Physics 1120: Newton's Laws Solutions

1. In the diagrams below, a ball is on a flat horizontal surface. The velocity and external forces acting on the ball are indicated. Describe qualitatively how motion the motion of the ball will change.

First determine the direction of the net force on the ball. From Newton's Second Law, \( \mathbf{F} = m\mathbf{a} \), so the direction of the net force and resulting acceleration are in the same direction. As time passes, the direction of the ball will tend to point in the direction of the acceleration.

2. Three forces \( \mathbf{F}_1, \mathbf{F}_2, \) and \( \mathbf{F}_3 \) act on a particle with mass 6.0 kg. The forces are:

   \[ \mathbf{F}_1 = 2i - 5j + 2k, \]
   \[ \mathbf{F}_2 = -4i + 8j + 1k, \]
   \[ \mathbf{F}_3 = 5i + 2j - 5k, \]

   where the forces are measured in Newtons. (a) What is the net force vector? (b) What is the magnitude of the net force? (c) What is the acceleration vector? (d) What is the magnitude of the acceleration vector?
(a) The \( i \), \( j \), and \( k \) components are independent and are added separately, so

\[
F_{\text{net}} = F_1 + F_2 + F_3 = 3\ i + 5\ j - 2\ k.
\]

(b) The magnitude of the net force is given by the 3D version of Pythagoras' Theorem,

\[
F_{\text{net}} = \left[ (F_x)^2 + (F_y)^2 + (F_z)^2 \right]^{1/2} = \left[ 3^2 + 5^2 + (-2)^2 \right]^{1/2} = 6.2 \text{ N}.
\]

(c) Since \( a = F_{\text{net}}/m \), \( a = 0.5000\ i + 0.8333\ j - 0.3333\ k \).

(d) Using the 3D version of Pythagoras' Theorem,

\[
a_{\text{net}} = \left[ (a_x)^2 + (a_y)^2 + (a_z)^2 \right]^{1/2} = \left[ 0.5^2 + 0.8333^2 + (-0.3333)^2 \right]^{1/2} = 1.03 \text{ m/s}^2.
\]

3. Three forces \( F_1 = (25.0\text{N}, 42.5^\circ) \), \( F_2 = (15.5\text{N}, 215^\circ) \), and \( F_3 = (20.5\text{N}, 155^\circ) \) accelerate an 8.75 kg mass. What is the net force acting on the mass? What is the magnitude and direction of the mass's acceleration? What would have to be the magnitude and direction of a fourth force \( F_4 \) so that the acceleration of the mass would be zero?

This involves a 2D vector addition, so it is appropriate to sketch the addition of the forces and then add the components:

\[
\begin{align*}
F_{1x} &= 25 \cos(42.5) = 18.432 \\
F_{2x} &= -15.5 \cos(35.0) = -12.697 \\
F_{3x} &= -20.5 \sin(65.0) = -18.579 \\
F_{\text{net}x} &= -12.844 \\
F_{1y} &= 25 \sin(42.5) = 16.890 \\
F_{2y} &= -15.5 \sin(35.0) = -8.8904 \\
F_{3y} &= -20.5 \cos(65.0) = 8.664 \\
F_{\text{net}y} &= 16.663
\end{align*}
\]
The magnitude of the net force is given by 
\[ F_{net} = \left[ (F_x)^2 + (F_y)^2 \right]^{1/2} = 21.039 \, \text{N} \]. The angle is determined by trigonometry to be 
\[ \theta = \tan^{-1}\left( \frac{|F_y/F_x|}{} \right) = 52.4^\circ \]. So putting the answer in the same form as that we were given, the net force is 
\[ F_{net} = (21.0 \, \text{N}, 127.6^\circ) \].

Since \( a = F_{net}/m \), 
\[ a = \left[ \frac{21.0 \, \text{N}}{8.75 \, \text{kg}} \right], 127.6^\circ = (2.40 \, \text{m/s}^2, 127.6^\circ) \].

We are asked to find \( F_4 \) such that \( F_1 + F_2 + F_3 + F_4 = F_{net} + F_4 = 0 \). In other words, we find to find \( F_4 = -F_{net} \). Equal and opposite vectors have the same magnitude but are \( 180^\circ \) apart, so 
\[ F_4 = (21.0 \, \text{N}, 127.6^\circ + 180^\circ) = (21.0 \, \text{N}, 307.6^\circ) \].

4. You are thrown from a bicycle and skid in a straight line to a complete stop on a rough gravel path. A measurement of the bloody skidmark reveals that it is \( 3.50 \, \text{m} \) long. What average force did the gravel exert on your anatomy? Assume that your mass is \( 70.0 \, \text{kg} \) and that your initial speed was \( 30 \, \text{km/h} \).

We are given enough kinematic data to determine the acceleration. Since \( F = ma \), we can determine the force on your anatomy. First we convert the initial velocity to proper units, \( v_0 = +30 \, \text{km/h} = +8.333 \, \text{m/s} \), where the direction of motion has been chosen to be positive as indicated by the plus sign. Next we calculate the magnitude of the acceleration,

\[ a = \left[ \frac{(v_f)^2 - (v_0)^2}{2\Delta x} \right] = -9.92 \, \text{m/s}^2 \],

where the minus sign indicates that the acceleration is in the direction opposite to the motion, in other words, slowing you down. This immediately tells us the direction of the force acting on you since the acceleration and the force causing the acceleration must point in the same direction according to Newton's Second Law.

The force is \( F = ma = (70 \, \text{kg})(-9.92 \, \text{m/s}^2) = -694 \, \text{N} \). Again the minus sign means the force is directed opposite to the direction of your initial motion.

5. The \( 80.0 \, \text{kg} \) male partner of a figure skating duo pushes his \( 60.0 \, \text{kg} \) female partner with a force of \( 70.0 \, \text{N} \). Find the acceleration of both partners.
According to Newton's Third Law, the woman must exert 70.0 N on the male skater but in the opposite direction, in other words, -70.0 N, where the minus sign indicates the direction opposite to the direction that the man pushes the woman.

To find the acceleration of the male skater we use Newton's Second Law,

\[ a_{\text{man}} = \frac{F_{\text{woman on man}}}{m_{\text{man}}} = \frac{-70 \text{ N}}{80.0 \text{ kg}} = -0.875 \text{ m/s}^2. \]

The minus sign indicates that the man moves backwards which is what one would expect.

Similarly the acceleration of the female skater is

\[ a_{\text{woman}} = \frac{F_{\text{man on woman}}}{m_{\text{woman}}} = \frac{+70 \text{ N}}{60.0 \text{ kg}} = +1.17 \text{ m/s}^2. \]

The plus sign indicates that the woman moves backwards (forward from the man's perspective) which is what one would expect.

In the diagram below, an object travels over a hill, down a valley, and around the inside of a loop-the-loop. At each of the specified points draw a free body diagram indicating the directions of the normal force, the weight, and the centripetal acceleration if it exists.

Recall that weight always acts down which, by convention, is taken to be the bottom of the page. Normal forces act normal to the surface, from the surface through the object. To have a centripetal acceleration, an object must be traveling partially or wholly in a circle. Since points A, D, and L are not curved there is no centripetal acceleration at these points. For the rest of the points, the direction of the centripetal acceleration is by definition towards the centre of the curve. There may also be a component of acceleration with is tangential to the curves but we are not asked about that.

In the diagram, normal forces are represented by black arrow, weight by blue arrows, and centripetal acceleration by green arrows.
7. What is the normal force on the object of mass $m_1$ shown in the diagram? What is the acceleration of the object? In the diagram $F_1 = 25 \text{ N}$, $F_2 = 15 \text{ N}$, and $m_1 = 20 \text{ kg}$.

This problem deals with forces and acceleration, which suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD). In the diagram we show the given forces $F_1$ and $F_2$. As well, since the object has mass, it has weight. Since the object touches the table top, there is a normal force from the table top through the object. We assume that the object will accelerate to the right.

Next we apply Newton's Second Law separately to the $i$ and $j$ components:

$i$

$F_x = m_1 a_x$

$F_1 \cos(35) + F_2 \cos(43) = m_1 a$

$j$

$F_y = m_1 a_y$

$N - m_1 g - F_1 \sin(35) + F_2 \sin(43) = 0$

Thus the normal force is:

$$N = m_1 g + F_1 \sin(35) - F_2 \sin(43) = (20)(9.81) + 25 \sin(35) - 15 \sin(43) = 200.3 \text{ N}.$$ 

The acceleration is:
\[ a = \frac{[F_1 \cos(35) + F_2 \cos(43)]}{m_1} = \frac{[25 \cos(35) + 15 \cos(43)]}{20} = 1.572 \text{ m/s}^2. \]

8. The apparent weight of a person in an elevator is \( 7/8 \) of his actual weight. What is the acceleration (including the direction) of the elevator?

This problem deals with a force, weight, and acceleration. That suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD). Since the person has mass, he has weight. Since the person touches the floor of the elevator, there is a normal force from the floor. We will assume that the person will accelerate upwards.

Applying Newton's Second Law yields,

\[ F_y = ma_y \]

\[ N - mg = ma. \]

This is not enough information to solve the problem, since we have one equation but two unknowns, \( N \) and \( a \). However, the apparent weight is just \( N \), so the first sentence of the problem is \( N = (7/8)mg \). Now we can find \( a \),

\[ a = \frac{(N - mg)}{m} = \frac{[(7/8)mg - mg]}{m} = \frac{-g}{8} = -1.23 \text{ m/s}^2. \]

The minus sign indicates that, contrary to our initial assumption, the elevator accelerates downwards at \( 1.23 \text{ m/s}^2 \).

9. A person is standing on a weigh scale in an elevator. When the elevator is accelerating upward with constant acceleration \( a \), the scale reads 867.0 N. When the elevator is accelerating downwards with the same constant acceleration \( a \), the scale reads 604.5 N. Determine the magnitude of the acceleration \( a \), the weight of the person, and the mass of the person.
This problem deals with a force, the scale reading, and acceleration. That suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD). Since the person has mass, he has weight. Since the person touches the scale, there is a normal force from the scale. The scale reading is a measure of this normal force. Since there are two cases, we draw an FBD for each case.

So we have two equation in two unknowns, m and a. First we add the two equations together to get \( N_1 - mg + N_2 - mg = 0 \). This becomes \( m = \frac{(N_1 + N_2)}{2g} = 75.0 \) kg.

Using the first equation, we get \( a = \frac{(N_2 - mg)}{m} = 1.75 \text{ m/s}^2 \).

10. A 1200-kg elevator is carrying an 80.0-kg passenger. Calculate the acceleration of the elevator (and thus of the passenger) if the tension in the cable pulling the elevator is (a) 15,000 N, (b) 12,557 N, and (c) 10,000 N. what is the apparent weight of the passenger in each case. Assume \( g = 9.81 \text{ m/s}^2 \).

This problem deals with a force, tension, and acceleration. That suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each object, the person and the elevator. Each has mass, so each has weight. Since the person stands on the elevator, there is a an equal but opposite normal force on each. The rope, and thus the tension, acts directly only on the elevator. Both object have the same acceleration if they remain in contact. We will assume that the acceleration is upwards.
Applying Newton's Second law:

\[
\begin{align*}
\text{elevator} & : F_y = ma_y \\
\text{passenger} & : F_y = ma_y \\
T - Mg - N &= +Ma \\
N - mg &= ma
\end{align*}
\]

So we have two equations in two unknowns, N and a. First we add the two equations together to get \( T - (M+m)g = (M+m)a \). This our equation for the acceleration is \( a = \frac{T}{M+m} - g \). The apparent weight, that is the normal force acting on the passenger, is given by \( N = mg + ma \). When we plug in the numbers we find:

<table>
<thead>
<tr>
<th>acceleration</th>
<th>apparent weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 1.91 m/s²</td>
<td>938 N</td>
</tr>
<tr>
<td>(b) 0</td>
<td>785 N</td>
</tr>
<tr>
<td>(c) -2.00 m/s²</td>
<td>625 N</td>
</tr>
</tbody>
</table>

The negative acceleration in part (c) indicates that the elevator is actually accelerating downwards.

11. A car is traveling over the crest of a small semi-circular hill of radius \( R = 750 \) m. How fast would it have to be traveling for it to leave the ground?

Notice that the car is traveling in part of a circle, that suggests a centripetal acceleration. For the car to leave the ground, the normal force between the car and the ground must be zero. Since this problem deals with a force, the normal, and acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for the car at the crest of the hill. The car has mass, so it has weight. There is the normal force. By definition, the centripetal acceleration points towards the centre of the hill; in this case down.
Applying Newton's Second Law, $F_y = m a_y$, we get $N - mg = -mv^2/R$. The car loses contact when $N = 0$, so our equation for the speed becomes $v = [gR]^{1/2} = [9.81750]^{1/2} = 85.8 \text{ m/s} = 309 \text{ km/h}$.

12. A rollercoaster is at the inside top of a circular loop of radius $R = 150 \text{ m}$. How fast must the rollercoaster be going if it isn't to fall off?

Notice that the rollercoaster car is traveling in part of a circle, that suggests a centripetal acceleration. For the car to leave the track, the normal force between the car and the track must be zero. Since this problem deals with a force, the normal, and acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for the car. The car has mass, so it has weight. There is the normal force from the track on the car. By definition, the centripetal acceleration points towards the centre of the track; in this case down.

Applying Newton's Second Law, $F_y = m a_y$, we get $N - mg = -mv^2/R$. The car loses contact when $N = 0$, so our equation for the speed at which the car leaves the track becomes $v = [gR]^{1/2} = [9.81150]^{1/2} = 38.4 \text{ m/s} = 138 \text{ km/h}$. If the car goes faster than this speed, it will remain safely in contact with the track.

13. In the diagram below, the mass of the object is $50.0\text{kg}$. What are the tensions in the ropes?
Since this problem deals with forces, the tensions, and we known that the acceleration is zero since nothing is moving, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for the block and the knot. We need a FBD for the knot since this is where the tensions all meet. A knot is massless since we are dealing with massless ropes. The block has mass, so it has weight. Each rope represents a different tension.

Applying Newton's Second law:

\[
\begin{align*}
\text{knot} & : & F_y &= ma_y \\
& & T_2 - T_3\cos(45) &= 0 \\
\text{block} & : & F_y &= ma_y \\
& & T_3\sin(45) - T_1 &= 0 \\
& & T_1 - mg &= 0
\end{align*}
\]

From the above we see that \(T_1 = mg = 509.81 = 490.5\) N, \(T_3 = T_1/\sin(45) = 693.7\), and \(T_2 = T_3\cos(45) = 490.5\) N.

14. In the diagram below, block A weighs 100N. The coefficient of static friction between the block and the surface on which it rests is 0.40. Find the maximum weight w for which the system will remain in equilibrium.

A system in equilibrium does not accelerate, so \(a = 0\). Since this problem deals with forces, friction and weight, and we known that the acceleration is zero, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each block and the knot. We need a FBD for the knot since this is where the tensions all meet. A knot is massless since we are dealing with massless ropes. Each block has mass, so each has weight. Each rope represents a different tension. Block A is on a table so there is a normal force from the table through A. If there were no friction, Block A would move forward. We deduce that the maximum static friction points left.
Besides our equations in the last row above, we also have the definition \( f_{s\, MAX} = \mu N \). Our equation in the second column tells us that \( N = mg = 100 \, N \), so we know \( f_{s\, MAX} = \mu N = 0.40 \times 100 \, N = 40 \, N \). The first equation tells us that \( T_1 = f_{s\, MAX} = 40 \, N \). The third equation yields, \( T_2 = T_1 / \cos(40) = 52.2 \, N \). The fourth equation yields \( T_3 = T_2 \sin(40) = 33.6 \, N \). The equation in the last column indicates that \( w = T_3 = 33.6 \, N \). So the maximum weight of the hanging block is 33.6 \, N.

15. A bicyclist is riding on the banked curve of a circular velodrome. The radius of curvature for the bicyclist's present position is \( R = 355 \, m \). The coefficient of static friction between the wheels and the path is \( \mu_s = 0.35 \). For which range of velocities will the bicyclist remain at the same height on the banked curve?

Notice that the cyclist traveling in part of a circle, that suggests a centripetal acceleration. By definition, the centripetal acceleration points towards the centre of the velodrome; in this case to the right. Since this problem deals with a force, friction, and acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for the cyclist. The cyclist has mass, so he has weight. There is the normal force from the incline on the cyclist. We are told that there is friction but it is not immediately clear which way it points. The fact that the problem asks for a range of
velocities suggests that we are to solve the problem both ways, one with friction up the incline and one with it down.

Keep in mind that even though the bicycle wheels are moving, they are not slipping, so it is static friction which applies. As well, experience tells us if the cyclist moves too fast, he will tend to move up the incline. In such a case friction would resist this tendency an point down the incline. Similarly if the cyclist slows down, friction will be up the incline.

Also we should choose a coordinate system where one axis point in the direction of the acceleration.

\[
\begin{align*}
F_x &= ma_x \\
F_y &= ma_y \\
N\sin(\theta) - f_{\text{MAX}}\cos(\theta) &= m(v_1^2)/R \\
N\cos(\theta) + f_{\text{MAX}}\sin(\theta) - mg &= 0
\end{align*}
\]

We also know that \( f_{\text{MAX}} = \mu_s N \). If we substitute this into our two equations above, we find

\[
N\sin(\theta) - \mu_s N\cos(\theta) = m(v_1^2)/R , \text{ and}
\]

\[
N\cos(\theta) + \mu_s N\sin(\theta) = mg.
\]

We have two equations in two unknowns, \( N \) and \( v_1 \). The second equation can be rewritten as \( N = mg / [\cos(\theta) + \mu_s \sin(\theta)] \). Using this in the first equation and rearranging yields:

\[
v_1 = \sqrt{\frac{gR}{\sin(\theta) - \mu_s \cos(\theta)} - \frac{\mu_s \cos(\theta)}{\cos(\theta) + \mu_s \sin(\theta)}}
\]

If we plug in the number we are given, we find \( v_1 = 18.7 \text{ m/s} = 67.2 \text{ km/h} \).

\[
\begin{align*}
F_x &= ma_x \\
F_y &= ma_y \\
N\sin(\theta) + f_{\text{MAX}}\cos(\theta) &= m(v_2^2)/R \\
N\cos(\theta) - f_{\text{MAX}}\sin(\theta) - mg &= 0
\end{align*}
\]
We also know that \( f_{s, \text{MAX}} = \mu_s N \). If we substitute this into our two equations above, we find

\[
N \sin(\theta) + \mu_s N \cos(\theta) = m(v_2^2)/R, \quad \text{and} \quad N \cos(\theta) - \mu_s N \sin(\theta) = mg.
\]

We have two equations in two unknowns, \( N \) and \( v_2 \). The second equation can be rewritten as \( N = mg / [\cos(\theta) - \mu_s \sin(\theta)] \). Using this in the first equation and rearranging yields:

\[
v_2 = \sqrt{\frac{gR(\sin(\theta) + \mu_s \cos(\theta))}{\cos(\theta) - \mu_s \sin(\theta)}}
\]

If we plug in the number we are given, we find \( v_2 = 58.3 \text{ m/s} = 210 \text{ km/h} \).

So as long as the cyclist keeps his speed between 18.7 m/s and 58.3 m/s, he will stay at the same radius.

16. A block weighing 100 N is placed on a plane with slope angle 30.0° and is connected to a hanging weight of mass \( m \) by a cord passing over a small frictionless pulley as shown below. The coefficient of static friction is 0.52, and the coefficient of kinetic friction is 0.20.

Since this problem deals with forces, weight and friction, and we known that the acceleration is zero since the velocity is constant, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each block. The tension in the rope is constant throughout since we are dealing with an ideal massless, frictionless pulley. Each block has mass, so each has weight. There is a normal for the block tough the incline.

(a) Here the block is moving up the incline, so we have kinetic friction down the incline.
(a) The force diagram for the block on the incline includes the normal force, friction force, tension force, and the weight. The forces can be resolved into horizontal and vertical components.

**incline**

- $F_x = ma_x$
- $T - f_k - Mg\sin(\theta) = 0$

**hanging weight**

- $F_y = ma_y$
- $N - Mg\cos(\theta) = 0$

By definition, $f_k = \mu_k N$. From the equation in the second column, we have $N = Mg\cos(\theta)$. Thus $f_k = \mu_k Mg\cos(\theta)$. The third equation gives $T = mg$. Putting all this into the first equation yields

$$mg - \mu_k Mg\cos(\theta) - Mg\sin(\theta) = 0.$$ 

Rearranging gives an expression for $m$,

$$m = M[\sin(\theta) + \mu_k \cos(\theta)] = 6.86 \text{ kg}.$$ 

(b) Here the block is moving down the incline, so the kinetic friction is up the incline.

**incline**

- $F_x = ma_x$
- $T + f_k - Mg\sin(\theta) = 0$

**hanging weight**

- $F_y = ma_y$
- $N - Mg\cos(\theta) = 0$

By definition, $f_k = \mu_k N$. From the equation in the second column, we have $N = Mg\cos(\theta)$. Thus $f_k = \mu_k Mg\cos(\theta)$. The third equation gives $T = mg$. Putting all this into the first equation yields

$$mg + \mu_k Mg\cos(\theta) - Mg\sin(\theta) = 0.$$
Rearranging gives an expression for $m$,

$$m = M[\sin(\theta) - \mu_k \cos(\theta)] = 3.33 \text{ kg}.$$ 

(c) The block doesn't move, so we must be dealing with static friction. The fact that we are asked for a range of values suggests that we must look at static friction point both up and down the incline, i.e. to redo (a) and (b) with static friction. Nothing will change except the coefficient of friction, so our results will be the same but with $\mu_k$ replaced by $\mu_s$. Thus our results are

$$m = M[\sin(\theta) + \mu_s \cos(\theta)] = 9.69 \text{ kg}, \text{ and } m = M[\sin(\theta) - \mu_s \cos(\theta)] = 0.51 \text{ kg}.$$ 

So the blocks will not move if the hanging mass is between 0.51 kg and 9.69 kg.

17. A force $F$ pushes on a 25-kg box as shown in the figure below. The coefficient of static friction between box and incline is $\mu_s = 0.20$. Find the range of values for which the block remains stationary.

Since this problem deals with forces, weight and friction, and we known that the acceleration is zero since the block not moving, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each case. Since the block has mass, it has weight. It is on the incline, so there is a normal. For a stationary object, we must be dealing with static friction. As the problem asks for a range of values, we must examine the cases where friction acts up the incline and down the incline.

$$F_x = m a_x$$
$$F_y = m a_y$$
\[ F_1 \cos(\tau) - f_s^{\text{MAX}} - mg \sin(\theta) = 0 \quad N - mg \cos(\theta) - F_1 \sin(\theta) = 0 \]

Since we know \( f_s^{\text{MAX}} = \mu_s N \), and the second equation says that \( N = mg \cos(\theta) + F_1 \sin(\theta) \), we have \( f_s^{\text{MAX}} = \mu_s [m \cos(\theta) + F_1 \sin(\theta)] \). Substituting this into the first equation yields,

\[ F_1 \cos(\theta) - \mu_s [m \cos(\theta) + F_1 \sin(\theta)] - mg \sin(\theta) = 0 \]

Collecting similar terms we get

\[ F_1 [\cos(\theta) - \mu_s \sin(\theta)] - mg [\sin(\theta) + \mu_s \cos(\theta)] = 0 \]

So we find that

\[ F_1 = mg [\sin(\theta) + \mu_s \cos(\theta)] / [\cos(\theta) - \mu_s \sin(\theta)] = 306 \text{ N}. \]

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18. A box of mass \( M \) rests on a ramp cart for which the coefficient of static friction is \( \mu_s \). The ramps makes an angle \( \theta \) with the horizontal. The cart can be given a maximum acceleration \( a_{\text{max}} \) up the ramp for which the box does not slide on the level surface of the ramp cart. Determine an equation for the magnitude of this acceleration, \( a_{\text{max}} \), in terms of the coefficient \( \mu_s \) and the angle \( \theta \). (HINT: choosing the \( x \)-axis along the incline is not your best choice here.)

\[ i \quad F_x = m a_x \]

\[ F_2 \cos(\theta) + f_s^{\text{MAX}} - mg \sin(\theta) = 0 \quad N - mg \cos(\theta) - F_2 \sin(\theta) = 0 \]

Since we know \( f_s^{\text{MAX}} = \mu_s N \), and the second equation says that \( N = mg \cos(\theta) + F_2 \sin(\theta) \), we have \( f_s^{\text{MAX}} = \mu_s [m \cos(\theta) - F_2 \sin(\theta)] \). Substituting this into the first equation yields,

\[ F_2 \cos(\theta) - \mu_s [m \cos(\theta) - F_2 \sin(\theta)] - mg \sin(\theta) = 0 \]

Collecting similar terms we get

\[ F_2 [\cos(\theta) + \mu_s \sin(\theta)] - mg [\sin(\theta) - \mu_s \cos(\theta)] = 0 \]

So we find that

\[ F_1 = mg [\sin(\theta) - \mu_s \cos(\theta)] / [\cos(\theta) + \mu_s \sin(\theta)] = 134 \text{ N}. \]

So when the applied force, \( F \), is between 134 N and 306 N, the block remains stationary.
Since this problem deals with a force, friction, and we have an acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for the block. Since the block has mass, it has weight. It is on the incline, so there is a normal. We have to decide which way the static friction points. This can be determined by noting that the block has a forward acceleration, since we know of no other forces friction must be the cause.

\[ F_x = ma_x \]
\[ f_{s \, \text{MAX}} = ma_{\text{MAX}} \cos(\theta) \]
\[ i \]
\[ F_y = ma_y \]
\[ N - mg = ma_{\text{MAX}} \sin(\theta) \]
\[ j \]

Since \( f_{s \, \text{MAX}} = \mu_s N \), and the second equation gives \( N = mg + ma_{\text{MAX}} \sin(\theta) \), we have \( f_{s \, \text{MAX}} = \mu_s [mg + ma_{\text{MAX}} \sin(\theta)] \). We substitute this into the first equation to get

\[ \mu_s [mg + ma_{\text{MAX}} \sin(\theta)] = ma_{\text{MAX}} \cos(\theta) \]

We divide \( m \) out, and rearrange to get

\[ a_{\text{MAX}} = \mu_s g / [\cos(\theta) - \mu_s \sin(\theta)] , \]

which is in the requested form.

19. What must be the acceleration of the cart pictured below in order that block A does not fall? The coefficient of friction between the block and the cart is \( \mu_s \).
Since this problem deals with a force, friction, and an acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each object. The cart and the block must have the same acceleration. As we will see, we only need an FBD for the block to solve the problem. We would need another FBD if the question asked what force was accelerating the cart. The block has mass, so it has weight. It is touching the cart, so there is a normal. We have to decide which way the static friction points. If the block is not to fall, it must be upwards.

\[
\begin{align*}
\text{i} & \quad \text{j} \\
F_x &= ma_x \\
F_y &= ma_y \\
N &= ma \\
f_{s\text{MAX}} - mg &= 0
\end{align*}
\]

We know that \(f_{s\text{MAX}} = \mu_s N\). The first equation gives us \(N\), so \(f_{s\text{MAX}} = \mu_s ma\). The second equation says, \(f_{s\text{MAX}} = mg\). So therefore \(\mu_s ma = mg\) or \(a = g/\mu_s\).

20. Three blocks with masses 6 kg, 9 kg, and 10 kg are connected as shown below. The coefficient of kinetic friction between the 10 kg block and the table is 0.2. Find the acceleration of the system. Find the tension in each cord.

Since this problem deals with forces, friction and tension, and acceleration, that suggests we should apply
Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each object. The three blocks must have the same acceleration, though in different directions. Each block has mass, so each has weight. The block on the table has a normal acting on it. The tension is the same throughout each rope as long as we deal with ideal massless, frictionless pulleys. Presumably, the 9-kg block will move downwards, therefore kinetic friction on the middle block points left.

\[
\begin{align*}
F_y &= m_ay \\
F_x &= ma_x \\
T_1 - m_1g &= m_1a \\
T_2 - T_1 - f_k &= m_2a \\
N - m_2g &= 0 \\
T_2 - m_3g &= -m_3a
\end{align*}
\]

We know that \(f_k = \mu_k N\). The third equation gives us \(N = m_2g\), so \(f_k = \mu_k m_2g\). The first equation gives us an expression for \(T_1\), \(T_1 = m_1g + m_1a\). The fourth equation gives an expression for \(T_2\), \(T_2 = m_3g - m_3a\). With these results, we can rewrite the second equation as

\[
[m_3g - m_3a] - [m_1g + m_1a] - \mu_k m_2g = m_2a.
\]

Collecting the terms involving \(a\), and rearranging yields

\[
a = g \left[ m_3 - m_1 - \mu_k m_2 \right] / \left[ m_3 + m_2 + m_1 \right].
\]

Substituting in the given values, one finds \(a = 0.3924 \text{ m/s}^2\).

Using this value for \(a\), \(T_1 = m_1g + m_1a = 61.2 \text{ N}\). As well, \(T_2 = m_3g - m_3a = 84.8 \text{ N}\).

21. Two blocks of mass \(m_1\) and \(m_2\) are sliding down an inclined plane making an angle \(\theta\) with the horizontal. The leading block has a coefficient of kinetic friction \(\mu_k\); the trailing block has coefficient \(2\mu_k\). A string connects the two blocks. The string makes an angle \(\phi\) with the ramp. Find the acceleration of the blocks. Find the tension in the string.
Since this problem deals with force, friction, and acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each object. The two blocks must have the same acceleration as they are connected by a string. Each block has mass, so each has weight. Each block is touching the incline so each has a normal acting on it. The tension is the same throughout each rope. Presumably, the blocks will move downwards, therefore kinetic friction is up the incline.

\[
\begin{align*}
F_x &= m_1 a_x \\
F_y &= m_1 a_y \\
T \cos \phi + m_1 g \sin \theta - f_1 &= m_1 a \\
N_1 - m_1 g \cos \theta - T \sin \phi &= 0
\end{align*}
\]

\[
\begin{align*}
F_x &= m_2 a_x \\
F_y &= m_2 a_y \\
m_2 g \sin \theta - T \cos \phi - f_2 &= m_2 a \\
N_2 - m_2 g \cos \theta + T \sin \phi &= 0
\end{align*}
\]

We also know that \( f_1 = \mu_k N_1 \) and \( f_2 = 2 \mu_k N_2 \). From our equations, we know \( N_1 = m_1 g \cos \theta + T \sin \phi \) and \( N_2 = m_2 g \cos \theta - T \sin \phi \). Therefore, \( f_1 = \mu_k [m_1 g \cos \theta + T \sin \phi] \) and \( f_2 = 2 \mu_k [m_2 g \cos \theta - T \sin \phi] \). Using these expressions, we can eliminate \( f_1 \) and \( f_2 \) from the expressions for \( a \). We have

\[
T \cos \phi + m_1 g \sin \theta - \mu_k [m_1 g \cos \theta + T \sin \phi] = m_1 a, \text{ and}
\]

\[
m_2 g \sin \theta - T \cos \phi - 2 \mu_k [m_2 g \cos \theta - T \sin \phi] = m_2 a.
\]
These can be rearranged to give

\[ T[\cos \phi - \mu g \sin \phi] + m_1 g[\sin \theta - \mu g \cos \theta] = m_1 a, \quad \text{and} \]

\[ -T[\cos \phi - 2\mu g \sin \phi] + m_2 g[\sin \theta - 2\mu g \cos \theta] = m_2 a. \]

To eliminate \( a \), we multiply the first equation by \( m_2 \), the second by \( m_1 \), and subtract the second equation from the first giving

\[ T\{m_1[\cos \phi - \mu g \sin \phi] + m_2[\cos \phi - 2\mu g \sin \phi]\} + m_1 m_2 g \mu g \cos \theta = 0. \]

So our expression for the tension is

\[ T = -m_1 m_2 g \mu g \cos \theta / \{m_1[\cos \phi - \mu g \sin \phi] + m_2[\cos \phi - 2\mu g \sin \phi]\}. \]

Substituting this back into the equations for \( a \), we get

\[ a = -m_2 g \mu g \cos \theta [\cos \phi - \mu g \sin \phi] / \{m_1[\cos \phi - \mu g \sin \phi] + m_2[\cos \phi - 2\mu g \sin \phi]\} \]

\[ + g[\sin \theta - \mu g \cos \theta], \]

or

\[ a = m_1 g \mu g \cos \theta [\cos \phi - 2\mu g \sin \phi] / \{m_1[\cos \phi - \mu g \sin \phi] + m_2[\cos \phi - 2\mu g \sin \phi]\} \]

\[ + g[\sin \theta - 2\mu g \cos \theta]. \]

22. If \( m_1 = 25.0 \, \text{kg} \) and \( m_2 = 12.0 \, \text{kg} \), calculate the normal force on the bottom mass if the force, \( F \), is (a) 100N, (b) 363N, and (c) 500N.

Since this problem deals with forces that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each object. The two blocks must have the same acceleration as they are in contact. Each block has mass, so each has weight. The top block is touching the bottom block, so it experiences a normal. An equal but opposite normal is exerted on the lower block. The table top also exerts a normal on the bottom block. The given force only acts on the
bottom block. The table top normal is nonzero only if the bottom block remains on the table, so we will assume that the acceleration is zero.

From the first equation we find \( N = m_2 g \). We substitute this into the second equation to get an expression for \( N_{\text{table}} \):

\[
N_{\text{table}} = m_1 g + m_2 g - F.
\]

Examining the three cases yields,

\[
\begin{array}{ccc}
F & N_{\text{table}} \\
(a) & 100 \text{ N} & 263 \text{ N} \\
(b) & 363 \text{ N} & 0 \text{ N} \\
(c) & 500 \text{ N} & -137 \text{ N}
\end{array}
\]

Since normal force can never be zero, case (c) is saying that the blocks has been lifted off the table, so that \( N_{\text{table}} = 0 \), and is actually accelerating upwards.

23. A 10-kg wooden block rests on a 5-kg, L-shaped piece of metal as shown below. The metal, in turn, rests on a frictionless surface. The coefficients of friction between the wood and the metal are \( \mu_s = 0.40 \) and \( \mu_k = 0.30 \). (a) What is the maximum force that can be applied if the 10-kg block is NOT to slide relative to the metal? (b) What is the corresponding acceleration of the piece of metal?
Since this problem deals with forces and acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each object. We are told that the block does not slip on the metal, that tells us that both objects have the same acceleration. It also tells us that the friction is static. Each block has mass, so each has weight. The top block is touching the bottom metal object, so it experiences a normal. An equal but opposite normal is exerted on the lower object. The table top also exerts a normal on the metal object. The pulley attached to the metal object acts as a force doubler. The tension in the rope, $F$, is the same throughout the rope. The tricky force is the static friction. Since the block accelerates to the right, some force must be the cause, only static friction is available so it must also point to the right. Since the friction is coming from the common surface between the block and metal object, an equal but opposite frictional force acts on the metal object.

Now we know $f_{s\text{MAX}} = \mu_s N$. Notice that $f_{s\text{MAX}}$, the friction between the block and object, is related to the normal that tells us how the block and the metal object are pressed together. Since the second equation tells us $N = mg$, we have $f_{s\text{MAX}} = \mu_s mg$. The fourth equation tells us that $N_{\text{table}} = N + Mg = (M+m)g$. Using all this information, the remaining equations can be written as

$$\mu_s mg - F = ma,$$

We now have two equations in two unknowns, $F$ and $a$. The first equation gives an expression for $F$,

$$F = \mu_s mg - ma.$$
Collecting the terms containing \( a \) and rearranging yields

\[
a = \frac{\mu_s mg}{(M + 2m)} = 1.57 \text{ m/s}^2.
\]

This is the maximum acceleration of the blocks if there is to be no slipping. Putting this expression for \( a \), into the expression for \( F \) yields

\[
F = \mu_s mg - m\left[\frac{\mu_s mg}{(M + 2m)}\right] = \mu_s mg\left[\frac{(M + m)}{(M + 2m)}\right] = 23.5 \text{ N}.
\]

The maximum force that can be applied is 23.5 N if the blocks are not to slip.

---

24. In the diagram below, a 10.0-kg block sits on top of a 20.0-kg block on top of a horizontal surface. The coefficients of friction between the two blocks are \( \mu_{s1} = 0.38 \) and \( \mu_{k1} = 0.19 \). The coefficients of friction between the bottom block and the surface are \( \mu_{s2} = 0.35 \) and \( \mu_{k2} = 0.10 \). (a) What is the maximum horizontal force that can be applied to the upper block such that it does not slip relative to the bottom block? What is the acceleration of the blocks? (b) What is the maximum horizontal force that can be applied to the lower block such that it does not slip relative to the upper block? What is the acceleration of the blocks?

Since this problem deals with forces and acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each object. We are told that the top block does not slip on the bottom block, that tells us that both objects have the same acceleration. It also tells us that the friction is static between the blocks. Each block has mass, so each has weight. The top block is touching the bottom block, so it experiences a normal. An equal but opposite normal is exerted on the lower block. The table top also exerts a normal on the bottom block. Let us assume that the applied force is to the right in each case. The bottom block slides over the table top, so there is kinetic friction between the table and lower block pointing to the left. The tricky force is the static friction and we must consider each case separately.

(a) When the upper block is pulled, the bottom block also accelerates to the right. Some force must be the cause. Only static friction is available so it must also point to the right. Since the friction is coming from the common surface between the block and metal object, an equal but opposite frictional force acts on the upper block.
Now we know \( f_{s\, \text{MAX}} = \mu s_1 N \). Notice that \( f_{s\, \text{MAX}} \), the friction between the blocks, is related to the normal that tells us how the block and the metal object are pressed together. Since the second equation tells us \( N = m_1 g \), we have \( f_{s\, \text{MAX}} = \mu s_1 m_1 g \).

We also know \( f_k = \mu k_2 N_{\text{table}} \). Notice that \( f_k \), the friction between the lower block and table, is related to the normal that tells us how the block and the table are pressed together. Since the fourth equation tells us \( N_{\text{table}} = N + m_2 g = (m_1 + m_2)g \), we have \( f_k = \mu k_2 (m_1 + m_2)g \).

The first and third equations then become

\[
F - \mu s_1 m_1 g = m_1 a, \quad \text{and} \quad \mu s_1 m_1 g - \mu k_2 (m_1 + m_2)g = m_2 a.
\]

Solving these equations yields,

\[
F = \mu s_1 m_1 g + m_1 [\mu s_1 m_1 g - \mu k_2 (m_1 + m_2)g]/m_2 = 41.2 \, \text{N}.
\]

And the equation for \( a \) is

\[
a = [\mu s_1 m_1 g - \mu k_2 (m_1 + m_2)g]/m_2 = 0.39 \, \text{m/s}^2.
\]

So the maximum force that can be applied to the upper block without the blocks slipping is 41.2 N and the resultant acceleration is 0.39 m/s\(^2\).

(b) When the lower block is pulled, the upper block also accelerates to the right. Some force must be the cause. Only static friction is available so it must also point to the right. Since the friction is coming from the common surface between the block and metal object, an equal but opposite frictional force acts on the lower block.
Now we know $f_{s\text{ MAX}} = \mu_s N$. Notice that $f_{s\text{ MAX}}$, the friction between the blocks, is related to the normal that tells us how the block and the metal object are pressed together. Since the second equation tells us $N = m_1 g$, we have $f_{s\text{ MAX}} = \mu_s m_1 g$.

We also know $f_k = \mu_k N_{\text{table}}$. Notice that $f_k$, the friction between the lower block and table, is related to the normal that tells us how the block and the table are pressed together. Since the fourth equation tells us $N_{\text{table}} = N + m_2 g = (m_1 + m_2) g$, we have $f_k = \mu_k (m_1 + m_2) g$.

The first and third equations then become

$$\mu_s m_1 g = m_1 a, \quad \text{and}$$

$$F - \mu_s m_1 g - \mu_k (m_1 + m_2) g = m_2 a.$$

Solving these equations yields,

$$a = s_1 g = 3.73 \text{ m/s}^2, \quad \text{and}$$

$$F = (\mu_s + \mu_k)(m_1 + m_2) g = 141 \text{ N}.$$

So the maximum force that can be applied to the upper block without the blocks slipping is 141 N and the resultant acceleration is 3.73 m/s$^2$.
(a) Find the maximum value of $F$, such that the blocks do not slip. Find the tension in the string.

(b) If $F$ is 100 N, find the initial acceleration of the lower block. Also find the tension in the string.

Since this problem deals with forces and acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each object. The tricky force is the static friction and we must consider each case separately.

(a) We are told that the top block does not slip, that tells us that both objects have the same zero acceleration. It also tells us that the friction is static between the blocks. Each block has mass, so each has weight. The top block is touching the bottom block, so it experiences a normal. An equal but opposite normal is exerted on the lower block. The table top also exerts a normal on the bottom block. When the lower block is pulled, the top surface of the bottom block exerts a static frictional force on the top block directed to the right. An equal but opposite frictional force acts on the lower block. The string exerts a tension force on the upper block. The bottom block does not slide over the table top, so there is static friction between the table and lower block pointing to the left.

Now we know $f_{sb\, \text{MAX}} = \mu_{sb}N$. Notice that $f_{sb\, \text{MAX}}$, the friction between the blocks, is related to the normal that tells us how the blocks are pressed together. Since the second equation tells us $N = m_1g$, we have $f_{sb\, \text{MAX}} = \mu_{sb}m_1g$.

We also know $f_{st\, \text{MAX}} = \mu_{st}N_{\text{table}}$. Notice that $f_{st\, \text{MAX}}$, the friction between the lower block and table, is related to the normal that tells us how the lower block and the table are pressed together. Since the fourth equation tells us $N_{\text{table}} = N + m_2g = (m_1 + m_2)g$, we have $f_{st\, \text{MAX}} = \mu_{st}(m_1 + m_2)g$.

The first and third equations then become

$$T - \mu_{sb}m_1g = 0,$$

and
\[ F - \mu_{sb}m_1g - \mu_{st}(m_1 + m_2)g = 0. \]

Solving these equations yields,

\[ T = \mu_{sb}m_1g = 23.5 \text{ N, and} \]

\[ F = \mu_{sb}m_1g + \mu_{st}(m_1 + m_2)g = 97.1 \text{ N}. \]

So the maximum force that can be applied to the upper block without either block slipping is 97.1 N.

(b) The bottom block now slips across both the top and bottom surfaces. This tells us that the friction is kinetic between the blocks and between the lower block and the table top. The kinetic friction between the blocks will act to slow the lower block, so it points to the left. Since they share a common surface, an equal but opposite kinetic friction acts on the top block. The kinetic friction between the lower block and the table top also acts to slow the lower block and thus also point to the left.

Now we know \( f_{kb} = \mu_{kb}N \). Notice that \( f_{kb} \), the friction between the blocks, is related to the normal that tells us how the blocks are pressed together. Since the second equation tells us \( N = m_1g \), we have \( f_{kb} = \mu_{kb}m_1g \).

We also know \( f_{kt} = \mu_{kt}N_{table} \). Notice that \( f_{kt} \), the friction between the lower block and table, is related to the normal that tells us how the lower block and the table are pressed together. Since the fourth equation tells us \( N_{table} = N + m_2g = (m_1 + m_2)g \), we have \( f_{kt} = \mu_{kt}(m_1 + m_2)g \).

The first and third equations then become

\[ T - \mu_{kb}m_1g = 0, \text{ and} \]

\[ F - \mu_{kb}m_1g - \mu_{kt}(m_1 + m_2)g = m_2a. \]

Solving these equations yields,

\[ T = \mu_{kb}m_1g = 14.7 \text{ N, and} \]
\[ a = \frac{[F - \mu_{kb}m_1g - \mu_{kl}(m_1 + m_2)g]}{m_2a} = 2.04 \text{ m/s}^2. \]

So a force of a 100 N will cause the lower block to accelerate at 2.04 m/s\(^2\) and cause a tension in the string of 14.7 N.

26. A 10-kg block is hanging from a spring with spring constant \( K = 1000 \text{ N/m} \). The spring is attached to the ceiling of an elevator. The elevator is currently moving upwards at 10 m/s and slowing down at 1.0 m/s\(^2\). How much is the spring stretched?

The elevator is moving upwards but slowing so the acceleration points downwards.

After drawing the free-body diagram, we apply Newton's Second Law and get \( kx - mg = -ma \). Solving for \( x \) we find

\[ x = \frac{m(g - a)}{K} = \frac{10(9.81 - 1.0)}{1000} = 0.088 \text{ m}. \]

The spring is stretched 8.8 cm.

27. A block of mass \( M \) is on an incline set at angle \( \theta \) and is not moving. A spring with spring constant \( K \) is connected to the upper side of the block and a fixed support rod. The coefficients of friction between the block and the incline are \( \mu_s \) and \( \mu_k \). What is the expression for the minimum amount that the spring could be stretched? What is the maximum amount that the spring could be stretched?

The fact that the spring is stretched in both cases tells us that the block would slide down the incline if the spring was not attached. The acceleration is zero and the friction is static since the block is not moving. Since we want the maximum and minimum extensions of the spring, we must be dealing with the maximum static friction. For the minimum extension of the spring, the friction must point in the same direction as the spring force. For the maximum extension, friction is in the opposite direction.

We draw our free body diagrams as shown in diagram (i) and (ii) below,
Applying Newton's Second Law to diagram (i), our equations are

\[ N_1 - mg \cos(\theta) = 0 \]  
\[ Kx_1 + f_{s1} = mg \sin(\theta) = 0 \]

We know \( f_{s1} = \mu_s N_1 = \mu_s mg \cos(\theta) \), where we have made use of equation (1). We plug this result into equation (2) and find

\[ x_1 = \frac{mg [\sin(\theta) - \mu_s \cos(\theta)]}{K} \]

Applying Newton's Second Law to diagram (ii), our equations are

\[ N_2 - mg \cos(\theta) = 0 \]  
\[ Kx_2 - f_{s2} = mg \sin(\theta) = 0 \]

We know \( f_{s2} = \mu_s N_2 = \mu_s mg \cos(\theta) \), where we have made use of equation (3). We plug this result into equation (4) and find

\[ x_2 = \frac{mg [\sin(\theta) + \mu_s \cos(\theta)]}{K} \]

---

28. Two blocks, A of mass 8 kg and B of mass 6 kg, are on flat surface. A spring with spring constant \( K = 1000 \) N/m joins them together. The coefficients of friction between the blocks and the incline are \( \mu_s = 0.17 \) and \( \mu_k = 0.12 \). A constant horizontal force \( F \) is pulling block A and, via the spring, block B.

(a) How much is the spring stretched if both blocks are moving at constant velocity? What is \( F \)?

(b) How much is the spring stretched if both blocks are accelerating at \( 2 \) m/s\(^2\)? What is \( F \)?
(a) Constant velocity means that there is no acceleration. We draw our free-body diagrams as in diagram (a) and apply Newton's Second Law. Note that the friction is kinetic since there is motion of the surfaces. The resulting equations are

\[ \begin{align*}
N_A - m_Ag &= 0 \quad (1) \\
N_B - m_Bg &= 0 \quad (2) \\
F - f_{KA} - Kx_1 &= 0 \quad (3) \\
Kx_1 - f_{KA}B &= 0 \quad (4)
\end{align*} \]

We know that \( f_{KA} = \mu_k N_A \) and \( f_{KB} = \mu_k N_B \). Equations (1) and (2) indicate that \( N_A = m_Ag \) and \( N_B = m_Bg \), so \( f_{KA} = \mu_k m_Ag \) and \( f_{KB} = \mu_k m_Bg \). We substitute this result into equation (4) and find

\[ x_1 = \frac{\mu_k m_Bg}{K} = 0.0071 \text{ m}. \]

With the above result and equation (3) we find

\[ F = \mu_k(m_A + m_B)g = 16.5 \text{ N}. \]

(b) Again, we draw our free-body diagrams as in diagram (b) and apply Newton's Second Law. Note that the friction is kinetic since there is motion of the surfaces. The resulting equations are

\[ \begin{align*}
N_A - m_Ag &= 0 \quad (5) \\
N_B - m_Bg &= 0 \quad (6) \\
F - f_{KA} - Kx_2 &= m_Aa \quad (7) \\
Kx_2 - f_{KA}B &= m_Ba \quad (8)
\end{align*} \]

We know that \( f_{KA} = \mu_k N_A \) and \( f_{KB} = \mu_k N_B \). Equations (5) and (6) indicate that \( N_A = m_Ag \) and \( N_B = m_Bg \), so \( f_{KA} = \mu_k m_Ag \) and \( f_{KB} = \mu_k m_Bg \). We substitute this result into equation (8) and find

\[ x_2 = \frac{m_Ba + \mu_k m_Bg}{K} = 0.0191 \text{ m}. \]

With the above result and equation (3) we find

\[ F = \mu_k(m_A + m_B)(g + a) = 19.8 \text{ N}. \]