 \times 10^6 \text{ N/C} . \]

The net electric field acting at point A due to the charge is \( 2.55 \times 10^6 \text{ N/C} \) to the left.

If we place a positive charge at point A, it will feel a force in the same direction as the net electric field. A negative charge would experience a force in the opposite direction, to the right.

Using \( F = qE \) to find the magnitude of the force, we get:
7. Find the net electric field at point A in the diagram below.

![Diagram showing electric field components](image)

We are asked to calculate the net electric field at a point due to electric charges. To do this we follow the following steps:

- determine the vector distance to the point of interest,
- determine the vector electric field due to a charge using \( E = kQ \frac{r}{r^3} \),
- check the direction of the field using the fact that field of a positive charge points outwards while the field due to a negative charge points in,
- sum all the \( i \) and \( j \) components separately,
- find the magnitude and direction of the net electric field.

Since A is the point of interest, the point where we wish to find the field, our vector distances are

<table>
<thead>
<tr>
<th>Vector Distance</th>
<th>Magnitude</th>
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</thead>
<tbody>
<tr>
<td>( r_1 = +i \ 3.0 + j \ 1.6 )</td>
<td>( r_1 = 3.4 )</td>
</tr>
<tr>
<td>( r_2 = +i \ 3.0 + j \ 2.8 )</td>
<td>( r_2 = 4.10366 )</td>
</tr>
<tr>
<td>( r_3 = +j \ 2.8 )</td>
<td>( r_3 = 2.8 )</td>
</tr>
</tbody>
</table>

The electric field due to charge 1 is
\[ \mathbf{E}_1 = k \frac{Q_1}{r_1^3} \mathbf{r}_1 = (8.99 \times 10^9 \text{N} \cdot \text{m/C}^2) \frac{5 \times 10^{-6} \text{C}}{(3.4 \text{m})^3} (i 3.0 \text{m} + j 1.6 \text{m}) = (i 3.4309 \text{m} + j 1.8298) \times 10^3 \text{N/C}. \]

Charge 1 is positive, so the field \( \mathbf{E}_1 \) is directed away from charge 1 along the line \( \mathbf{r}_1 \).

The electric field due to charge 2 is

\[ \mathbf{E}_2 = k \frac{Q_2}{r_2^3} \mathbf{r}_2 = (8.99 \times 10^9 \text{N} \cdot \text{m/C}^2) \frac{5 \times 10^{-6} \text{C}}{(4.1037 \text{m})^3} (i 3.0 \text{m} + j 2.8 \text{m}) = (i 1.9513 \text{m} + j 1.8212) \times 10^3 \text{N/C}. \]

Charge 2 is positive, so the field \( \mathbf{E}_2 \) is directed away from charge 2 along the line \( \mathbf{r}_2 \).

The electric field due to charge 3 is

\[ \mathbf{E}_3 = k \frac{Q_3}{r_3^3} \mathbf{r}_3 = (8.99 \times 10^9 \text{N} \cdot \text{m/C}^2) \frac{2 \times 10^{-6} \text{C}}{(2.8 \text{m})^3} (j 2.8 \text{m}) = j 2.2934 \times 10^3 \text{N/C}. \]

Charge 3 is positive, so the field \( \mathbf{E}_3 \) is directed away from charge 3 upwards along \( \mathbf{r}_3 \).

We sketch the electric fields

\[ \mathbf{E}_{\text{net}} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 \]

\[ = i 5.3822 \times 10^3 \text{N/C} + j 5.9444 \times 10^3 \text{N/C} \]

We use the Pythagorean Theorem to find magnitude of the net electric field,

\[ E_{\text{net}} = \sqrt{(E_x^2 + E_y^2)} = \sqrt{(5.3822 \times 10^3 \text{N/C})^2 + (5.9444 \times 10^3 \text{N/C})^2} = 8.0190 \times 10^3 \text{N/C} . \]
We use trigonometry to find the angle $\alpha$,

$$\alpha = \arctan\left(\frac{|E_y|}{E_x}\right) = \arctan\left(\frac{5.9444}{5.3822}\right) = 47.84^\circ .$$

The net electric field acting at point A due to the charges is $8.019 \times 10^3$ N/C at $47.84^\circ$ above horizontal.

8. A small ball of mass $m = 0.015$ kg is suspended floating in an electric field of magnitude $E = 5000$ N/C. (a) If the electric field is pointing straight up into the air, what is the charge on the ball? (b) If $E$ points straight down?

(a) A charge in an electric field experiences a Coulomb force $F = |q|E$. The ball has mass, so weight acts on it. The ball is not moving so it has no acceleration. This means the weight must be balanced by the Coulomb force. Since the Coulomb force and the electric field are in the same direction, this indicates that the charge is positive.

We apply Newton’s Second Law and find $|q|E - mg = 0$. Upon rearranging we find

$$|q| = \frac{mg}{E} = \frac{(0.015)(9.81)}{5000} = 2.94 \times 10^{-5} \text{ C}.$$

The unknown charge is $+29.4 \mu\text{C}$.

(b) Here the electric field is straight down. However the Coulomb force must still point up to balance the weight. Since the Coulomb force and the electric field are in opposite directions, this indicates that the charge is negative.

We apply Newton’s Second Law and find $|q|E - mg = 0$. Upon rearranging we find
\[ |q| = \frac{mg}{E} = (0.015)(9.81)/(5000) = 2.94 \times 10^{-5} \text{ C}. \]

The unknown charge is \(-29.4 \mu\text{C}\).

9. A ball of mass \(m = 0.010 \text{ kg}\) and charge \(q\) is tied by a very light string to the ceiling. The effects of a uniform electric field \(E = 5000 \text{ N/C}\) has caused the charged ball to move to one side so that the string makes an angle of \(\theta = 37^\circ\) with the vertical. Draw the free body diagram of the floating ball. Determine the magnitude of the charge \(q\). How can you tell the sign (positive or negative) of the charge?

The charge is hanging at an angle because it feels a Coulomb force from being in an electric field, \(F = |q|E\). The Coulomb force is clearly to the right because that is the way the charge moved. Since the Coulomb force and \(E\) are in opposite directions, the charge must be negative.

Since we have the Coulomb force, the weight, the tension, and know that the acceleration is zero since the body is not moving, we apply Newton’s Second Law.

\[
\begin{align*}
\Sigma F_x &= m a_x \\
|q|E - T \sin(\theta) &= 0 \\
\Sigma F_y &= m a_y \\
T \cos(\theta) - mg &= 0
\end{align*}
\]

From the second column, we have \(T = mg / \cos(\theta)\). We can substitute this in to the equation in the first column to get, \(|q|E - (mg / \cos(\theta))\sin(\theta) = 0\). Solving for \(|q|\) yields

\[
|q| = \frac{mg \tan(\theta)}{E} = (0.010 \text{ kg})(9.81 \text{ m/s}^2) \tan(37^\circ)/(5000 \text{ N/C}) = 1.4785 \times 10^{-5} \text{ C}.
\]

The unknown charge is thus, \(q = -14.8 \mu\text{C}\).