

Mathematics Problem of the Week (245)

This week's winner is:

Matthew Potma

Contact Lin Hammill (Surrey Fir 348) or Judy Bicep (Richmond,3335) for your prize or email MathProblem@kpu.ca.

Also submitting correct solutions to problem 245 were:

Robert Hill, Zephram Tripp, Suzanne Pearce and David Luna

Problem 245 solution:

Solution

Let the number of missing pages be n and the first missing page be $p + 1$. Then pages $p + 1$ to $p + n$ are missing. Then the sum of the digits from $p + 1$ to $p + n$ is 2896.

To sum these digits consider:

$$\begin{array}{c} (p+1) + (p+2) + (p+3) + \dots + (p+n-2) + (p+n-1) + (p+n) \\ \left[\begin{array}{c} (p+3) + (p+n-2) \\ = (p+1) + (p+n) \end{array} \right] \\ (p+2) + (p+n-1) = (p+1) + (p+n) \\ (p+1) + (p+n) \end{array}$$

Notice that n must be even because there are two pages on each leaf of a book. Thus we have $n/2$ pairs, each with sum $(p + 1) + (p + n)$. Also note that the prime factorization of 2896 is $2896 = 2^4 \cdot 181$. Hence:

$$((p+1) + (p+n)) \frac{n}{2} = (2p + n + 1) \frac{n}{2} = 2896 = 2^4 \cdot 181.$$

Multiplying by 2: $(2p + n + 1)n = 2^5 \cdot 181$

Since n must be even and $2p + n + 1$ must be odd (since $2p$ and n are even),

$$n = 2^5 = 32 \text{ and } 2p + n + 1 = 181 \text{ so that } p = 74.$$

Thus 32 pages are missing and they are pages 75 to 106.