## Chapter 2

## Equations and Inequalities

### 2.1 The Rectangular Coordinate Systems and Graphs

## Verbal

1. Is it possible for a point plotted in the Cartesian coordinate system to not lie in one of the four quadrants? Explain.
Answers may vary. Yes. It is possible for a point to be on the $x$-axis or on the $y$-axis and therefore is considered to NOT be in one of the quadrants.
2.     - 
3. Describe in your own words what the $y$-intercept of a graph is.

The $y$-intercept is the point where the graph crosses the $y$-axis.
4.

## Algebraic

For each of the following exercises, find the $x$-intercept and the $y$-intercept without graphing. Write the coordinates of each intercept.
5. $y=-3 x+6$
$y=-3 x+6 \quad$ Substitute 0 for $y$
$0=-3 x+6 \quad$ Subtract 6 from both sides
$-6=-3 x \quad$ Divide both sides by -3
$x=2 \quad$ Substitute 0 for $x$
$y=-3(0)+6 \quad$ Multiply
$y=6$
The $x$-intercept is $(2,0)$ and the $y$-intercept is $(0,6)$.
6. -
7. $3 x-2 y=6$
$3 x-2 y=6 \quad$ Substitute 0 for $x$
$3(0)-2 y=6$ Multiply
$-2 y=6 \quad$ Divide both sides by -2
$y=-3$
$3 x-2 y=6$ Substitute 0 for $y$
$3 x-2(0)=6$ Multiply
$3 x=6 \quad$ Divide both sides by 3
$x=2$
The $x$-intercept is $(2,0)$ and the $y$-intercept is $(0,-3)$.
8. -
9. $3 x+8 y=9$
$3 x+8 y=9 \quad$ Substitute 0 for $x$
$3(0)+8 y=9 \quad$ Multiply
$8 y=9$ Divide both sides by 8
$y=\frac{9}{8}$
$3 x+8(0)=9 \quad$ Substitute 0 for $y$
$3 x=9 \quad$ Divide both sides by 3
$x=3$
The $x$-intercept is $(3,0)$ and the $y$-intercept is $\left(0, \frac{9}{8}\right)$.
10. -

For each of the following exercises, solve the equation for $y$ in terms of $x$.
11. $4 x+2 y=8$
$4 x+2 y=8 \quad$ Subtract $4 x$ from both sides
$2 y=8-4 x \quad$ Divide both sides by 2
$y=4-2 x$
12. -
13. $2 x=5-3 y$
$2 x=5-3 y \quad$ Subtract 5 from both sides
$2 x-5=-3 y$ Divide both sides by -3
$-\frac{2 x-5}{3}=y \quad$ Rewrite
$y=\frac{5-2 x}{3}$
14. -
15. $5 y+4=10 x$
$5 y+4=10 x$ Subtract 4 from both sides
$5 y=10 x-4$ Divide both sides by 5
$y=2 x-\frac{4}{5}$
16. -

For each of the following exercises, find the distance between the two points. Simplify your answers, and write the exact answer in simplest radical form for irrational answers.
17. $(-4,1)$ and $(3,-4)$

$$
d=\sqrt{(-4-3)^{2}+(1-(-4))^{2}} \quad \text { Subtract }
$$

$$
d=\sqrt{(-7)^{2}+(5)^{2}} \quad \text { Square the }-7 \text { and } 5
$$

$$
d=\sqrt{49+25} \quad \text { Add }
$$

$$
d=\sqrt{74}
$$

18.     - 
19. $(5,0)$ and $(5,6)$
$d=\sqrt{(6-0)^{2}+(5-5)^{2}} \quad$ Subtract
$d=\sqrt{(6)^{2}+(0)^{2}} \quad$ Square the 6
$d=\sqrt{36}=6$
20.     - 
21. Find the distance between the two points given using your calculator, and round your answer to the nearest hundredth.
$(19,12)$ and $(41,71) d \approx 62.97$

For each of the following exercises, find the coordinates of the midpoint of the line segment that joins the two given points.
22. -
23. $(-1,1)$ and $(7,-4)$

$$
\begin{aligned}
& \left(\frac{-1+7}{2}, \frac{1-4}{2}\right) \quad \text { Add } \\
& \left(\frac{6}{2}, \frac{-3}{2}\right) \text { Simplify } \\
& \left(3, \frac{-3}{2}\right)
\end{aligned}
$$

24.     - 
25. $(0,7)$ and $(4,-9)$
$\left(\frac{0+4}{2}, \frac{7+(-9)}{2}\right) \quad$ Add
$\left(\frac{4}{2}, \frac{-2}{2}\right) \quad$ Simplify
$(2,-1)$
26.     - 

## Graphical

For each of the following exercises, identify the information requested.
27. What are the coordinates of the origin?
$(0,0)$
28. -
29. If a point is located on the $x$-axis, what is the $y$-coordinate?

$$
y=0
$$

For each of the following exercises, plot the three points on the given coordinate plane. State whether the three points you plotted appear to be collinear (on the same line).
30. -
31. $(-1,2)(0,4)(2,1)$

32. -
33. Name the coordinates of the points graphed.

## Section 2.1


34. -

For each of the following exercises, construct a table and graph the equation by plotting at least three points.
35. $y=\frac{1}{3} x+2$

| $x$ | $y$ |
| :---: | :---: |
| -3 | 1 |
| 0 | 2 |
| 3 | 3 |
| 6 | 4 |

Section 2.1

36. -
37. $2 y=x+3$

| $X$ | $y$ |
| :--- | :--- |
| -3 | 0 |
| 0 | 1.5 |
| 3 | 3 |



## Numeric

For each of the following exercises, find and plot the $x$ - and $y$-intercepts, and graph the straight line based on those two points.
38.
39. $x-2 y=8$

40. -
41. $3 y=-2 x+6$

42. -

For each of the following exercises, use the graph.

43. Find the distance between the two endpoints using the distance formula. Round to three decimal places.
$d=\sqrt{(5-(-3))^{2}+(2-(-3))^{2}} \quad$ Subtract
$d=\sqrt{(8)^{2}+(5)^{2}} \quad$ Square the 8 and 5
$d=\sqrt{64+25} \quad$ Add
$d=\sqrt{89} \quad$ Evaluate
$d=8.246$
44. -
45. Find the distance that $(-3,4)$ is from the origin.
$d=\sqrt{(-3-0)^{2}+(4-0)^{2}} \quad$ Subtract
$d=\sqrt{(-3)^{2}+(4)^{2}} \quad$ Square the -3 and 4
$d=\sqrt{9+16} \quad$ Add
$d=\sqrt{25} \quad 25$ is a perfect square
$d=5$
46. -
47. Which point is closer to the origin?

## Technology

For the following exercises, use your graphing calculator to input the linear graphs in the $\mathrm{Y}=$ graph menu.
After graphing it, use the $2^{\text {nd }}$ CALC button and 1 :value button, hit enter. At the lower part of the screen you will see " $x=$ " and a blinking cursor. You may enter any number for $x$ and it will display the $y$ value for any $x$ value you input. Use this and plug in $x=0$, thus finding the $y$ intercept, for each of the following graphs.
48. -
49. $\mathrm{Y}_{1}=\frac{3 x-8}{4}$
$x=0 \quad y=-2$
50. -

For the following exercises, use your graphing calculator to input the linear graphs in the $\mathrm{Y}=$ graph menu.
After graphing it, use the $2^{\text {nd }}$ CALC button and 2:zero button, hit enter. At the lower part of the screen you will see "left bound?" and a blinking cursor on the graph of the line. Move this cursor to the left of the $x$-intercept, hit ENTER. Now it says "right bound?" Move the cursor to the right of the $x$-intercept, hit enter. Now it says "guess?" Move your cursor to the left somewhere in between the left and right bound near the $x$-intercept. Hit enter. At the bottom of your screen it will display the coordinates of the $x$-intercept or the "zero" to the $y$-value. Use this to find the $x$ intercept.
Note: With linear/straight line functions the zero is not really a "guess," but it is necessary to enter a "guess" so it will search and find the exact $x$-intercept between your right and left boundaries. With other types of functions (more than one $x$-intercept), they may be irrational numbers so "guess" is more appropriate to give it the correct limits to find a very close approximation between the left and right boundaries.

$$
\text { 51. } \begin{aligned}
\mathrm{Y}_{1} & =-8 x+6 \\
x & =0.75 \quad y=0
\end{aligned}
$$

52.     - 
53. $\mathrm{Y}_{1}=\frac{3 x+5}{4}$ Round your answer to the nearest thousandth.
$x=-1.667 \quad y=0$

## Extensions

54.     - 
55. If the road was made in the previous exercise, how much shorter would the man's oneway trip be every day?
$15-11.2=3.8 \mathrm{mi}$ shorter
56.     - 
57. After finding the two midpoints in the previous exercise, find the distance between the two midpoints to the nearest thousandth.
$d=\sqrt{(4.5-(-1))^{2}+(1.5-4)^{2}} \quad$ Subtract
$d=\sqrt{(5.5)^{2}+(-2.5)^{2}} \quad$ Square the 5.5 and -2.5
$d=\sqrt{30.25+6.25} \quad$ Add
$d=\sqrt{36.5}$ Evaluate
6.042
58.     - 
59. In the previous exercise, find the coordinates of the midpoint for each diagonal.
$\left(\frac{-6+10}{2}, \frac{5+(-1)}{2}\right) \quad$ Add
$\left(\frac{4}{2}, \frac{4}{2}\right) \quad$ Simplify
$(2,2)$
$\left(\frac{-6+10}{2}, \frac{-1+5}{2}\right) \quad$ Add
$\left(\frac{4}{2}, \frac{4}{2}\right) \quad$ Simplify
$(2,2)$
Midpoint of each diagonal is the same point $(2,2)$. Note this is a characteristic of rectangles, but not other quadrilaterals.

## Real-World Applications

60.     - 
61. If San Jose's coordinates are $(76,-12)$, find the distance between San Jose and San Francisco to the nearest mile.
$d=\sqrt{(53-76)^{2}+(17-(-12))^{2}}$
$d=\sqrt{(-23)^{2}+(29)^{2}}$
$d=\sqrt{529+841}$
$d=\sqrt{1370}$
$d=37.0135110466$
37 mi
62.     - 
63. A man on the top of a building wants to have a guy wire extend to a point on the ground 20 ft from the building. To the nearest foot, how long will the wire have to be if the building is 50 ft tall?


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## Chapter 2

## Equations and Inequalities

### 2.2 Linear Equations in One Variable

## Verbal

1. What does it mean when we say that two lines are parallel?

It means they have the same slope.
2. -
3. How do we recognize when an equation, for example $y=4 x+3$, will be a straight line (linear) when graphed?

The exponent of the $x$ variable is 1 . It is called a first-degree equation.
4. -
5. When solving the following equation:

$$
\frac{2}{x-5}=\frac{4}{x+1}
$$

explain why we must exclude $x=5$ and $x=-1$ as possible solutions from the solution set.

If we insert either value into the equation, they make an expression in the equation undefined (zero in the denominator).

## Algebraic

For the following exercises, solve the equation for $x$.
6. -
7. $4 x-3=5$
$4 x-3=5$ Add 3 to both sides
$4 x=8 \quad$ Divide both sides by 4
$x=2$
8. -
9. $12-5(x+3)=2 x-5$
$12-5(x+3)=2 x-5 \quad$ Distribute the -5
$12-5 x-15=2 x-5 \quad$ Combine like terms
$-5 x-3=2 x-5 \quad$ Subtract 2 x from both sides
$-7 x-3=-5$ Add 3 to both sides
$-7 x=-2 \quad$ Divide both sides by -7
$x=\frac{2}{7}$
10. -
11. $\frac{x}{3}-\frac{3}{4}=\frac{2 x+3}{12}$
$\frac{x}{3}-\frac{3}{4}=\frac{2 x+3}{12} \quad$ Rewrite with a common denominator
$\frac{4}{4} \cdot \frac{x}{3}-\frac{3}{3} \cdot \frac{3}{4}=\frac{2 x+3}{12} \quad$ Multiply
$\frac{4 x}{12}-\frac{9}{12}=\frac{2 x+3}{12} \quad$ Divide both sides by 12
$4 x-9=2 x+3$ Subtract 2 x from both sides
$2 x-9=3$ Add 9 to both sides
$2 x=12$ Divide both sides by 2
$x=6$
12. -
13. $3(2 x-1)+x=5 x+3$
$3(2 x-1)+x=5 x+3 \quad$ Distribute the 3
$6 x-3+x=5 x+3$ Combine like terms
$7 x-3=5 x+3 \quad$ Subtract 5 x from both sides
$2 x-3=3$ Add 3 to both sides
$2 x=6 \quad$ Divide both sides by 2
$x=3$
14. -

$$
\begin{aligned}
& \text { 15. } \frac{x+2}{4}-\frac{x-1}{3}=2 \\
& \frac{x+2}{4}-\frac{x-1}{3}=2 \quad \text { Multiply both sides by } 12 \\
& 12 \cdot \frac{x+2}{4}-12 \cdot \frac{x-1}{3}=24 \quad \text { Simplify } \\
& 3(x+2)-4(x-1)=24 \quad \text { Distribute the } 3 \text { and }-4 \\
& 3 x+6-4 x+4=24 \quad \text { Combine like terms } \\
& -x+10=24 \quad \text { Subtract } 10 \text { from both sides } \\
& -x=14 \quad \text { Divide both sides by }-1 \\
& x=-14
\end{aligned}
$$

For the following exercises, solve each rational equation for $x$. State all $x$-values that are excluded from the solution set.
16. -
17. $2-\frac{3}{x+4}=\frac{x+2}{x+4}$

$$
\begin{aligned}
& 2-\frac{3}{x+4}=\frac{x+2}{x+4} \quad \text { Multiply both sides by } \mathrm{x}+4 \\
& 2(x+4)-\frac{3}{x+4} \cdot x+4=\frac{x+2}{x+4} \cdot x+4 \quad \text { Simplify }
\end{aligned}
$$

$$
2 x+8-3=x+2 \quad \text { Combine like terms }
$$

$$
2 x+5=x+2 \quad \text { Subtract } \mathrm{x} \text { from both sides }
$$

$$
x+5=2 \quad \text { Subtract } 5 \text { from both sides }
$$

$$
x \neq-4 ; x=-3
$$

18.     - 
19. $\frac{3 x}{x-1}+2=\frac{3}{x-1}$
$\frac{3 x}{x-1}+2=\frac{3}{x-1} \quad$ Multiply both sides by $\mathrm{x}-1$
$\frac{3 x}{x-1} \cdot x-1+2(x-1)=\frac{3}{x-1} \cdot x-1 \quad$ Simplify
$3 x+2 x-2=3 \quad$ Combine like terms
$5 x-2=3$ Add 2 to both sides
$5 x=5$ Divide both sides by 5
$x \neq 1$; when we solve this we get $x=1$, which is excluded, therefore NO solution
20.     - 
21. $\frac{1}{x}=\frac{1}{5}+\frac{3}{2 x}$
$\frac{1}{x}=\frac{1}{5}+\frac{3}{2 x} \quad$ Multiply both sides by 10 x
$10 x \cdot \frac{1}{x}=10 x \cdot \frac{1}{5}+10 x \cdot \frac{3}{2 x} \quad$ Simplify
$10=2 x+15 \quad$ Subtract 15 from both sides
$-5=2 x \quad$ Divide both sides by 2
$x \neq 0 ; x=\frac{-5}{2}$

For the following exercises, find the equation of the line using the point-slope formula.
Write all the final equations using the slope-intercept form.
22.
23. $(1,2)$ with a slope of $\frac{-4}{5}$
$m=-\frac{4}{5} \quad$ Use the slope and point in the point-slope formula
$y-2=-\frac{4}{5}(x-1) \quad$ Distribute the $-4 / 5$
$y-2=-\frac{4}{5} x+\frac{4}{5} \quad$ Add 2 to both sides
$y=\frac{-4}{5} x+\frac{14}{5}$
24. -
25. $y$-intercept is 2 , and $(4,-1)$
$(0,2)$ and $(4,-1)$ Use the points to find the slope
$m=\frac{-1-2}{4-0} \quad$ Subtract
$m=\frac{-3}{4} \quad$ Use the slope and the $y$-intercept in the slope-intercept formula
$y=\frac{-3}{4} x+2$
26. -
27. $(1,3)$ and $(5,5)$
$m=\frac{5-3}{5-1} \quad$ Subtract
$m=\frac{2}{4} \quad$ Simplify
$m=\frac{1}{2}$ Use the slope and a point in the point-slope formula
$y-3=\frac{1}{2}(x-1) \quad$ Distribute the $1 / 2$
$y-3=\frac{x}{2}-\frac{1}{2} \quad$ Add 3 to both sides
$y=\frac{1}{2} x+\frac{5}{2}$
28. -
29. perpendicular to $3 y=x-4$ and passes through the point $(-2,1)$
$3 y=x-4 \quad$ Divide both sides by 3
$y=\frac{x}{3}-\frac{4}{3}$ The slope of a perpendicular line is the negative reciprocal of the given slope $m=-3$ Use the slope and the point in the point-slope formula
$y-1=-3(x-(-2)) \quad$ Distribute the -3
$y-1=-3 x-6 \quad$ Add 1 to both sides
$y=-3 x-5$
For the following exercises, find the equation of the line using the given information.
30. -
31. $(1,7)$ and $(3,7)$
$m=\frac{7-7}{3-1} \quad$ Subtract
$m=\frac{0}{2}=0$ Use the slope and a point in the point-slope formula
$y-7=0(x-1) \quad$ Multiply
$y-7=0 \quad$ Add 7 to both sides
$y=7$
32. -
33. The slope equals zero and it passes through the point $(1,-4)$.

$$
y=-4
$$

34.     - 
35. $(-1,3)$ and $(4,-5)$
$m=\frac{-5-3}{4-(-1)} \quad$ Subtract
$m=\frac{-8}{5} \quad$ Use the slope and a point in the point-slope formula $y-3=-\frac{8}{5}(x-(-1)) \quad$ Rewrite
$y-3=-\frac{8}{5}(x+1) \quad$ Distribute the $-8 / 5$
$y-3=-\frac{8}{5} x-\frac{8}{5} \quad$ Add 3 to both sides
$y=-\frac{8}{5} x+\frac{7}{5}$

## Graphical

For the following exercises, graph the pair of equations on the same axes, and state whether they are parallel, perpendicular, or neither.
36. -

## Section 2.2

37. $\begin{aligned} & 3 x-2 y=5 \\ & 6 y-9 x=6\end{aligned}$


Parallel
38. -
39. $\begin{aligned} & x=4 \\ & y=-3\end{aligned}$


## Perpendicular

## Numeric

For the following exercises, find the slope of the line that passes through the given points.
40. -
41. $(-3,2)$ and $(4,-7)$

$$
\begin{aligned}
m & =\frac{-7-2}{4-(-3)} \quad \text { Subtract } \\
m & =\frac{-9}{7}
\end{aligned}
$$

42.     - 
43. $(-1,-2)$ and $(3,4)$

$$
\begin{aligned}
m & =\frac{4-(-2)}{3-(-1)} \quad \text { Subtract } \\
m & =\frac{6}{4} \quad \text { Simplify } \\
m & =\frac{3}{2}
\end{aligned}
$$

44.     - 

For the following exercises, find the slope of the lines that pass through each pair of points and determine whether the lines are parallel or perpendicular.
45. $\begin{gathered}(-1,3) \text { and }(5,1) \\ (-2,3) \text { and }(0,9)\end{gathered}$

$$
\begin{aligned}
& m_{1}=\frac{1-3}{5-(-1)} \quad \text { Subtract } \\
& m_{1}=\frac{-2}{6} \quad \text { Simplify } \\
& m_{1}=-\frac{1}{3} \\
& m_{2}=\frac{9-3}{0-(-2)} \quad \text { Subtract } \\
& m_{2}=\frac{6}{2} \quad \text { Simplify } \\
& m_{2}=3 \\
& m_{1}=\frac{-1}{3}, \quad m_{2}=3 ; \quad \text { Perpendicular. }
\end{aligned}
$$

46.     - 

## Technology

For the following exercises, express the equations in slope intercept form (rounding each number to the thousandths place). Enter this into a graphing calculator as Y1, then adjust the ymin and ymax values for your window to include where the $y$-intercept occurs. State your ymin and ymax values.
47. $0.537 x-2.19 y=100$

$$
y=0.245 x-45.662 . \text { Answers may vary. } y_{\min }=-50, y_{\max }=-40
$$

48.     - 
49. $\frac{200-30 y}{x}=70$

$$
y=-2.333 x+6.667 . \text { Answers may vary. } y_{\min }=-10, y_{\max }=10
$$

## Extensions

50.     - 
51. Starting with the standard form of an equation $\mathrm{A} x+\mathrm{B} y=\mathrm{C}$, solve this expression for $y$ in terms of $A, B, C$, and $x$. Then put the expression in slope-intercept form.
$\mathrm{A} x+\mathrm{B} y=\mathrm{C} \quad$ Subtract Ax from both sides
$B y=C-A x \quad$ Divide both sides by B
$y=\frac{C-A x}{B} \quad$ Rewrite
$y=\frac{-A}{B} x+\frac{C}{B}$
52.     - 
53. Given that the following coordinates are the vertices of a rectangle, prove that this truly is a rectangle by showing the slopes of the sides that meet are perpendicular.
$(-1,1),(2,0),(3,3)$, and $(0,4)$

The slope for $(-1,1)$ to $(0,4)$ is 3 .
The slope for $(-1,1)$ to $(2,0)$ is $\frac{-1}{3}$.
The slope for $(2,0)$ to $(3,3)$ is 3 .
The slope for $(0,4)$ to $(3,3)$ is $\frac{-1}{3}$.
Yes they are perpendicular.
54. -

## Real-World Applications

55. The slope for a wheelchair ramp for a home has to be $\frac{1}{12}$. If the vertical distance from the ground to the door bottom is 2.5 ft , find the distance the ramp has to extend from the home in order to comply with the needed slope.

$2.5=\frac{1}{12} x$
$x=30$
30 ft
56.     - 

For the following exercises, use this scenario: The cost of renting a car is $\$ 45 / \mathrm{wk}$ plus $\$ 0.25 / \mathrm{mi}$ traveled during that week. An equation to represent the cost would be $y=45+.25 x$, where $x$ is the number of miles traveled.
57. What is your cost if you travel 50 mi ?
$y=45+0.25(50)$ Multiply
$y=45+12.5 \quad$ Add
$y=57.5$
$\$ 57.50$
58. -
59. Suppose you have a maximum of $\$ 100$ to spend for the car rental. What would be the maximum number of miles you could travel?
$100>45+0.25 x$ Subtract 45 from both sides
$55>0.25 x \quad$ Divide both sides by 0.25
$220>x$
220 mi
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## Chapter 2

## Equations and Inequalities

### 2.3 Models and Applications

## Verbal

1. To set up a model linear equation to fit real-world applications, what should always be the first step?
Answers may vary. Possible answers: We should define in words what our variable is representing. We should "declare" the variable. A heading.
2.     - 
3. If the total amount of money you had to invest was $\$ 2,000$ and you deposit $x$ amount in one investment, how can you represent the remaining amount?
$2,000-x$
4.     - 
5. If Bill was traveling $v \mathrm{mi} / \mathrm{h}$, how would you represent Daemon's speed if he was traveling $10 \mathrm{mi} / \mathrm{h}$ faster?
$v+10$

## Real-World Applications

For the following exercises, use the information to find a linear algebraic equation model to use to answer the question being asked.
6. -
7. Beth and Ann are joking that their combined ages equal Sam's age. If Beth is twice Ann's age and Sam is 69 yr old, what are Beth and Ann's ages?
$B=$ Beth's age and $A=$ Ann's age
$B=2 A$
$A+B=69 \quad$ Substitute equations
$A+2 A=69$ Combine like terms
$3 A=69$ Divide both sides by 3
$A=23$ Substitute 23 for A in equation 1
$B=2(23) \quad$ Multiply
$B=46$
Ann: 23; Beth: 46
8. -

For the following exercises, use this scenario: Two different telephone carriers offer the following plans that a person is considering. Company A has a monthly fee of $\$ 20$ and charges of $\$ 0.05 / \mathrm{min}$ for calls. Company B has a monthly fee of $\$ 5$ and charges $\$ 0.10 / \mathrm{min}$ for calls.
9. Find the model of the total cost of Company A's plan, using $m$ for the minutes.
$20+0.05 m$
10. -
11. Find out how many minutes of calling would make the two plans equal.
$20+0.05 m=5+0.10 m$ Subtract 0.05 m from both sides
$20=5+0.05 m \quad$ Subtract 5 from both sides
$15=0.05 m \quad$ Divide both sides by 0.05
$m=300$
300 min
12.

For the following exercises, use this scenario: A wireless carrier offers the following plans that a person is considering. The Family Plan: $\$ 90$ monthly fee, unlimited talk and text on up to 8 lines, and data charges of $\$ 40$ for each device for up to 2 GB of data per device. The Mobile Share Plan: $\$ 120$ monthly fee for up to 10 devices, unlimited talk and text for all the lines, and data charges of $\$ 35$ for each device up to a shared total of 10 GB of data. Use $P$ for the number of devices that need data plans as part of their cost.
13. Find the model of the total cost of the Family Plan.
$90+40 P$
14. -
15. Assuming they stay under their data limit, find the number of devices that would make the two plans equal in cost.
$90+40 P=120+35 P$ Subtract 35 P from both sides
$90+5 P=120 \quad$ Subtract 90 from both sides
$5 P=30 \quad$ Divide both sides by 5
$P=6$
6 devices
16. -

For the following exercises, use this scenario: A retired woman has $\$ 50,000$ to invest but needs to make $\$ 6,000$ a year from the interest to meet certain living expenses. One bond investment pays $15 \%$ annual interest. The rest of it she wants to put in a CD that pays $7 \%$.
17. If we let $x$ be the amount the woman invests in the $15 \%$ bond, how much will she be able to invest in the CD?
50,000-x
18. -
19. Two planes fly in opposite directions. One travels $450 \mathrm{mi} / \mathrm{h}$ and the other $550 \mathrm{mi} / \mathrm{h}$. How long will it take before they are $4,000 \mathrm{mi}$ apart?
$450 x+550 x=4000$ Combine like terms
$1000 x=4000 \quad$ Divide both sides by 1000
$x=4$
4 h
20. -
21. Fiora starts riding her bike at $20 \mathrm{mi} / \mathrm{h}$. After a while, she slows down to $12 \mathrm{mi} / \mathrm{h}$, and maintains that speed for the rest of the trip. The whole trip of 70 mi takes her 4.5 h . For what distance did she travel at $20 \mathrm{mi} / \mathrm{h}$ ?
$20 h+12(4.5-h)=70 \quad$ Distribute the 12
$20 h+54-12 h=70 \quad$ Combine like terms
$8 h+54=70 \quad$ Subtract 54 from boht sides
$8 h=16$ Divide both sides by 8
$h=2$
She traveled for 2 h at $20 \mathrm{mi} / \mathrm{h}$, or 40 miles.
22. -
23. Paul has $\$ 20,000$ to invest. His intent is to earn $11 \%$ interest on his investment. He can invest part of his money at $8 \%$ interest and part at $12 \%$ interest. How much does Paul need to invest in each option to make get a total $11 \%$ return on his $\$ 20,000$ ?

| $20000(.11)=2200$ | Set up equation with this as the $11 \%$ |
| :--- | :--- |
| $0.08 x+0.12(20000-x)=2200$ | Distribute the 0.12 |
| $0.08 x+2400-0.12 x=2200$ | Combine like terms |
| $-0.04 x+2400=2200$ | Subtract 2400 from both sides |
| $-0.04 x=-200$ | Divide both sides by -0.04 |
| $x=5000$ |  |
| $\$ 5,000$ at $8 \%$ and $\$ 15,000$ at $12 \%$ |  |

For the following exercises, use this scenario: A truck rental agency offers two kinds of plans. Plan A charges $\$ 75 /$ wk plus $\$ .10 /$ mi driven. Plan B charges $\$ 100 /$ wk plus $\$ .05 / \mathrm{mi}$ driven. 24. -
25. Write the model equation for the cost of renting a truck with plan $B$.

$$
B=100+.05 x
$$

26.     - 
27. If Tim knows he has to travel 300 mi , which plan should he choose?
$A=75+0.1(300)$
$A=75+30$
$A=105$
$B=100+0.05(300)$
$B=100+15$
$B=115$
Plan A

For the following exercises, use the given formulas to answer the questions.
28. -
29. The formula $F=\frac{m v^{2}}{R}$ relates force $(F)$, velocity $(v)$, mass $(m)$, and resistance $(R)$. Find $R$ when $m=45, v=7$, and $F=245$.
$245=\frac{(45)(7)^{2}}{R} \quad$ Square the 7
$245=\frac{(45)(49)}{R} \quad$ Multiply
$245=\frac{2205}{R} \quad$ Multiply both sides by R
$245 R=2205$ Divide both sides by 245
$R=9$
30. -
31. Sum $=\frac{1}{1-r}$ is the formula for an infinite series sum. If the sum is 5 , find $r$.
$5=\frac{1}{1-r} \quad$ Multiply both sides by 1-r
$5(1-r)=1 \quad$ Distribute the 5
$5-5 r=1 \quad$ Subtract 5 from both sides
$-5 r=-4 \quad$ Divide both sides by -5
$r=\frac{4}{5}$ or 0.8

For the following exercises, solve for the given variable in the formula. After obtaining a new version of the formula, you will use it to solve a question.
32. -
33. Use the formula from the previous question to find the width, $W$, of a rectangle whose length is 15 and whose perimeter is 58 .

$$
W=\frac{P-2 L}{2}=\frac{58-2(15)}{2}=14
$$

34. Solve for $f: \frac{1}{p}+\frac{1}{q}=\frac{1}{f}$
35. Use the formula from the previous question to find $f$ when $p=8$ and $q=13$.
36.     - 
37. Use the formula from the previous question to find $m$ when the coordinates of the point are $(4,7)$ and $b=12$. $m=\frac{-5}{4}$
38.     - 
39. Solve for $h$ : $A=\frac{1}{2} h\left(b_{1}+b_{2}\right)$
$A=\frac{1}{2} h\left(b_{1}+b_{2}\right) \quad$ Multiply both sides by 2
$2 A=h\left(b_{1}+b_{2}\right) \quad$ Divide both sides by $b_{1}+b_{2}$
$h=\frac{2 A}{b_{1}+b_{2}}$
40.     - 
41. Find the dimensions of an American football field. The length is 200 ft more than the width, and the perimeter is $1,040 \mathrm{ft}$. Find the length and width. Use the perimeter formula $P=2 L+2 W$.
$L=$ length and $W=$ width
$L=200+W$
$1040=2 L+2 W \quad$ Substitute equations
$1040=2(200+W)+2 W \quad$ Distribute the 2
$1040=400+2 W+2 W \quad$ Combine like terms
$640=4 W \quad$ Divide both sides by 4
$W=160$ Substitute 160 for W in equation 1
$L=200+160 \quad$ Add
$L=360$
length $=360 \mathrm{ft}$; width $=160 \mathrm{ft}$
42.     - 
43. Using the formula in the previous exercise, find the distance that Susan travels if she is moving at a rate of $60 \mathrm{mi} / \mathrm{h}$ for 6.75 h .
$d=60(6.75)$
$d=405$
405 mi
44.     - 
45. If the area model for a triangle is $A=\frac{1}{2} b h$, find the area of a triangle with a height of 16 in. and a base of 11 in .
$A=\frac{1}{2}(11)(16)$
$A=8(11)$
$A=88$
$A=88 \mathrm{in} .{ }^{2}$
46.     - 
47. Use the formula from the previous question to find the height to the nearest tenth of a triangle with a base of 15 and an area of 215 .
$h=\frac{2(215)}{15}$
$h=\frac{430}{15}$
$h=28.7$
28.7
48.     - 
49. Solve for $h: V=\pi r^{2} h$
$V=\pi r^{2} h \quad$ Divide both sides by $\pi$
$\frac{V}{\pi}=r^{2} h \quad$ Divide both sides by $\mathrm{r}^{2}$
$h=\frac{V}{\pi r^{2}}$
50.     - 
51. Solve for $r: V=\pi r^{2} h$
$V=\pi r^{2} h \quad$ Divide both sides by $\pi$
$\frac{V}{\pi}=r^{2} h \quad$ Divide both sides by h
$\frac{V}{\pi h}=r^{2} \quad$ Take the square root of both sides
$r=\sqrt{\frac{V}{\pi h}}$
52.     - 
53. The formula for the circumference of a circle is $C=2 \pi r$. Find the circumference of a circle with a diameter of 12 in . (diameter $=2 r$ ). Use the symbol $\pi$ in your final answer. $C=2 r \pi$ when $2 \mathrm{r}=\mathrm{d}=12$ $C=12 \pi$
54.     - 

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## Chapter 2

## Equations and Inequalities

### 2.4 Complex Numbers

## Verbal

1. Explain how to add complex numbers.

Add the real parts together and the imaginary parts together.
2. -
3. Give an example to show that the product of two imaginary numbers is not always imaginary.
Possible answer: $i$ times $i$ equals -1 , which is not imaginary.
4. -

## Algebraic

For the following exercises, evaluate the algebraic expressions.
5. If $y=x^{2}+x-4$, evaluate $y$ given $x=2 i$.
$y=x^{2}+x-4 \quad$ Substitute $2 i$ for $x$
$y=(2 i)^{2}+(2 i)-4 \quad$ Square the $2 i$
$y=4 i^{2}+2 i-4 \quad$ Evaluate $i^{2}$
$y=4(-1)+2 i-4 \quad$ Multiply
$y=-4+2 i-4 \quad$ Combine like terms
$y=-8+2 i$
$-8+2 i$
6. -
7. If $y=x^{2}+3 x+5$, evaluate $y$ given $x=2+i$.
$y=x^{2}+3 x+5$ Substitute $2+i$ for $x$
$y=(2+i)^{2}+3(2+i)+5$ Expand; Distribute the 3
$y=(2+i)(2+i)+6+3 i+5$ Use the FOIL method; combine like terms
$y=4+2 i+2 i+(-1)+11+3 i \quad$ Combine like terms
$y=14+7 i$
$14+7 i$
8. -
9. If $y=\frac{x+1}{2-x}$, evaluate $y$ given $x=5 i$.
$y=\frac{x+1}{2-x}$ Substitute $5 i$ for $x$
$y=\frac{(5 i)+1}{2-(5 i)}$ Rewrite; multiply top and bottom by the conjugate of the denominator
$y=\frac{1+5 i}{2-5 i} \cdot \frac{2+5 i}{2+5 i} \quad$ Use the FOIL method
$y=\frac{2+5 i+10 i+25\left(i^{2}\right)}{4+10 i-10 i-25\left(i^{2}\right)} \quad$ Combine like terms; evaluate $i^{2}$
$y=\frac{2+15 i+25(-1)}{4-25(-1)} \quad$ Multiply; combine like terms
$y=\frac{-23+15 i}{29} \quad$ Rewrite
$-\frac{23}{29}+\frac{15}{29} i$
10. -

## Graphical

For the following exercises, plot the complex numbers on the complex plane.
11. $1-2 i$

12. -
13. $i$

14. -

## Numeric

For the following exercises, perform the indicated operation and express the result as a simplified complex number.
15. $(3+2 i)+(5-3 i)$
$(3+2 i)+(5-3 i)$ Group the real and imaginary terms
$(3+5)+i(2-3) \quad$ Combine like terms
$8+i(-1) \quad$ Multiply
$8-i$
16. -
17. $(-5+3 i)-(6-i)$
$(-5+3 i)-(6-i) \quad$ Distribute the -1
$(-5+3 i)+(-6+i) \quad$ Group the real and imaginary terms
$(-5-6)+i(3+1) \quad$ Combine like terms
$-11+i(4) \quad$ Multiply
$-11+4 i$
18. -
19. $(-4+4 i)-(-6+9 i)$
$(-4+4 i)-(-6+9 i)$ Distribute the -1
$(-4+4 i)+(6-9 i) \quad$ Group the real and imaginary terms
$(-4+6)+i(4-9) \quad$ Combine like terms
$2+i(-5)$ Multiply
$2-5 i$
20. -
21. $(5-2 i)(3 i)$
$(5-2 i)(3 i)$ Distribute the $3 i$
$15 i-6\left(i^{2}\right) \quad$ Evaluate $i^{2}$
15i-6(-1) Multiply; rewrite
$6+15 i$
22. -
23. $(-2+4 i)(8)$
$(-2+4 i)(8)$ Distribute the 8
$-16+32 i$
24. -
25. $(-1+2 i)(-2+3 i)$
$(-1+2 i)(-2+3 i)$ Use the FOIL method
$2-3 i-4 i+6\left(i^{2}\right)$ Combine like terms; evaluate $i^{2}$
$2-7 i+6(-1)$ Multiply; combine like terms
$-4-7 i$
26. -
27. $(3+4 i)(3-4 i)$
$(3+4 i)(3-4 i)$ Use the FOIL method
$9-12 i+12 i-16\left(i^{2}\right)$ Combine like terms; evaluate $i^{2}$
9-16(-1) Multiply; combine like terms
25
28. -
29. $\frac{6-2 i}{3}$
$\frac{6-2 i}{3}$ Separate the numerators
$\frac{6}{3}-\frac{2 i}{3} \quad$ Simplify
$2-\frac{2}{3} i$
30. -
31. $\frac{6+4 i}{i}$
$\frac{6+4 i}{i}$ Multiply top and bottom by the conjugate of the denominator
$\frac{6+4 i}{i} \cdot \frac{-i}{-i} \quad$ Multiply
$\frac{-6 i-4\left(i^{2}\right)}{-\left(i^{2}\right)} \quad$ Evaluate $i^{2}$
$\frac{-6 i-4(-1)}{-(-1)} \quad$ Multiply
$\frac{4-6 i}{1}$ Rewrite
$4-6 i$
32. -
33. $\frac{3+4 i}{2-i}$
$\frac{3+4 i}{2-i}$ Multiply top and bottom by the conjugate of the denominator
$\frac{3+4 i}{2-i} \cdot \frac{2+i}{2+i}$ Use the FOIL method
$\frac{6+3 i+8 i+4\left(i^{2}\right)}{4+2 i-2 i-\left(i^{2}\right)} \quad$ Combine like terms; evaluate $i^{2}$
$\frac{6+11 i+4(-1)}{4-(-1)}$ Multiply; combine like terms
$\frac{2+11 i}{5}$ Rewrite
$\frac{2}{5}+\frac{11}{5} i$
34. -
35. $\sqrt{-9}+3 \sqrt{-16}$
$\sqrt{-9}+3 \sqrt{-16}$ Multiplication property of radicals
$\sqrt{-1} \sqrt{9}+3 \sqrt{-1} \sqrt{16} \quad 9$ and 16 are perfect squares; evaluate
$3 i+3(4 i)$ Multiply
$3 i+12 i$ Add
$15 i$
36. -
37. $\frac{2+\sqrt{-12}}{2}$
$\frac{2+\sqrt{-12}}{2}$ Multiplication property of radicals
$\frac{2+\sqrt{4} \sqrt{3} \sqrt{-1}}{2} 4$ is a perfect square; evaluate
$\frac{2+2 i \sqrt{3}}{2}$ Separate the numerator
$\frac{2}{2}+\frac{2 i \sqrt{3}}{2} \quad$ Simplify
$1+i \sqrt{3}$
38. -
39. $i^{8}$
$i^{8} \quad$ Rewrite
$\left(i^{4}\right)^{2} \quad$ Evaluate
(1) ${ }^{2}$ Evaluate

1
40. -
41. $i^{22}$
$i^{22} \quad$ Rewrite using exponential properties
$\left(i^{20}\right)\left(i^{2}\right) \quad$ Rewrite using exponential properties; evaluate $i^{2}$
$\left(i^{4}\right)^{5}(-1) \quad$ Evaluate $i^{4}$
$(1)^{5}(-1) \quad$ Multiply
-1

## Technology

For the following exercises, use a calculator to help answer the questions.
42. -
43. Evaluate $(1-i)^{k}$ for $k=2,6$, and 10 . Predict the value if $k=14$.

128i
44. -
45. Show that a solution of $x^{6}+1=0$ is $\frac{\sqrt{3}}{2}+\frac{1}{2} i$.

$$
\left(\frac{\sqrt{3}}{2}+\frac{1}{2} i\right)^{6}=-1
$$

46.     - 

## Extensions

For the following exercises, evaluate the expressions, writing the result as a simplified complex number.
47. $\frac{1}{i}+\frac{4}{i^{3}}$
$\frac{1}{i}+\frac{4}{i^{3}} \quad$ Evaluate $i^{3}$
$\frac{1}{i}+\frac{4}{-i} \quad$ Rewrite
$\frac{1}{i}-\frac{4}{i}$ Subtract
$\frac{-3}{i}$ Multiply top and bottom by the conjugate of the denominator
$\frac{-3}{i} \cdot \frac{-i}{-i} \quad$ Multiply
$\frac{3 i}{-\left(i^{2}\right)}$ Evaluate $\mathrm{i}^{2}$
$\frac{3 i}{-(-1)}$ Simplify
$3 i$
48. -
49. $i^{7}\left(1+i^{2}\right)$
$i^{7}\left(1+i^{2}\right) \quad$ Evaluate $\mathrm{i}^{2}$
$i^{7}(1+(-1))$ Add
$i^{7}(0)$ Multiply
0
50. -
51. $\frac{(2+i)(4-2 i)}{(1+i)}$
$\frac{(2+i)(4-2 i)}{(1+i)}$ Use the FOIL method
$\frac{8-4 i+4 i-2\left(i^{2}\right)}{1+i}$ Combine like terms; evaluate $i^{2}$
$\frac{8-2(-1)}{1+i}$ Multiply and combine like terms
$\frac{10}{1+i}$ Multiply top and bottom by $1+i$
$\frac{10}{1+i} \cdot \frac{1-i}{1-i} \quad$ Multiply
$\frac{10-10 i}{1-i+i-\left(i^{2}\right)}$ Combine like terms; evaluate $i^{2}$
$\frac{10-10 i}{1-(-1)}$ Add
$\frac{10-10 i}{2}$ Rewrite
$\frac{10}{2}-\frac{10 i}{2}$ Simplify
5-5i
52. -
53. $\frac{(3+i)^{2}}{(1+2 i)^{2}}$
$\frac{(3+i)^{2}}{(1+2 i)^{2}} \quad$ Expand
$\frac{(3+i)(3+i)}{(1+2 i)(1+2 i)}$ Use the FOIL method
$\frac{9+3 i+3 i+\left(i^{2}\right)}{1+2 i+2 i+4\left(i^{2}\right)}$ Combine like terms; evaluate $i^{2}$
$\frac{9+6 i+(-1)}{1+4 i+4(-1)} \quad$ Multiply; combine like terms
$\frac{8+6 i}{-3+4 i}$ Multiply top and bottom by the conjugate of the denominator
$\frac{8+6 i}{-3+4 i} \cdot \frac{-3-4 i}{-3-4 i}$ Multiply
$\frac{-24-32 i-18 i-24\left(i^{2}\right)}{9+12 i-12 i-16\left(i^{2}\right)} \quad$ Combine like terms; evaluate $i^{2}$
$\frac{-24-50 i-24(-1)}{9-16(-1)}$ Multiply; combine like terms
$\frac{-50 i}{25}$ Simplify
$-2 i$
54. -
55. $\frac{4+i}{i}+\frac{3-4 i}{1-i}$
$\frac{4+i}{i}+\frac{3-4 i}{1-i} \quad$ Multiply top and bottom by the conjugate of the denominator $\frac{4+i}{i} \cdot \frac{-i}{-i}+\frac{3-4 i}{1-i} \cdot \frac{1+i}{1+i}$ Use the FOIL method
$\frac{-4 i-\left(i^{2}\right)}{-\left(i^{2}\right)}+\frac{3+3 i-4 i-4\left(i^{2}\right)}{1+i-i-\left(i^{2}\right)}$ Combine like terms; evaluate $i^{2}$
$\frac{-4 i-(-1)}{-(-1)}+\frac{3-i-4(-1)}{1-(-1)} \quad$ Combine like terms
$\frac{1-4 i}{1}+\frac{7-i}{2}$ Multiply top and bottom by 2
$\frac{2}{2} \cdot(1-4 i)+\frac{7-i}{2} \quad$ Multiply
$\frac{2-8 i}{2}+\frac{7-i}{2}$ Add
$\frac{9-9 i}{2}$ Rewrite
$\frac{9}{2}-\frac{9}{2} i$
56. -

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## Chapter 2

## Equations and Inequalities

### 2.5 Quadratic Equations

## Verbal

1. How do we recognize when an equation is quadratic?

It is a second-degree equation (the highest variable exponent is 2 ).
2. -
3. When we solve a quadratic equation by factoring, why do we move all terms to one side, having zero on the other side?
We want to take advantage of the zero property of multiplication in the fact that if $a \cdot b=0$ then it must follow that each factor separately offers a solution to the product being zero: $a=0$ or $\mathrm{b}=0$.
4. -
5. Describe two scenarios where using the square root property to solve a quadratic equation would be the most efficient method.
One, when no linear term is present (no $x$ term), such as $x^{2}=16$. Two, when the equation is already in the form $(a x+b)^{2}=d$.

## Algebraic

For the following exercises, solve the quadratic equation by factoring.
6. -
7. $x^{2}-9 x+18=0$
$x^{2}-9 x+18=0 \quad$ Rewrite $-9 x$ as $-6 x-3 x$
$\left(x^{2}-6 x\right)+(-3 x+18)=0 \quad$ Factor out the GCF of each expression
$x(x-6)-3(x-6)=0 \quad$ Use the distributive property
$(x-3)(x-6)=0 \quad$ Use the zero-product property
$x-3=0$
$x=3$
$x-6=0$
$x=6$
$x=6, x=3$
8. -
9. $6 x^{2}+17 x+5=0$
$6 x^{2}+17 x+5=0$
$\left(6 x^{2}+15 x\right)+(2 x+5)=0 \quad$ Factor out the GCF of each expression
$3 x(2 x+5)+(2 x+5)=0 \quad$ Use the distributive property
$(3 x+1)(2 x+5)=0$
$3 x+1=0$
$3 x=-1$
$x=-\frac{1}{3}$
$2 x+5=0$
$2 x=-5$
$x=-\frac{5}{2}$
$x=\frac{-5}{2}, x=\frac{-1}{3}$
10. -
11. $3 x^{2}-75=0$
$3 x^{2}-75=0 \quad$ Factor out the GCF of the expression
$3\left(x^{2}-25\right)=0 \quad$ Divide both sides by 3
$x^{2}-25=0 \quad$ Difference of squares
$(x-5)(x+5)=0 \quad$ Use the zero-product property
$x-5=0 \quad$ Solve for $x$
$x=5$
$x+5=0$
$x=-5$
$x=5, x=-5$
12. -

$$
\begin{aligned}
& \text { 13. } 4 x^{2}=9 \\
& 4 x^{2}=9 \quad \text { Write in standard form by subtracting } 9 \text { from both sides } \\
& 4 x^{2}-9=0 \quad \text { Difference of squares } \\
& (2 x-3)(2 x+3)=0 \quad \text { Use the zero-product property } \\
& 2 x-3=0 \\
& 2 x=3 \\
& x=\frac{3}{2} \\
& 2 x+3=0 \\
& 2 x=-3 \\
& x=-\frac{3}{2} \\
& x=\frac{-3}{2}, x=\frac{3}{2} \\
& \text { 14. - } \\
& \text { 15. } 5 x^{2}=5 x+30 \\
& 5 x^{2}=5 x+30 \\
& 5 x^{2}-5 x-30=0 \\
& \left(5 x^{2}-15 x\right)+(10 x-30)=0 \\
& 5 x(x-3)+10(x-3)=0 \quad \text { Use the distributive property } \\
& (5 x+10)(x-3)=0 \\
& 5 x+10=0 \\
& 5 x=-10 \\
& x=-2 \\
& x-3=0 \\
& x=3 \\
& x=-2, x=3
\end{aligned}
$$

$$
\begin{array}{ll}
\text { 17. } \begin{array}{ll}
7 x^{2}+3 x=0 & \\
7 x^{2}+3 x=0 & \text { Factor out the GCF of the expression } \\
x(7 x+3)=0 & \text { Use the zero-product property } \\
x=0 & \text { Solve for } x \\
7 x+3=0 & \\
7 x=-3 & \\
x=-\frac{3}{7} & \\
x=0, x=\frac{-3}{7} &
\end{array} \\
\text { 18. - }
\end{array}
$$

For the following exercises, solve the quadratic equation by using the square root property.
19. $x^{2}=36$
$x^{2}=36 \quad$ Take the square roots of both sides
$\sqrt{x^{2}}=\sqrt{36}$ Use the square root property
$x= \pm 6$
$x=-6, x=6$
20. -
21. $(x-1)^{2}=25$
$(x-1)^{2}=25 \quad$ Take the square root of both sides
$\sqrt{(x-1)^{2}}=\sqrt{25} \quad$ Use the square root property
$x-1= \pm 5 \quad$ Add 1 to both sides
$x=1 \pm 5 \quad$ Simplify
$x=6$ and $x=-4$
$x=6, x=-4$
22. -
23. $(2 x+1)^{2}=9$

$$
\begin{array}{ll}
(2 x+1)^{2}=9 & \text { Take the square root of both sides } \\
\sqrt{(2 x+1)^{2}}=\sqrt{9} & \text { Use the square root property } \\
2 x+1= \pm 3 & \text { Subtract } 1 \text { from both sides } \\
2 x=-1 \pm 3 & \text { Divide both sides by } 2 \\
x=\frac{-1 \pm 3}{2} & \text { Simplify } \\
x=1 \text { and } x=-2 & \\
x=1, x=-2 &
\end{array}
$$

24.     - 

For the following exercises, solve the quadratic equation by completing the square. Show each step.

| 25. $x^{2}-9 x-22=0$ |  |
| :---: | :---: |
| $x^{2}-9 x-22=0$ | Use the associative property |
| $\left(x^{2}-9 x\right)-22=0$ | Create a third term |
| $\left(x-9 x+\frac{81}{4}\right)-22-\frac{81}{4}=0$ | Rewrite and simplify |
| $\left(x-\frac{9}{2}\right)^{2}-\frac{169}{4}=0$ | Add 169/4 to both sides |
| $\left(x-\frac{9}{2}\right)^{2}=\frac{169}{4}$ | Take the square root of both sides |
| $\sqrt{\left(x-\frac{9}{2}\right)^{2}}=\sqrt{\frac{169}{4}}$ | Use the square root property |
| $x-\frac{9}{2}= \pm \frac{13}{2}$ | Add 9/2 to both sides |
| $x=\frac{9}{2} \pm \frac{13}{2}$ | Simplify |
| $\begin{gathered} x=11 \text { and } x=-2 \\ x=-2, x=11 \end{gathered}$ |  |

26.     - 
27. $x^{2}-6 x=13$
$x^{2}-6 x=13$
$\left(x^{2}-6 x+9-9\right)=13 \quad$ Rewrite
$\left(x^{2}-6 x+9\right)-9=13 \quad$ Rewrite
$(x-3)^{2}-9=13$
Add 9 to both sides
$(x-3)^{2}=22$
Take the square root of both sides
$\sqrt{(x-3)^{2}}=\sqrt{22} \quad$ Use the square root property
$x-3= \pm \sqrt{22} \quad$ Add 3 to both sides
$x=3 \pm \sqrt{22}$
$x=3 \pm \sqrt{22}$
28.     - 
29. $2+z=6 z^{2}$
$2+z=6 z^{2} \quad$ Rewrite and subtract z from both sides
$6 z^{2}-z=2 \quad$ Factor out a 6
$6\left(z^{2}-\frac{z}{6}\right)=2 \quad$ Create an extra term
$6\left(z^{2}-\frac{z}{6}+\frac{1}{144}-\frac{1}{144}\right)=2 \quad$ Rewrite
$6\left(z^{2}-\frac{z}{6}+\frac{1}{144}\right)-\frac{6}{144}=2 \quad$ Rewrite
$6\left(z-\frac{1}{12}\right)^{2}-\frac{6}{144}=2 \quad$ Add $6 / 144$ to both sides
$6\left(z-\frac{1}{12}\right)^{2}=\frac{294}{144} \quad$ Divide both sides by 6
$\left(z-\frac{1}{12}\right)^{2}=\frac{49}{144} \quad$ Take the square root of both sides
$\sqrt{\left(z-\frac{1}{12}\right)^{2}}=\sqrt{\frac{49}{144}} \quad$ Use the square root property
$z-\frac{1}{12}= \pm \frac{7}{12} \quad$ Add $1 / 12$ to both sides
$z=\frac{1}{12} \pm \frac{7}{12} \quad$ Simplify
$z=\frac{2}{3}$ and $z=-\frac{1}{2}$
$z=\frac{2}{3}, z=-\frac{1}{2}$
30.     - 

$\begin{array}{ll}\text { 31. } 2 x^{2}-3 x-1=0 & \\ 2 x^{2}-3 x-1=0 & \text { Factor out the } 2 \\ 2\left(x^{2}-\frac{3}{2} x\right)-1=0 & \text { Create an extra term } \\ 2\left(x^{2}-\frac{3}{2} x+\frac{9}{16}-\frac{9}{16}\right)-1=0 & \text { Rewrite } \\ 2\left(x^{2}-\frac{3}{2} x+\frac{9}{16}\right)-1-\frac{9}{8}=0 & \text { Rewrite } \\ 2\left(x-\frac{3}{4}\right)^{2}-\frac{17}{8}=0 & \text { Add } \frac{17}{8} \text { to both sides } \\ 2\left(x-\frac{3}{4}\right)^{2}=\frac{17}{8} & \text { Divide both sides by } 2 \\ \left(x-\frac{3}{4}\right)^{2}=\frac{17}{16} & \text { Take the square root of both sides } \\ \sqrt{\left(x-\frac{3}{4}\right)^{2}}=\sqrt{\frac{17}{16}} & \text { Use the square root property } \\ x-\frac{3}{4}= \pm \frac{\sqrt{17}}{4} & \text { Add } 3 / 4 \text { to both sides } \\ x=\frac{3}{4} \pm \frac{\sqrt{17}}{4} & \text { Simplify } \\ x=\frac{3 \pm \sqrt{17}}{4} & \\ x=\frac{3 \pm \sqrt{17}}{4} & \end{array}$
For the following exercises, determine the discriminant, and then state how many solutions there are and the nature of the solutions. Do not solve.
32. -
33. $x^{2}+4 x+7=0$
$b^{2}-4 a c \quad$ Substitute the correct values
$(4)^{2}-4(1)(7) \quad$ Multiply
16-28 Subtract
$-12<0 \quad$ Compare to zero
Not real
34.
35. $9 x^{2}-30 x+25=0$
$b^{2}-4 a c$
Substitute the correct values
$(-30)^{2}-4(9)(25)$
Multiply
900-900
Subtract
$0=0$
Compare to zero
One rational
36.
37. $6 x^{2}-x-2=0$
$b^{2}-4 a c \quad$ Substitute the correct values
$(-1)^{2}-4(6)(-2) \quad$ Multiply
$1+48 \quad$ Add
$49>0 \quad$ Compare to zero; 49 is a perfect square
Two real; rational

For the following exercises, solve the quadratic equation by using the quadratic formula. If the solutions are not real, state No Real Solution.
38. -
39. $x^{2}+x=4$
$x^{2}+x=4 \quad$ Subtract 4 from both sides
$x^{2}+x-4=0 \quad$ Substitute the correct values
$x=\frac{-(1) \pm \sqrt{(1)^{2}-4(1)(-4)}}{2(1)} \quad$ Multiply
$x=\frac{-1 \pm \sqrt{1+16}}{2} \quad$ Add
$x=\frac{-1 \pm \sqrt{17}}{2}$
$x=\frac{-1 \pm \sqrt{17}}{2}$
40. -
41. $3 x^{2}-5 x+1=0$
$3 x^{2}-5 x+1=0 \quad$ Substitute the correct values
$x=\frac{-(-5) \pm \sqrt{(-5)^{2}-4(3)(1)}}{2(3)} \quad$ Multiply
$x=\frac{5 \pm \sqrt{25-12}}{6} \quad$ Subtract
$x=\frac{5 \pm \sqrt{13}}{6}$
$x=\frac{5 \pm \sqrt{13}}{6}$
42. -
43. $4+\frac{1}{x}-\frac{1}{x^{2}}=0$
$4+\frac{1}{x}-\frac{1}{x^{2}}=0 \quad$ Multiply both sides by $\mathrm{x}^{2}$
$4 x^{2}+x-1=0 \quad$ Substitute the correct values
$x=\frac{-(1) \pm \sqrt{(1)^{2}-4(4)(-1)}}{2(4)} \quad$ Multiply
$x=\frac{-1 \pm \sqrt{1+}}{8}$
$x=\frac{-1 \pm \sqrt{17}}{8}$
$x=\frac{-1 \pm \sqrt{17}}{8}$

## Technology

For the following exercises, enter the expressions into your graphing utility and find the zeroes to the equation (the $x$-intercepts) by using $2^{\text {nd }}$ CALC 2:zero. Recall finding zeroes will ask left bound (move your cursor to the left of the zero,enter), then right bound (move your cursor to the right of the zero,enter), then guess (move your cursor between the bounds near the zero, enter).
Round your answers to the nearest thousandth.
44. -
45. $\mathrm{Y}_{1}=-3 x^{2}+8 x-1$

$$
x \approx 0.131 \text { and } x \approx 2.535
$$

46.     - 
47. To solve the quadratic equation $x^{2}+5 x-7=4$, we can graph these two equations $\mathrm{Y}_{1}=x^{2}+5 x-7$
$Y_{2}=4$
and find the points of intersection. Recall $2^{\text {nd }}$ CALC 5:intersection. Do this and find the solutions to the nearest tenth.

$$
x \approx-6.7 \text { and } x \approx 1.7
$$

48.     - 

## Extensions

49. Beginning with the general form of a quadratic equation, $a x^{2}+b x+c=0$, solve for $x$ by using the completing the square method, thus deriving the quadratic formula.

$$
a x^{2}+b x+c=0 \quad \text { Subtract both sides by } \mathrm{c} \text { and then divide both sides by a }
$$

$$
x^{2}+\frac{b}{a} x=\frac{-c}{a} \quad \text { Create an extra term on both sides }
$$

$$
x^{2}+\frac{b}{a} x+\frac{b^{2}}{4 a^{2}}=\frac{-c}{a}+\frac{b^{2}}{4 a^{2}} \quad \text { Rewrite }
$$

$$
\left(x+\frac{b}{2 a}\right)^{2}=\frac{b^{2}-4 a c}{4 a^{2}} \quad \text { Take the square root of both sides }
$$

$$
x+\frac{b}{2 a}= \pm \sqrt{\frac{b^{2}-4 a c}{4 a^{2}}} \quad \text { Subtract }-\frac{b}{2 a} \text { from both sides }
$$

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

50.     - 
51. A person has a garden that has a length 10 feet longer than the width. Set up a quadratic equation to find the dimensions of the garden if its area is $119 \mathrm{ft} .^{2}$. Solve the quadratic equation to find the length and width.
$x(x+10)=119$
Use the distributive property
$x^{2}+10 x=119 \quad$ Subtract 119 from both sides
$x^{2}+10 x-119=0 \quad$ Rewrite 10 x as $17 \mathrm{x}-7 \mathrm{x}$
$\left(x^{2}+17 x\right)+(-7 x-119)=0 \quad$ Factor out the GCF of each expression
$x(x+17)-7(x+17)=0 \quad$ Use the distributive property
$(x-7)(x+17)=0 \quad$ Use the zero-product property
$x-7=0 \quad$ Solve for x
$x=7$
$7+10=17$
$x(x+10)=119 ; 7 \mathrm{ft}$. and 17 ft.
52. 
53. Suppose that an equation is given $p=-2 x^{2}+280 x-1000$, where $x$ represents the number of items sold at an auction and $p$ is the profit made by the business that ran the auction. How many items sold would make this profit a maximum? Solve this by graphing the expression in your graphing utility and finding the maximum using $2^{\text {nd }}$ CALC maximum. To obtain a good window for the curve, set $x[0,200]$ and $y$ [0,10000].
maximum at $x=70$

## Real World Applications

54.     - 
55. The cost function for a certain company is $C=60 x+300$ and the revenue is given by $R=100 x-0.5 x^{2}$. Recall that profit is revenue minus cost. Set up a quadratic equation and find two values of $x$ (production level) that will create a profit of $\$ 300$.
The quadratic equation would be:
$\left(100 x-0.5 x^{2}\right)-(60 x+300)=300 \quad$ Distribute the minus sign
$100 x-0.5 x^{2}-60 x-300=300 \quad$ Combine like terms; Subtract 300 from both sides
$-0.5 x^{2}+40 x-600=0 \quad$ Rewrite 40 x as $30 \mathrm{x}+10 \mathrm{x}$
$\left(-0.5 x^{2}+30 x\right)+(10 x-600)=0 \quad$ Factor out the GCF of each expression
$-0.5 x(x-60)+10(x-60)=0 \quad$ Use the distributive property
$(-0.5 x+10)(x-60)=0 \quad$ Use the zero-product property
$-0.5 x+10=0 \quad$ Solve for x
$-0.5 x=-10$
$x=20$
$x-60=0$
$x=60$
The two values of $x$ are 20 and 60 .
56.     - 
57. A vacant lot is being converted into a community garden. The garden and the walkway around its perimeter have an area of $378 \mathrm{ft}^{2}$. Find the width of the walkway if the garden
is 12 ft . wide by 15 ft . long.


$$
\begin{array}{ll}
A=(12+2 x)(15+2 x) \\
180+24 x+30 x+4 x^{2}=A & \text { Use the FOIL method } \\
4 x^{2}+54 x+180=A & \text { Combine like terms } \\
4 x^{2}+54 x+180=378 \\
4 x^{2}+54 x-198=0 & \text { Substitute } 378 \text { for A } \\
\left(4 x^{2}+66 x\right)+(-12 x-198)=0 & \text { Subtract } 378 \text { from both sides } \\
2 x(2 x+33)-6(2 x+33)=0 & \text { Fewrite } 54 \mathrm{x} \text { as } 66 \mathrm{x}-12 \mathrm{x} \\
(2 x-6)(2 x+33)=0 & \text { Use the distributive property } \\
2 x-6=0 & \text { Use the zero-product property } \\
2 x=6 & \text { Solve for } \mathrm{x} \\
x=3 & \\
3 \text { feet } & \\
58 .- &
\end{array}
$$

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## Chapter 2

## Equations and Inequalities

2.6 Other Types of Equations

## Verbal

1. In a radical equation, what does it mean if a number is an extraneous solution?

This is not a solution to the radical equation, it is a value obtained from squaring both sides and thus changing the signs of an equation which has caused it not to be a solution in the original equation.
2. -
3. Your friend tries to calculate the value $-9^{\frac{3}{2}}$ and keeps getting an ERROR message. What mistake is he or she probably making?
He or she is probably trying to enter negative 9 , but taking the square root of -9 is not a real number. The negative sign is in front of this, so your friend should be taking the square root of 9 , cubing it, and then putting the negative sign in front, resulting in -27 .
4. -
5. Explain how to change a rational exponent into the correct radical expression.

A rational exponent is a fraction: the denominator of the fraction is the root or index number and the numerator is the power to which it is raised.

## Algebraic

For the following exercises, solve the rational exponent equation. Use factoring where necessary.
6. -
7. $x^{\frac{3}{4}}=27$
$x^{\frac{3}{4}}=27 \quad$ Rewrite
$\sqrt[4]{x^{3}}=27 \quad$ Raise both sides to the 4th power
$\left(\sqrt[4]{x^{3}}\right)^{4}=27^{4} \quad$ Take the cube root of both sides
$\sqrt[3]{x^{3}}=\sqrt[3]{27^{4}} \quad$ Evaluate the cube root
$x=3^{4} \quad$ Evaluate
$x=81$
8. -
9. $(x-1)^{\frac{3}{4}}=8$

$$
\begin{array}{ll}
(x-1)^{\frac{3}{4}}=8 & \text { Rewrite } \\
\sqrt[4]{(x-1)^{3}}=8 & \text { Raise both sides to the 4th power } \\
\left(\sqrt[4]{(x-1)^{3}}\right)^{4}=8^{4} & \text { Take the cube root of both sides } \\
\sqrt[3]{(x-1)^{3}}=\sqrt[3]{8^{4}} & \text { Evaluate the cube root } \\
x-1=2^{4} & \text { Evaluate } \\
x-1=16 & \text { Add 1 to both sides } \\
x=17 & \\
\text { 10. }- \\
\text { 11. } x^{\frac{2}{3}}-5 x^{\frac{1}{3}}+6=0 &
\end{array}
$$

10.     - 

$$
\begin{array}{ll}
x^{\frac{2}{3}}-5 x^{\frac{1}{3}}+6=0 & \text { Rewrite }-5 x^{\frac{1}{3}} \text { as }-3 x^{\frac{1}{3}}-2 x^{\frac{1}{2}} \\
\left(x^{\frac{2}{3}}-3 x^{\frac{1}{3}}\right)+\left(-2 x^{\frac{1}{3}}+6\right)=0 & \text { Factor out the GCF of each expression } \\
x^{\frac{1}{3}}\left(x^{\frac{1}{3}}-3\right)-2\left(x^{\frac{1}{3}}-3\right)=0 & \text { Use the distributive property } \\
\left(x^{\frac{1}{3}}-2\right)\left(x^{\frac{1}{3}}-3\right)=0 & \text { Use the zero-product property } \\
x^{\frac{1}{3}}-2=0 & \text { Solve for } x \\
\left(x^{\frac{1}{3}}\right)^{3}=2^{3} & \\
x=8 & \\
x^{\frac{1}{3}}-3=0 & \\
x^{\frac{1}{3}}=3 & \\
\left(x^{\frac{1}{3}}\right)^{3}=3^{3} & \\
x=27 & \\
x=8, x=27 &
\end{array}
$$

12.     - 

For the following exercises, solve the following polynomial equations by grouping and factoring.

$$
\begin{aligned}
& \text { 13. } x^{3}+2 x^{2}-x-2=0 \\
& x^{3}+2 x^{2}-x-2=0 \quad \text { Group the terms to create two expressions } \\
& \left(x^{3}+2 x^{2}\right)+(-x-2)=0 \quad \text { Factor out the GCF of each expression } \\
& x^{2}(x+2)-(x+2)=0 \quad \text { Use the distributive property } \\
& \left(x^{2}-1\right)(x+2)=0 \quad \text { Difference of squares } \\
& (x+1)(x-1)(x+2)=0 \quad \text { Use the zero-product property } \\
& x+1=0 \\
& \text { Solve for } x \\
& x=-1 \\
& x-1=0 \\
& x=1 \\
& x+2=0 \\
& x=-2 \\
& x=-2,1,-1 \\
& \text { 14. - } \\
& \text { 15. } 4 y^{3}-9 y=0 \\
& 4 y^{3}-9 y=0 \quad \text { Factor out the } \mathrm{y} \\
& y\left(4 y^{2}-9\right)=0 \quad \text { Difference of squares } \\
& y(2 y-3)(2 y+3)=0 \quad \text { Use the zero-product property } \\
& y=0 \quad \text { Solve for } y \\
& 2 y-3=0 \\
& 2 y=3 \\
& y=\frac{3}{2} \\
& 2 y+3=0 \\
& 2 y=-3 \\
& y=-\frac{3}{2} \\
& y=0, \frac{3}{2}, \frac{-3}{2}
\end{aligned}
$$

16.     - 
17. $m^{3}+m^{2}-m-1=0$
$m^{3}+m^{2}-m-1=0 \quad$ Group the terms to create two expressions
$\left(m^{3}+m^{2}\right)+(-m-1)=0 \quad$ Factor out the GCF of each expression
$m^{2}(m+1)-(m+1)=0 \quad$ Use the distributive property
$\left(m^{2}-1\right)(m+1)=0 \quad$ Difference of squares
$(m-1)(m+1)(m+1)=0 \quad$ Rewrite
$(m-1)(m+1)^{2}=0 \quad$ Use the zero-product property
$m-1=0 \quad$ Solve for $m$
$m=1$
$m+1=0$
$m=-1$
$m=1,-1$
18.     - 
19. $5 x^{3}+45 x=2 x^{2}+18$
$5 x^{3}+45 x=2 x^{2}+18 \quad$ Rewrite
$5 x^{3}-2 x^{2}+45 x-18=0 \quad$ Group the terms to create two expressions
$\left(5 x^{3}-2 x^{2}\right)+(45 x-18)=0 \quad$ Factor out the GCF of each expression
$x^{2}(5 x-2)+9(5 x-2)=0 \quad$ Use the distributive property
$\left(x^{2}+9\right)(5 x-2)=0 \quad$ Use the zero-product property
$5 x-2=0 \quad$ Solve for $x$
$5 x=2$
$x=\frac{2}{5}$
$x^{2}+9=0$
$x^{2}=-9$
$\sqrt{x^{2}}=\sqrt{-9} \quad$ The square root of a negative number is not a real number $x=\frac{2}{5}$

For the following exercises, solve the radical equation. Be sure to check all solutions to eliminate extraneous solutions.
20. -

$$
\begin{aligned}
& \text { 21. } \sqrt{x-7}=5 \\
& \sqrt{x-7}=5 \quad \text { Square both sides } \\
& (\sqrt{x-7})^{2}=5^{2} \quad \text { Evaluate } \\
& x-7=25 \quad \text { Add } 7 \text { to both sides } \\
& x=32 \quad \text { Substitute to check answer } \\
& \sqrt{32-7}=5 \\
& \sqrt{25}=5 \\
& 5=5 \\
& x=32 \\
& \text { 22. - } \\
& \text { 23. } \sqrt{3 t+5}=7 \\
& (\sqrt{3 t+5})^{2}=7^{2} \quad \text { Evaluate } \\
& 3 t+5=49 \quad \text { Subtract } 5 \text { from both sides } \\
& 3 t=44 \quad \text { Divide both sides by } 3 \\
& t=\frac{44}{3} \quad \text { Substitute to check answer } \\
& \sqrt{3\left(\frac{44}{3}\right)+5}=7 \\
& \sqrt{44+5}=7 \\
& \sqrt{49}=7 \\
& 7=7 \\
& t=\frac{44}{3}
\end{aligned}
$$

24.     - 
25. $\sqrt{12-x}=x$
$\sqrt{12-x}=x$
$(\sqrt{12-x})^{2}=x^{2}$
$12-x=x^{2}$
$x^{2}+x-12=0$
$\left(x^{2}+4 x\right)+(-3 x-12)=0$
$x(x+4)-3(x+4)=0$
$(x-3)(x+4)=0$
$x-3=0$
$x=3$
$x+4=0$
$x=-4$
$\sqrt{12-3}=3$
$\sqrt{9}=3$
$3=3$
$\sqrt{12-(-4)}=-4$ $x=3$
26.     - 
27. $\sqrt{3 x+7}+\sqrt{x+2}=1$

Square both sides
Evaluate
Rewrite
Rewrite $x$ as $4 x-3 x$
Factor out the GCF of each expression
Use the distributive property
Use the zero-product property
Solve for x

Substitute to check answers

Note that the square root is equal to a negative value

| $\sqrt{3 x+7}+\sqrt{x+2}=1$ | Square both sides |
| :--- | :--- |
| $(\sqrt{3 x+7}+\sqrt{x+2})^{2}=1^{2}$ | Expand |
| $(\sqrt{3 x+7}+\sqrt{x+2})(\sqrt{3 x+7}+\sqrt{x+2})=1$ | Use the FOIL method |
| $3 x+7+2 \sqrt{3 x^{2}+7 x+6 x+14}+x+2=1$ | Combine like terms |
| $4 x+9+2 \sqrt{3 x^{2}+13 x+14}=1$ | Rewrite |
| $2 \sqrt{3 x^{2}+13 x+14}=-4 x-8$ | Square both sides |
| $\left(2 \sqrt{3 x^{2}+13 x+14}\right)^{2}=(-4 x-8)^{2}$ | Use the FOIL method |
| $4\left(3 x^{2}+13 x+14\right)=16 x^{2}+32 x+32 x+64$ | Distribute the 4; Combine like terms |
| $12 x^{2}+52 x+56=16 x^{2}+64 x+64$ | Rewrite |
| $4 x^{2}+12 x+8=0$ | Factor out the 4 |
| $4\left(x^{2}+3 x+2\right)=0$ | Divide both sides by 4 |
| $x^{2}+3 x+2=0$ | Factor |
| $(x+2)(x+1)=0$ | Use the zero-product property |
| $x+2=0$ | Solve for $x$ |
| $x=-2$ |  |
| $x+1=0$ |  |
| $x=-1$ |  |
| $\sqrt{3(-2)+7}+\sqrt{(-2)+2}=1$ |  |
| $\sqrt{1}+\sqrt{0}=1$ |  |
| $1+0=1$ |  |
| $1=1$ |  |
| $\sqrt{3(-1)+7}+\sqrt{(-1)+2}=1$ |  |
| $\sqrt{4}+\sqrt{1}=1$ |  |
| $3 \neq 1=1$ |  |
| $x=-2$ |  |
| - |  |
|  |  |

28.     - 

For the following exercises, solve the equation involving absolute value.
29. $|3 x-4|=8$

$$
\begin{aligned}
& 3 x-4=8 \text { and } 3 x-4=-8 \\
& 3 x-4=8 \\
& 3 x=12 \\
& x=4 \\
& 3 x-4=-8 \\
& 3 x=-4 \\
& x=-\frac{4}{3} \\
& x=4, \frac{-4}{3}
\end{aligned}
$$

30.     - 
31. $|1-4 x|-1=5$

$$
|1-4 x|-1=5
$$

$$
|1-4 x|=6
$$

$$
1-4 x=6 \text { and } 1-4 x=-6
$$

$$
-4 x=5
$$

$$
x=-\frac{5}{4}
$$

$$
1-4 x=-6
$$

$$
-4 x=-7
$$

$$
x=\frac{7}{4}
$$

$$
x=\frac{-5}{4}, \frac{7}{4}
$$

32.     - 
33. $|2 x-1|-7=-2$
$|2 x-1|=5$
$2 x-1=5$ and $2 x-1=-5$
$2 x-1=5$
$2 x=6$
$x=3$
$2 x-1=-5$
$2 x=-4$
$x=-2$
$x=3,-2$
34.     - 
35. $|x+5|=0$
$x+5=0$
$x=-5$
36.     - 

For the following exercises, solve the equation by identifying the quadratic form. Use a substitute variable and find all real solutions by factoring.

```
37. \(x^{4}-10 x^{2}+9=0\)
    Use \(u=x^{2}\) and rewrite
    \(u^{2}-10 u+9=0 \quad\) Factor
    \((u-9)(u-1)=0 \quad\) Use the zero-product property
    \(u-9=0 \quad\) Solve for u then x
    \(u=9\)
    \(x^{2}=9\)
    \(\sqrt{x^{2}}=\sqrt{9}\)
    \(x= \pm 3\)
    \(u-1=0\)
    \(u=1\)
    \(x^{2}=1\)
    \(\sqrt{x^{2}}=\sqrt{1}\)
    \(x= \pm 1\)
    \(x=1,-1,3,-3\)
```

38.     - 
39. $\left(x^{2}-1\right)^{2}+\left(x^{2}-1\right)-12=0$

Use $u=x^{2}-1$ and rewrite
$u^{2}+u-12=0$
$(u+4)(u-3)=0$
$u+4=0$
$u=-4$
$x^{2}-1=-4$
$x^{2}=-3$
$\sqrt{x^{2}}=\sqrt{-3}$
$u-3=0$
$u=3$
$x^{2}-1=3$
$x^{2}=4$
$\sqrt{x^{2}}=\sqrt{4}$
$x= \pm 2$
$x=2,-2$
40. -
41. $(x-3)^{2}-4=0$

Use $u=x-3$ and rewrite
$u^{2}-4=0$
Difference of squares
$(u-2)(u+2)=0$
Use the zero-product property
$u-2=0$
$u=2$
$x-3=2$
$x=5$
$u+2=0$
$u=-2$
$x-3=-2$
$x=1$
$x=1,5$

Factor
Use the zero-product property
Solve for $u$ then $x$

Solve for $u$ then $x$

## Extensions

For the following exercises, solve for the unknown variable.
42. -
43. $\sqrt{|x|^{2}}=x$
$\sqrt{|x|^{2}}=x \quad$ Square both sides
$\left(\sqrt{|x|^{2}}\right)^{2}=x^{2} \quad$ Evaluate
$|x|^{2}=x^{2} \quad$ Note that no matter what the choice of x , the equation is satisfied
All real numbers
44. -
45. $\left|x^{2}+2 x-36\right|=12$
$x^{2}+2 x-36=12$ and $x^{2}+2 x-36=-12$
$x^{2}+2 x-36=12$
$x^{2}+2 x-48=0$
$(x+8)(x-6)=0$
$x+8=0$
Factor
Use the zero-product property Solve for x
$x=-8$
$x-6=0$
$x=6$
$x^{2}+2 x-36=-12$
$x^{2}+2 x-24=0$
Factor
$(x+6)(x-4)=0$
Use the zero-product property
$x+6=0$
$x=-6$
$x-4=0$
$x=4$
$x=4,6,-6,-8$

## Real-World Applications

For the following exercises, use the model for the period of a pendulum, $T$, such that $T=2 \pi \sqrt{\frac{L}{g}}$, where the length of the pendulum is $L$ and the acceleration due to gravity is $g$.
46. -
47. If the gravity is $32 \frac{\mathrm{ft}}{\mathrm{s}^{2}}$ and the period equals 1 s , find the length to the nearest in. ( $12 \mathrm{in} .=$ 1 ft ). Round your answer to the nearest in.
$T=2 \pi \sqrt{\frac{L}{g}}$
$1=2 \pi \sqrt{\frac{L}{32}}$
$\frac{1}{2 \pi}=\sqrt{\frac{L}{32}}$
$\left(\frac{1}{2 \pi}\right)^{2}=\left(\sqrt{\frac{L}{32}}\right)^{2}$
$\frac{1}{4 \pi^{2}}=\frac{L}{32}$
$L=\frac{32}{4 \pi^{2}}$
$L=0.810569469138 \mathrm{ft}$.
$L \approx 10$ in
10 in.
For the following exercises, use a model for body surface area, BSA, such that $B S A=\sqrt{\frac{w h}{3600}}$, where $w=$ weight in kg and $h=$ height in cm .
48. -
49. Find the weight of a $177-\mathrm{cm}$ male to the nearest kg whose $B S A=2.1$.

$$
\begin{aligned}
& B S A=\sqrt{\frac{w h}{3600}} \\
& 2.1=\sqrt{\frac{177 w}{3600}} \\
& (2.1)^{2}=\left(\sqrt{\frac{177 w}{3600}}\right)^{2} \\
& 4.41=\frac{177 w}{3600} \\
& 15876=177 w \\
& w=89.69491525423 \mathrm{~kg} \\
& 90 \mathrm{~kg}
\end{aligned}
$$

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## Section 2.7

## Chapter 2 <br> Equations and Inequalities <br> 2.7 Linear Inequalities and Absolute Value Inequalities

## Verbal

1. When solving an inequality, explain what happened from Step 1 to Step 2:

Step $1-2 x>6$
Step $2 x<-3$
When we divide both sides by a negative it changes the sign of both sides so the sense of the inequality sign changes.
2. -
3. When writing our solution in interval notation, how do we represent all the real numbers? $(-\infty, \infty)$
4.
5. Describe how to graph $y=|x-3|$

We start by finding the $x$-intercept, or where the function $=0$. Once we have that point, which is $(3,0)$, we graph to the right the straight line graph $y=x-3$, and then when we draw it to the left we plot positive $y$ values, taking the absolute value of them.

## Algebraic

For the following exercises, solve the inequality. Write your final answer in interval notation.
6. -
7. $3 x+2 \geq 7 x-1$
$3 x+2 \geq 7 x-1 \quad$ Subtract 7 x from both sides
$-4 x+2 \geq-1 \quad$ Subtract 2 from both sides
$-4 x \geq-3 \quad$ Divide both sides by -4 , flip the inequality symbol
$x \leq \frac{3}{4}$
$\left(-\infty, \frac{3}{4}\right]$
8. -


$$
\begin{array}{ll}
\text { 13. } \begin{array}{ll}
\frac{x+3}{8}-\frac{x+5}{5} \geq \frac{3}{10} & \\
\frac{x+3}{8}-\frac{x+5}{5} \geq \frac{3}{10} & \text { Make sure both sides have a common denominator } \\
\frac{5}{5}\left(\frac{x+3}{8}\right)-\frac{8}{8}\left(\frac{x+5}{5}\right) \geq \frac{12}{40} & \text { Multiply } \\
\frac{5 x+15}{40}-\frac{8 x+40}{40} \geq \frac{12}{40} & \text { Subtract } \\
\frac{-3 x-25}{40} \geq \frac{12}{40} & \text { Multiply both sides by } 40 \\
-3 x-25 \geq 12 & \begin{array}{l}
\text { Factor out the }-1 \\
-(3 x+25) \geq 12 \\
3 x+25 \leq-12
\end{array} \\
\begin{array}{ll}
\text { Divide both sides by }-1 ; \text { flip the inequality symbol } \\
3 x \leq-37 & \text { Subtract } 25 \text { from both sides } \\
x \leq-\frac{37}{3} & \text { Divide by } 3
\end{array} \\
\begin{array}{ll}
\left.-\infty,-\frac{37}{3}\right] &
\end{array}
\end{array} \begin{array}{l}
\text { 14. }-
\end{array}
\end{array}
$$

For the following exercises, solve the inequality involving absolute value. Write your final answer in interval notation.
15. $|x+9| \geq-6$
$|x+9| \geq-6$ Note that an absolute value expression is always greater than or equal to zero
All real numbers $(-\infty, \infty)$
16. -
17. $|3 x-1|>11$
$|3 x-1|>11$
$3 x-1>11$ and $3 x-1<-11$
$3 x-1>11$
Add 1 to both sides
$3 x>12$
Divide both sides by 3
$x>4$
$3 x-1<-11$
Add 1 to both sides
$3 x<-10$
Divide both sides by 3
19. $|x-2|+4 \geq 10$

$$
|x-2|+4 \geq 10
$$

$$
|x-2| \geq 6
$$

$$
x-2 \geq 6 \text { and } x-2 \leq-6
$$

$$
x-2 \geq 6
$$

$$
x \geq 8
$$

$$
x-2 \leq-6
$$

$$
x \leq-4
$$

$$
(-\infty,-4] \cup[8,+\infty)
$$

20.     - 
21. $|x-7|<-4$
$|x-7|<-4$ Note that an absolute value expression is never less than zero No solution
22.     - 
23. $\left|\frac{x-3}{4}\right|<2$
$\frac{x-3}{4}<2$ and $\frac{x-3}{2}>-2$
$\frac{x-3}{4}<2$
$x-3<8$
$x<11$
$\frac{x-3}{2}>-2$
$x-3>-4$
$x>-1$
$(-5,11)$

For the following exercises, describe all the $x$-values within or including a distance of the given values.
24. -

25 . Distance of 3 units from the number 9
$[6,12]$
26. -
27. Distance of 11 units from the number 1
$[-10,12]$
For the following exercises, solve the compound inequality. Express your answer using inequality signs, and then write your answer using interval notation.
28. -
29. $3 x+1>2 x-5>x-7$
$3 x+1>2 x-5>x-7$
$3 x+1>2 x-5$ and $2 x-5>x-7$
$3 x+1>2 x-5$
Subtract x from both sides
$x+1>-5$
$x>-6$
$2 x-5>x-7 \quad$ Subtract x from both sides
$x-5>-7$
Add 5 to both sides
$x>-2$
$x>-6$ and $x>-2 \quad$ Take the intersection of two sets.
$x>-2, \quad(-2,+\infty)$

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30.     - 
31. $2 x-5<-11$ or $5 x+1 \geq 6$
$x<-3$ or $x \geq 1 \quad$ Take the union of the two sets.
$(-\infty,-3) \cup[1, \infty)$
32. 

## Graphical

For the following exercises, graph the function. Observe the points of intersection and shade the $x$-axis representing the solution set to the inequality. Show your graph and write your final answer in interval notation.
33. $|x-1|>2$
$(-\infty,-1) \cup(3, \infty)$

34. -
35. $|x+7| \leq 4$
$[-11,-3]$

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36. -
37. $|x-2|<0$

It is never less than zero. No solution.


For the following exercises, graph both straight lines (left-hand side being yl and right-hand side being y2) on the same axes. Find the point of intersection and solve the inequality by observing where it is true comparing the $y$-values of the lines.
38. -
39. $x-2>2 x+1$

Where the blue line is above the orange line; point of intersection is $x=-3$.
$(-\infty,-3)$

40. -
41. $\frac{1}{2} x+1>\frac{1}{2} x-5$

Where the blue line is above the orange line; always. All real numbers.

$$
(-\infty,-\infty)
$$


42. -

## Numeric

For the following exercises, write the set in interval notation.
43. $\{x \mid-1<x<3\}$
$(-1,3)$
44. -
45. $\{x \mid x<4\}$
$(-\infty, 4)$
46. -

For the following exercises, write the interval in set-builder notation.
47. $(-\infty, 6)$

$$
\{x \mid x<6\}
$$

48.     - 
49. $[-3,5)$

$$
\{x \mid-3 \leq x<5\}
$$

50. 

For the following exercises, write the set of numbers represented on the number line in interval notation.
51.

$(-2,1]$
52. -
53.


$$
(-\infty, 4]
$$

## Technology

For the following exercises, input the left-hand side of the inequality as a Y1 graph in your graphing utility. Enter y2 = the right-hand side. Entering the absolute value of an expression is found in the MATH menu, Num, $1: \mathrm{abs}$ (. Find the points of intersection, recall ( $2^{\text {nd }}$ CALC 5:intersection, $1^{\text {st }}$ curve, enter, $2^{\text {nd }}$ curve, enter, guess, enter). Copy a sketch of the graph and shade the $x$-axis for your solution set to the inequality. Write final answers in interval notation.
54. -

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55. $\frac{-1}{2}|x+2|<4$

Where the blue is below the orange; always. All real numbers. $(-\infty,+\infty)$.

56. -
57. $|x-4|<3$

Where the blue is below the orange; $(1,7)$.

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58. -

## Extensions

59. Solve $|3 x+1|=|2 x+3|$
$|3 x+1|=|2 x+3|$
$3 x+1=2 x+3$ and $-3 x-1=2 x+3$
$3 x+1=2 x+3$
Subtract $2 x$ from both sides
$x+1=3$
Subtract 1 from both sides
$x=2$
$-3 x-1=2 x+3$
Subtract $2 x$ from both sides
$-5 x-1=3$
Add 1 to both sides
$-5 x=4$
Divide both sides by -5
$x=-\frac{4}{5}$
$x=2, \frac{-4}{5}$
60.     - 
61. $\frac{x-5}{x+7} \leq 0, x \neq-7$
$\frac{x-5}{x+7} \leq 0$
$x-5 \leq 0$
$x \leq 5$
$x+7 \geq 0$
$x \geq-7$
$(-7,5]$
62.     - 

## Real-World Applications

63. In chemistry the volume for a certain gas is given by $V=20 T$, where $V$ is measured in cc and $T$ is temperature in ${ }^{\circ} \mathrm{C}$. If the temperature varies between $80^{\circ} \mathrm{C}$ and $120^{\circ} \mathrm{C}$, find the set of volume values.
Solution:
$80 \leq T \leq 120$
$1,600 \leq 20 T \leq 2,400$
$[1,600,2,400]$
64.     - 

## Chapter 2 Review Exercises

## Section 2.1

For the following exercises, find the $x$-intercept and the $y$-intercept without graphing.

1. $4 x-3 y=12$
$4 x-3(0)=12$
$4 x=12$
$x=3$
$4(0)-3 y=12$
$-3 y=12$
$y=-4$
$x$-intercept: $(3,0) ; y$-intercept: $(0,-4)$
2.     - 

For the following exercises, solve for $y$ in terms of $x$, putting the equation in slopeintercept form.
3. $5 x=3 y-12$
$3 y=5 x+12$
$y=\frac{5}{3} x+4$
4. -

For the following exercises, find the distance between the two points.
5. $(-2,5)(4,-1)$
$\sqrt{(4-(-2))^{2}+(-1-5)^{2}}$
$\sqrt{(6)^{2}+(-6)^{2}}$
$\sqrt{36+36}$
$\sqrt{72}$
$\sqrt{36 \cdot 2}$
$\sqrt{36} \sqrt{2}$
$6 \sqrt{2}$
6. -
7. Find the distance between the two points $(-71,432)$ and $(511,218)$ using your calculator, and round your answer to the nearest thousandth.

For the following exercises, find the coordinates of the midpoint of the line segment that joins the two given points.
8. -
9. $(-13,5)$ and $(17,18)$

$$
\left(\frac{-13+17}{2}, \frac{5+18}{2}\right)
$$

$$
\left(\frac{4}{2}, \frac{23}{2}\right)
$$

$$
\text { midpoint is }\left(2, \frac{23}{2}\right)
$$

For the following exercises, construct a table and graph the equation by plotting at least three points.
10. -
11. $4 x-3 y=6$

| $x$ | $y$ |
| :--- | :--- |
| 0 | -2 |
| 3 | 2 |
| 6 | 6 |



## Section 2.2

For the following exercises, solve for $x$.
12. -
13. $3(x+2)-10=x+4$
$3 x+6-10=x+4$
$3 x-4=x+4$
$3 x=x+8$
$2 x=8$
$x=4$
14.
15. $12-5(x+1)=2 x-5$
$12-5 x-5=2 x-5$
$7-5 x=2 x-5$
$-5 x=2 x-12$
$-7 x=-12$
$x=\frac{12}{7}$
16. -

For the following exercises, solve for $x$. State all $x$-values that are excluded from the solution set.
17. $\frac{x}{x^{2}-9}+\frac{4}{x+3}=\frac{3}{x^{2}-9}$

$$
\begin{aligned}
& \frac{x}{x^{2}-9}+\frac{x-3}{x-3} \cdot \frac{4}{x+3}=\frac{3}{x^{2}-9} \\
& \frac{x}{x^{2}-9}+\frac{4(x-3)}{(x-3)(x+3)}=\frac{3}{x^{2}-9} \\
& \frac{x}{x^{2}-9}+\frac{4 x-12}{x^{2}-9}=\frac{3}{x^{2}-9} \\
& \frac{5 x-12}{x^{2}-9}=\frac{3}{x^{2}-9} \\
& 5 x-12=3 \\
& 5 x=15 \\
& x=3 \\
& x \neq 3,-3
\end{aligned}
$$

No solution
18. -

For the following exercises, find the equation of the line using the point-slope formula.
19. Passes through these two points: $(-2,1),(4,2)$.
$\frac{2-1}{4-(-2)}$
$\frac{1}{4+2}$
$\frac{1}{6}$
$y-2=\frac{1}{6}(x-4)$
$y-2=\frac{x}{6}-\frac{2}{3}$
$y=\frac{x}{6}-\frac{2}{3}+2$
$y=\frac{1}{6} x+\frac{4}{3}$
20.
21. Passes through the point $(-3,4)$ and is parallel to the graph $y=\frac{2}{3} x+5$.
$y-4=\frac{2}{3}(x-(-3))$
$y-4=\frac{2}{3}(x+3)$
$y-4=\frac{2}{3} x+2$
$y=\frac{2}{3} x+2+4$
$y=\frac{2}{3} x+6$
22. -

## Section 2.3

For the following exercises, write and solve an equation to answer each question.
23. The number of males in the classroom is five more than three times the number of females. If the total number of students is 73 , how many of each gender are in the class?
$f=$ female and $m=$ male
$m=3 f+5$
$73=m+f$
$73=(3 f+5)+f$
$73=4 f+5$
$68=4 f$
$f=17$
$73=m+17$
$m=56$
females 17, males 56
24. -
25. A truck rental is $\$ 25$ plus $\$ .30 / \mathrm{mi}$. Find out how many miles Ken traveled if his bill was $\$ 50.20$.
$p=0.30 m+25$
$50.20=0.30 m+25$
$25.20=0.30 \mathrm{~m}$
$m=84$
84 mi

## Section 2.4

For the following exercises, use the quadratic equation to solve.
26. -
27. $2 x^{2}+3 x+7=0$
$x=\frac{-3 \pm \sqrt{(3)^{2}-4(2)(7)}}{2(2)}$
$x=\frac{-3 \pm \sqrt{9-56}}{4}$
$x=\frac{-3 \pm \sqrt{-47}}{4}$
$x=\frac{-3 \pm i \sqrt{47}}{4}$
$x=\frac{-3}{4} \pm \frac{i \sqrt{47}}{4}$
For the following exercises, name the horizontal component and the vertical component.
28. -
29. $-2-i$
horizontal component -2 ; vertical component -1

For the following exercises, perform the operations indicated.
30. -
31. $(2+3 i)-(-5-8 i)$
$(2+3 i)+(5+8 i)$
$(2+5)+i(3+8)$
$7+11 i$
32. -
33. $\sqrt{-16}+4 \sqrt{-9}$
$\sqrt{-1} \sqrt{16}+4 \sqrt{-1} \sqrt{9}$
$4 i+4(3)(i)$
$4 i+12 i$
$16 i$
34. -
35. $(3-5 i)^{2}$

$$
(3-5 i)(3-5 i)
$$

$$
9-15 i-15 i+25 i^{2}
$$

$$
9-30 i+25(-1)
$$

$$
9-25-30 i
$$

$$
-16-30 i
$$

36.     - 
37. $\sqrt{-2}(\sqrt{-8}-\sqrt{5})$
$\sqrt{-1} \sqrt{2}(\sqrt{-1} \sqrt{4} \sqrt{2}-\sqrt{5})$
$\sqrt{2} i(2 \sqrt{2} i-\sqrt{5})$
$2 \sqrt{4} i^{2}-\sqrt{10} i$
$-4-i \sqrt{10}$
38.     - 
39. $\frac{3+7 i}{i}$

$$
\begin{aligned}
& \frac{3+7 i}{i} \cdot \frac{-i}{-i} \\
& \frac{-3 i-7 i^{2}}{-\left(i^{2}\right)} \\
& \frac{-3 i-7(-1)}{-(-1)} \\
& \frac{7-3 i}{1} \\
& x=7-3 i
\end{aligned}
$$

## Section 2.5

For the following exercises, solve the quadratic equation by factoring.
40. -
41. $3 x^{2}+18 x+15=0$

$$
\begin{aligned}
& \left(3 x^{2}+3 x\right)+(15 x+15)=0 \\
& 3 x(x+1)+15(x+1)=0 \\
& (3 x+15)(x+1)=0 \\
& 3 x+15=0 \\
& 3 x=-15 \\
& x=-5 \\
& x+1=0 \\
& x=-1 \\
& x=-1,-5
\end{aligned}
$$

42.     - 
43. $7 x^{2}-9 x=0$

$$
x(7 x-9)=0
$$

$$
x=0
$$

$$
7 x-9=0
$$

$$
7 x=9
$$

$$
x=\frac{9}{7}
$$

$$
x=0, \frac{9}{7}
$$

For the following exercises, solve the quadratic equation by using the square-root property.
44. -
45. $(x-4)^{2}=36$

$$
\begin{aligned}
& \sqrt{(x-4)^{2}}=\sqrt{36} \\
& x-4= \pm 6 \\
& x=4 \pm 6 \\
& x=4+6 \\
& x=10 \\
& x=4-6 \\
& x=-2 \\
& x=10,-2
\end{aligned}
$$

For the following exercises, solve the quadratic equation by completing the square. 46. -
47. $4 x^{2}+2 x-1=0$

$$
\begin{aligned}
& 4\left(x^{2}+\frac{1}{2} x\right)-1=0 \\
& 4\left(x^{2}+\frac{1}{2} x+\frac{1}{16}-\frac{1}{16}\right)-1=0 \\
& 4\left(x^{2}+\frac{1}{2} x+\frac{1}{16}\right)-1-\frac{1}{4}=0 \\
& 4\left(x+\frac{1}{4}\right)^{2}-\frac{5}{4}=0 \\
& 4\left(x+\frac{1}{4}\right)^{2}=\frac{5}{4}
\end{aligned}
$$

$$
\left(x+\frac{1}{4}\right)^{2}=\frac{5}{16}
$$

$$
\sqrt{\left(x+\frac{1}{4}\right)^{2}}=\sqrt{\frac{5}{16}}
$$

$$
x+\frac{1}{4}= \pm \frac{\sqrt{5}}{4}
$$

$$
x=-\frac{1}{4} \pm \frac{\sqrt{5}}{4}
$$

$$
x=\frac{-1 \pm \sqrt{5}}{4}
$$

For the following exercises, solve the quadratic equation by using the quadratic formula. If the solutions are not real, state No real solution.
48. -
49. $15 x^{2}-x-2=0$

$$
x=\frac{2}{5}, \frac{-1}{3}
$$

For the following exercises, solve the quadratic equation by the method of your choice. 50. -
51. $x^{2}=10 x+3$

$$
\begin{aligned}
& x^{2}-10 x-3=0 \\
& \left(x^{2}-10 x\right)-3=0 \\
& \left(x^{2}-10 x+25-25\right)-3=0 \\
& \left(x^{2}-10 x+25\right)-3-25=0 \\
& (x-5)^{2}-28=0 \\
& (x-5)^{2}=28 \\
& \sqrt{(x-5)^{2}}=\sqrt{28} \\
& x-5= \pm 2 \sqrt{7}
\end{aligned}
$$

## Section 2.6

For the following exercises, solve the equations.
52. -
53. $x^{\frac{1}{2}}-4 x^{\frac{1}{4}}=0$
$x^{\frac{1}{4}}\left(x^{\frac{1}{4}}-4\right)=0$
$x^{\frac{1}{4}}=0$
$x=0$
$x^{\frac{1}{4}}-4=0$
$x^{\frac{1}{4}}=4$
$x=256$
$x=0,256$
54. -
55. $3 x^{5}-6 x^{3}=0$

$$
\begin{aligned}
& 3 x^{3}\left(x^{2}-2\right)=0 \\
& 3 x^{3}(x-\sqrt{2})(x+\sqrt{2})=0 \\
& 3 x^{3}=0 \\
& x^{3}=0 \\
& x=0 \\
& x-\sqrt{2}=0 \\
& x=\sqrt{2} \\
& x+\sqrt{2}=0 \\
& x=-\sqrt{2} \\
& x=0, \pm \sqrt{2}
\end{aligned}
$$

56.     - 
57. $\sqrt{3 x+7}+\sqrt{x+2}=1$

$$
\begin{aligned}
& (\sqrt{3 x+7}+\sqrt{x+2})^{2}=1^{2} \\
& (\sqrt{3 x+7}+\sqrt{x+2})(\sqrt{3 x+7}+\sqrt{x+2})=1 \\
& 3 x+7+2 \sqrt{(3 x+7)(x+2)}+x+2=1 \\
& 4 x+9+2 \sqrt{3 x^{2}+6 x+7 x+14}=1 \\
& 4 x+9+2 \sqrt{3 x^{2}+13 x+14}=1 \\
& 2 \sqrt{3 x^{2}+13 x+14}=-4 x-8 \\
& \sqrt{3 x^{2}+13 x+14}=-2 x-4 \\
& \left(\sqrt{3 x^{2}+13 x+14}\right)^{2}=(-2 x-4)^{2} \\
& 3 x^{2}+13 x+14=(-2 x-4)(-2 x-4) \\
& 3 x^{2}+13 x+14=4 x^{2}+8 x+8 x+16 \\
& 3 x^{2}+13 x+14=4 x^{2}+16 x+16 \\
& -x^{2}-3 x-2=0 \\
& x^{2}+3 x+2=0 \\
& (x+2)(x+1)=0 \\
& x+2=0 \\
& x=-2 \\
& x+1=0 \\
& x=-1 \\
& \sqrt{3(-1)+7}+\sqrt{(-1)+2}=1 \\
& \sqrt{-3+7}+\sqrt{1}=1 \\
& \sqrt{4}+1=1 \\
& 2+1=1 \\
& 3 \neq 1 \\
& \sqrt{3(-2)+7}+\sqrt{(-2)+2}=1 \\
& \sqrt{-6+7}+0=1 \\
& \sqrt{1}=1 \\
& 1=1 \\
& x=-2 \\
& \\
& x
\end{aligned}
$$

58.     - 
59. $|2 x+3|-5=9$

$$
\begin{aligned}
& |2 x+3|=14 \\
& 2 x+3=14 \text { and } 2 x+3=-14 \\
& 2 x+3=14 \\
& 2 x=11 \\
& x=\frac{11}{2} \\
& 2 x+3=-14 \\
& 2 x=-17 \\
& x=-\frac{17}{2} \\
& x=\frac{11}{2}, \frac{-17}{2}
\end{aligned}
$$

## Section 2.7

For the following exercises, solve the inequality. Write your final answer in interval notation.
60. -
61. $-2 x+5>x-7$
$-3 x+5>-7$
$-3 x>-12$
$x<4$
$(-\infty, 4)$
62. -
63. $|3 x+2|+1 \leq 9$
$|3 x+2| \leq 8$
$3 x+2 \leq 8$ and $3 x+2 \geq-8$
$3 x+2 \leq 8$
$3 x \leq 6$
$x \leq 2$
$3 x+2 \geq-8$
$3 x \geq-10$
$x \geq-\frac{10}{3}$
$\left[\frac{-10}{3}, 2\right]$
64.
65. $|x-3|<-4$
$|x-3|<-4$ Note that an absolute value expression can never be less than zero.
No solution

For the following exercises, solve the compound inequality. Write your answer in interval notation.
66. -

$$
\text { 67. } \begin{aligned}
& 3 y<1-2 y<5+y \\
& 3 y<1-2 y \text { and } 1-2 y<5+y \\
& 3 y<1-2 y \\
& 5 y<1 \\
& y<\frac{1}{5} \\
& 1-2 y<5+y \\
& 1<5+3 y \\
&-4<3 y \\
&-\frac{4}{3}<y \\
&\left(-\frac{4}{3}, \frac{1}{5}\right)
\end{aligned}
$$

For the following exercises, graph as described.
68. -
69. Graph both straight lines (left-hand side being y1 and right-hand side being y2) on the same axes. Find the point of intersection and solve the inequality by observing where it is true comparing the $y$-values of the lines. See the interval where the inequality is true.
$x+3<3 x-4$
Where the blue is below the orange line; point of intersection is $x=3.5$. $(3.5, \infty)$

Chapter 2 Review Exercises


## Chapter 2 Practice Test

1. Graph the following: $2 y=3 x+4$.

$$
y=\frac{3}{2} x+2
$$

| $x$ | $y$ |
| :--- | :--- |
| 0 | 2 |
| 2 | 5 |
| 4 | 8 |


2. -
3. Find the $x$ - and $y$-intercepts of this equation, and sketch the graph of the line using just the intercepts plotted.

## Chapter 2 Practice Test

$$
\begin{aligned}
& 3 x-4 y=12 \\
& 3(0)-4 y=12 \\
& -4 y=12 \\
& y=-3 \\
& 3 x-4(0)=12 \\
& 3 x=12 \\
& x=4 \\
& (0,-3)(4,0)
\end{aligned}
$$


4. -
5. Write the interval notation for the set of numbers represented by $\{x \mid x \leq 9\}$. $(-\infty, 9]$
6. -
7. Solve for $x: 3(2 x-5)-3(x-7)=2 x-9$.
$3(2 x-5)-3(x-7)=2 x-9$
$6 x-15-3 x+21=2 x-9$
$3 x+6=2 x-9$
$x=-15$
8. -
9. Solve for $x: \frac{5}{x+4}=4+\frac{3}{x-2}$.
$\frac{5}{x+4}=4+\frac{3}{x-2}$
$\frac{x-2}{x-2} \cdot \frac{5}{x+4}=4 \frac{(x+4)(x-2)}{(x+4)(x-2)}+\frac{3}{x-2} \cdot \frac{x+4}{x+4}$
$\frac{5(x-2)}{(x-2)(x+4)}=4 \frac{x^{2}-2 x+4 x-8}{x^{2}-2 x+4 x-8}+\frac{3(x+4)}{(x-2)(x+4)}$
$\frac{5 x-10}{x^{2}+2 x-8}=4 \frac{x^{2}+2 x-8}{x^{2}+2 x-8}+\frac{3 x+12}{x^{2}+2 x-8}$
$\frac{5 x-10}{x^{2}+2 x-8}=\frac{4 x^{2}+8 x-32+3 x+12}{x^{2}+2 x-8}$
$5 x-10=4 x^{2}+11 x-20$
$4 x^{2}+6 x-10=0$
$\left(4 x^{2}+10 x\right)+(-4 x-10)=0$
$2 x(2 x+5)-2(2 x+5)=0$
$(2 x-2)(2 x+5)=0$
$2 x-2=0$
$2 x=2$
$x=1$
$2 x+5=0$
$2 x=-5$
$x=-\frac{5}{2}$
$x \neq-4,2 ; x=\frac{-5}{2}, 1$
10. -
11. Solve for $x$. Write the answer in simplest radical form.

$$
\frac{x^{2}}{3}-x=\frac{-1}{2}
$$

$$
\begin{aligned}
& \frac{x^{2}}{3}-x=\frac{-1}{2} \\
& 2 x^{2}-6 x=-3 \\
& 2 x^{2}-6 x+3=0 \\
& \frac{-(-6) \pm \sqrt{(-6)^{2}-4(2)(3)}}{2(2)} \\
& \frac{6 \pm \sqrt{36-24}}{4} \\
& \frac{6 \pm \sqrt{12}}{4} \\
& \frac{6 \pm 2 \sqrt{3}}{4} \\
& x=\frac{3 \pm \sqrt{3}}{2}
\end{aligned}
$$

12.     - 
13. Solve: $|2 x+3|<5$.
$|2 x+3|<5$
$2 x+3<5$ and $2 x+3>-5$
$2 x+3<5$
$2 x<2$
$x<1$
$2 x+3>-5$
$2 x>-8$
$x>-4$
$(-4,1)$
14.     - 

For the following exercises, find the equation of the line with the given information.
15. Passes through the points $(-4,2)$ and $(5,-3)$.
$m=\frac{-3-2}{5-(-4)}$
$m=\frac{-5}{9}$
$y-2=-\frac{5}{9}(x-(-4)) y=\frac{-5}{9} x-\frac{2}{9}$
$y-2=-\frac{5}{9} x-\frac{20}{9}$
$y=-\frac{5}{9} x-\frac{2}{9}$
16. -
17. Passes through the point $(2,1)$ and is perpendicular to $y=\frac{-2}{5} x+3$.

$$
\begin{aligned}
& m=\frac{5}{2} \\
& y-1=\frac{5}{2}(x-2) \\
& y-1=\frac{5}{2} x-5 \\
& y=\frac{5}{2} x-4
\end{aligned}
$$

18.     - 
19. Simplify: $\sqrt{-4}+3 \sqrt{-16}$.

$$
\begin{aligned}
& \sqrt{-4}+3 \sqrt{-16} \\
& \sqrt{-1} \sqrt{4}+3 \sqrt{-1} \sqrt{16} \\
& 2 i+3 i(4) \\
& 2 i+12 i
\end{aligned}
$$

$$
14 i
$$

20.     - 
21. Divide: $\frac{4-i}{2+3 i}$.

$$
\begin{aligned}
& \frac{4-i}{2+3 i} \\
& \frac{4-i}{2+3 i} \cdot \frac{2-3 i}{2-3 i} \\
& \frac{(4-i)(2-3 i)}{(2+3 i)(2-3 i)} \\
& \frac{8-12 i-2 i+3 i^{2}}{4-6 i+6 i-9 i^{2}} \\
& \frac{8-14 i+3(-1)}{4-9(-1)} \\
& \frac{8-14 i-3}{4+9} \\
& \frac{5-14 i}{13} \\
& \frac{5}{13}-\frac{14}{13} i
\end{aligned}
$$

22.     - 
23. Solve:

$$
\begin{aligned}
& (3 x-1)^{2}-1=24 \\
& (3 x-1)^{2}=25 \\
& \sqrt{(3 x-1)^{2}}=\sqrt{25} \\
& 3 x-1= \pm 5 \\
& 3 x=1 \pm 5 \\
& x=\frac{1 \pm 5}{3} \\
& x=\frac{1+5}{3} \\
& x=\frac{6}{3} \\
& x=2 \\
& x=\frac{1-5}{3} \\
& x=-\frac{4}{3} \\
& x=2, \frac{-4}{3}
\end{aligned}
$$

24.     - 
25. Solve:

$$
\begin{aligned}
& 4 x^{2}-4 x-1=0 \\
& \frac{-(-4) \pm \sqrt{(-4)^{2}-4(4)(-1)}}{2(4)} \\
& \frac{4 \pm \sqrt{16+16}}{8} \\
& \frac{4 \pm \sqrt{32}}{8} \\
& \frac{4 \pm \sqrt{16} \sqrt{2}}{8} \\
& \frac{4 \pm 4 \sqrt{2}}{8} \\
& x=\frac{1}{2} \pm \frac{\sqrt{2}}{2}
\end{aligned}
$$

26.     - 
27. Solve:

$$
\begin{aligned}
& 2+\sqrt{12-2 x}=x \\
& \sqrt{12-2 x}=x-2 \\
& (\sqrt{12-2 x})^{2}=(x-2)^{2} \\
& 12-2 x=(x-2)(x-2) \\
& 12-2 x=x^{2}-2 x-2 x+4 \\
& 12-2 x=x^{2}-4 x+4 \\
& x^{2}-2 x-8=0 \\
& (x-4)(x+2)=0 \\
& x-4=0 \\
& x=4 \\
& x+2=0 \\
& x=-2 \\
& 2+\sqrt{12-2(4)}=4 \\
& 2+\sqrt{12-8}=4 \\
& 2+\sqrt{4}=4 \\
& 2+2=4 \\
& 4=4 \\
& 2+\sqrt{12-2(-2)}=-2 \\
& 2+\sqrt{12+4}=-2 \\
& 2+\sqrt{16}=-2 \\
& 2+4=-2 \\
& 6 \neq-2 \\
& 4
\end{aligned}
$$

28.     - 

For the following exercises, find the real solutions of each equation by factoring.
29. $2 x^{3}-x^{2}-8 x+4=0$

$$
\begin{aligned}
& \left(2 x^{3}-x^{2}\right)+(-8 x+4)=0 \\
& x^{2}(2 x-1)-4(2 x-1)=0 \\
& \left(x^{2}-4\right)(2 x-1)=0 \\
& (x-2)(x+2)(2 x-1)=0 \\
& x-2=0 \\
& x=2 \\
& x+2=0 \\
& x=-2 \\
& 2 x-1=0 \\
& 2 x=1 \\
& x=\frac{1}{2} \\
& x=\frac{1}{2}, 2,-2
\end{aligned}
$$

30.     - 

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