Kwantlen Polytechnic University

Mathematics Problem of the Week (252)

This week's winner is:

Victor Blancard

Contact Lin Hammill (Surrey Fir 348) or Judy Bicep (Richmond,3335) for your prize or email MathProblem@kpu.ca**.**

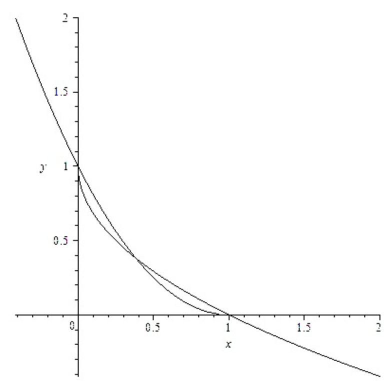
Also submitting a corrrect solution was

Burhan Akram

Problem 252 solution:

$$\begin{aligned} x + \sqrt{y} &= 1 \\ \sqrt{x} + y &= 1 \end{aligned}$$ with $x \geq 0,\ y \geq 0$ since the solutions must be real.

By inspection it is easy to see that two solutions are $(x,y) = (1,0)$ and $(x,y) = (0,1)$. Solving each equation for y ($y = (1-x)^2,\quad y = 1-\sqrt{x}$) and graphing confirms that there is a third solution.



Since $x + \sqrt{y} = 1$ and $\sqrt{x} + y = 1$, $x + \sqrt{y} = \sqrt{x} + y$. Clearly a possible solution to this is $x = y$. Thus:

$$x + \sqrt{x} = 1$$
$$x + \sqrt{x} - 1 = 0$$
$$\left(\sqrt{x}\right)^2 + \sqrt{x} - 1 = 0$$

Solving the quadratic yields $\sqrt{x} = \dfrac{-1 \pm \sqrt{5}}{2}$. Since $\sqrt{x} > 0$ by definition, we have

$$x = \left(\frac{-1 + \sqrt{5}}{2}\right)^2 = \frac{3 - \sqrt{5}}{2}.$$

Thus the third solution is $(x,y) = \left(\dfrac{3 - \sqrt{5}}{2}, \dfrac{3 - \sqrt{5}}{2}\right)$