Chapter 6
Exponential and Logarithmic Functions
6.1 Exponential Functions

Section Exercises
Verbal

1. Explain why the values of an increasing exponential function will eventually overtake the values of an increasing linear function.
   Linear functions have a constant rate of change. Exponential functions increase at a rate proportional to their size. So, the bigger they get, the faster they increase, which makes them even bigger, which makes them increase even faster, etc.
2. -
3. The Oxford Dictionary defines the word *nominal* as a value that is "stated or expressed but not necessarily corresponding exactly to the real value."[1] Develop a reasonable argument for why the term *nominal rate* is used to describe the annual percentage rate of an investment account that compounds interest.
   When interest is compounded, the percentage of interest earned on principal ends up being greater than the annual percentage rate for the investment account, because interest is earned on interest. Thus, the annual percentage rate does not necessarily correspond to the real interest earned, which is the very definition of *nominal*.

Algebraic

For the following exercises, identify whether the statement represents an exponential function. Explain.

4. -
5. A population of bacteria decreases by a factor of $\frac{1}{8}$ every 24 hours.
   Exponential functions increase/decrease proportionally, or by a percentage, over the domain, therefore this would be an exponential function; the population decreases by a proportional rate .
6. -
7. For each training session, a personal trainer charges his clients $\$5$ less than the previous training session.
   Since this is decreasing at a constant rate, $5, over the domain this would be linear decay, not exponential; the charge decreases by a constant amount each visit, so the statement represents a linear function. .

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[1] Oxford Dictionary. http://oxforddictionaries.com/us/definition/american_english/nomina.

8. -

For the following exercises, consider this scenario: For each year t, the population of a forest of trees is represented by the function $A(t) = 115(1.025)^t$. In a neighboring forest, the population of the same type of tree is represented by the function $B(t) = 82(1.029)^t$. (Round answers to the nearest whole number.)

9. Which forest's population is growing at a faster rate?
   The base number that is raised to the exponent is the proportion or percentage increase, therefore the larger base would be the faster growth rate. The larger base is 1.029 compared to 1.025, therefore $B(t) = 82(1.029)^t$ is growing at the faster rate.
10. -
11. Assuming the population growth models continue to represent the growth of the forests, which forest will have a greater number of trees after 20 years? By how many?
    Evaluating both functions at t = 20 will yield $A(20) = 115(1.025)^{20} \approx 188.44$ and $B(20) = 82(1.029)^{20} \approx 145.25$, therefore after 20 years forest A – forest B, or 188 − 145 = 43. In 20 years Forest A will have 43 more trees than Forest B.
12. -
13. Discuss the above results from the previous four exercises. Assuming the population growth models continue to represent the growth of the forests, which forest will have the greater number of trees in the long run? Why? What are some factors that might influence the long-term validity of the exponential growth model?
    Answers may vary. Initially Forest A has a greater number of trees (115 compared to 82) so for a short period of time forest A will have more trees than Forest B, however in the long run Forest B will surpass growth of Forest A because it is growing at a faster rate. The difference in their rates 1.025 and 1.029 will not start making this change until about 97 years. One way to see this is to graph both functions in your graphing calculator, and find where the point of intersection is. At that point B begins to overtake A in number. Long term validity of the models has several environmental issues that may affect the growth rate , such as what happened in Colorado with a bore infestation or extreme fires destroying several thousands of trees. Answers will vary. Some factors that might influence the long-term validity of the exponential growth model are drought, an epidemic that culls the population, and other environmental and biological factors.

For the following exercises, determine whether the equation represents exponential growth, exponential decay, or neither. Explain.

14. -

15. $y = 220(1.06)^x$

Exponential functions increase/decrease proportionally, or by a percentage, over the domain, therefore this would be an exponential function. It is exponential GROWTH because the base that is the rate of proportion is greater than 1.

16. -

17. $y = 11,701(0.97)^t$

Exponential functions increase/decrease proportionally, or by a percentage, over the domain, therefore this would be an exponential function. It is exponential DECAY because the base that is the rate of proportion is .97 AND LESS THAN 1. Also a reminder that the base must be greater than 0; Thus, exponential decay; The decay factor, 0.97, is between 0 and 1.

For the following exercises, find the formula for an exponential function that passes through the two points given.

18. -

19. $(0,2000)$ and $(2,20)$

All exponential functions are in the form $f(x) = a(b)^x$, so input $x = 0$ and $f(0) = 2000$,

$a(b)^0 = 2000$

$a(1) = 2000$

$a = 2000,$

Since $a = 2000,$ we solve for b using the second coordinates, when $x = 2,$ $f(2) = 20$

$2000(b)^2 = 20 \quad \text{dividing by } 2000$

$b^2 = 0.01 \quad \text{take the square root of both sides}$

$b = 0.1$

Since $a = 2000$ and $b = 0.1$ our equation is $f(x) = 2000(0.1)^x$

20. -

21. $(-2,6)$ and $(3,1)$

Because we don't have the initial value (when x = 0) we substitute both points into an equation of the form $f(x) = ab^x$, and then solve the system for a and b. Substituting $(-2,6)$ gives a first equation of $6 = ab^{-2}$ substituting $(3,1)$ gives a second equation of $1 = ab^3$. We first solve for b:

$6 = ab^{-2} \to a = 6b^2$

$1 = ab^3 \to a = b^{-3}$

$6b^2 = b^{-3}$

$6 = b^{-5}$

$\left(\frac{1}{6}\right) = b^5$

$\left(\frac{1}{6}\right)^{\frac{1}{5}} = b$

Now use b to solve for a:

$1 = ab^3$

$1 = a\left(\frac{1}{6}\right)^{\frac{3}{5}}$

$a = \left(\frac{1}{6}\right)^{-\frac{3}{5}}$

Finally, substitute your values for a and b into $f(x) = ab^x$:

$f(x) = \left(\frac{1}{6}\right)^{-\frac{3}{5}}\left(\frac{1}{6}\right)^{\frac{x}{5}} \approx 2.93(0.699)^x$

22. -

For the following exercises, determine whether the table could represent a function that is linear, exponential, or neither. If it appears to be exponential, find a function that passes through the points.

23.

x	1	2	3	4
$f(x)$	70	40	10	-20

Since this is decreasing at a constant rate, -30 for each increase of 1 over the domain this would be LINEAR decay, not exponential.

24. -

25.

x	1	2	3	4
$m(x)$	80	61	42.9	25.61

To find the rate of proportion we may choose any m(x) value and divide it by a preceding $m(x)$. If you divide 61/80 =.7625, dividing 42.9/61 = .7032787 or 25.61/42.9 = .59697, therefore this function is not increasing by a percentage rate over the domain. It is not exponential. It is NOT linear because it is NOT decreasing by a constant rate. Therefore it is NEITHER.

26. -
27.

x		1	2	3	4
$g(x)$		–3.25	2	7.25	12.5

Since this is increasing at a constant rate, 2- (-3.25) = 5.25 for each increase of 1 over the domain this would be linear growth, not exponential

For the following exercises, use the compound interest formula, $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$.

- $A(t)$ is the account value,
- t is measured in years,
- P is the starting amount of the account, often called the principal, or more generally present value,
- r is the annual percentage rate (APR) expressed as a decimal, and
- n is the number of compounding periods in one year.

28. -
29. What was the initial deposit made to the account in the previous exercise?
    In the compound interest formula the P represents the initial deposit which is $\$10,250$.
30. -
31. An account is opened with an initial deposit of $6,500 and earns 3.6% interest compounded semi-annually. What will the account be worth in 20 years?
    Because our initial deposit is P, $P = \$6,500$. The interest rate is 3.6 % written as a decimal is 0.036. Since it is compounded semi-annually $n = 2$ and 20 is t, the number of years it has accumulated. Inputting the correct values into the formula we have
    $6,500\left(1+\frac{0.036}{2}\right)^{2(20)}$ Entering this value into your calculator make sure to use parentheses. From left to right input the following 6,500 (1 + 0.036 ÷ 2) ^ (2*20) , your answer should be $\$13,268.58$ rounded to the nearest cent.
32. -
33. Solve the compound interest formula for the principal, P .

    Starting with the formula, since multiplication is occurring between P *and* $\left(1+\frac{r}{n}\right)^{nt}$ we would be dividing both sides by $\left(1+\frac{r}{n}\right)^{nt}$

$$\frac{A(t)}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{P\left(1+\frac{r}{n}\right)^{nt}}{\left(1+\frac{r}{n}\right)^{nt}}$$ then rewriting the left hand side with a negative exponent we

write $A(t)\cdot\left(1+\frac{r}{n}\right)^{-nt} = P$. Therefore $P = A(t)\cdot\left(1+\frac{r}{n}\right)^{-nt}$

34. -

35. How much more would the account in the previous two exercises be worth if it were earning interest for 5 more years?

    We found that the initial deposit 5 years ago was $11,002. We will find the compounded amount using this $P = \$11,002,$ $n = 12,$ and $r = 0.055$ and $t = 10$ years. When we find that we will take the difference from the amount given in exercise 34 which was

    $14,472.74. $A(10) = 11,002\left(1+\frac{0.055}{12}\right)^{12(10)} = \$19,045.30$. Taking the difference

    $\$19,045.30 - \$14,472.74 = \$4,572.56$

36. -

37. Use the formula found in the previous exercise to calculate the interest rate for an account that was compounded semi-annually, had an initial deposit of $9,000 and was worth $\$13,373.53$ after 10 years.

    [$r = n\left[\left(\frac{A(t)}{P}\right)^{\frac{1}{nt}} - 1\right]$ where $P = 9000,$ $A(10) = 13,373.53,$ semi-annually $n = 2,$ and

    $t = 10$ years $r = 2\left[\left(\frac{\$13,373.53}{\$9,000}\right)^{\frac{1}{2(10)}} - 1\right] = 4\%$

38. -

For the following exercises, determine whether the equation represents continuous growth, continuous decay, or neither. Explain.

39. $y = 3742(e)^{0.75t}$

    The coefficient to t in the exponent is the growth rate , when that is positive then it is an exponential, or continuous GROWTH function; the growth rate is greater than 0.

40. -

41. $y = 2.25(e)^{-2t}$

The coefficient to t in the exponent is the growth rate , when that is negative then it is an exponential, or continuous DECAY function ; the growth rate is less than 0.

42. -

43. How much less would the account from Exercise 42 be worth after 30 years if it were compounded monthly instead?

For monthly compounding we use the previous formula $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$. with $n = 12$.

$A(30) = 12,000\left(1+\frac{.072}{12}\right)^{12(30)} = \$103,384.23$. Taking the difference

$\$104,053.65 - \$103,384.23 = \$669.42$

Numeric

For the following exercises, evaluate each function. Round answers to four decimal places, if necessary.

44. -

45. $f(x) = -4^{2x+3}$, for $f(-1)$

This means to input -1 into the function for x (which is in the exponent).

$f(-1) = -4^{2(-1)+3} = -4$

46. -

47. $f(x) = -2e^{x-1}$, for $f(-1)$

This means to input -1 into the function for x (which is in the exponent).

$f(-1) = -2e^{(-1-1)} \approx -.2707$

48. -

49. $f(x) = 1.2e^{2x} - 0.3$, for $f(3)$

This means to input 3 into the function for x (which is in the exponent).

$f(3) = 1.2e^{2(3)} - 0.3 \approx 483.8146$

50. -

Technology

For the following exercises, use a graphing calculator to find the equation of an exponential function given the points on the curve.

51. $(0, 3)$ and $(3, 375)$

First press **[STAT]**, **[EDIT]**, **[1: Edit…],** and clear the lists **L1** and **L2**. Next, in the **L1** column, enter the *x*-coordinates, 0 and 3. Do the same in the **L2** column for the *y*-coordinates, 3 and 375
Now press **[STAT]**, **[CALC]**, **[0: ExpReg]** and press **[ENTER]**. The display will read

$ExpReg$
$y = a*b \wedge x$
$a = 3$
$b = 5$

The exponential equation is $y = 3 \cdot 5^x$.

52. -
53. $(20, 29.495)$ and $(150, 730.89)$
First press **[STAT]**, **[EDIT]**, **[1: Edit…],** and clear the lists **L1** and **L2**. Next, in the **L1** column, enter the *x*-coordinates, 20 and 150. Do the same in the **L2** column for the *y*-coordinates, 29.495 and 730.89. Now press **[STAT]**, **[CALC]**, **[0: ExpReg]** and press **[ENTER]**. The display is

$ExpReg$
$y = a*b \wedge x$
$a = 17.99993433$
$b = 1.02500002$

The exponential equation is $y \approx 18 \cdot 1.025^x$.

54. -

55. $(11, 310.035)$ and $(25, 3563652)$
First press **[STAT]**, **[EDIT]**, **[1: Edit…],** and clear the lists **L1** and **L2**. Next, in the **L1** column, enter the *x*-coordinates,11 and 25. Do the same in the **L2** column for the *y*-coordinates, 310.035 and 3563652. Now press **[STAT]**, **[CALC]**, **[0: ExpReg]** and press **[ENTER]**. The display is

$ExpReg$
$y = a*b \wedge x$
$a = .2000000002$
$b = 1.950000001$

The exponential equation is $y \approx 0.2 \cdot 1.95^x$

Extensions

56. -
57. Repeat the previous exercise to find the formula for the APY of an account that compounds daily. Use the results from this and the previous exercise to develop a

function $I(n)$ for the APY of any account that compounds n times per year.
Using the same logic in the previous exercise but compounding it daily would mean that $n = 365.$ It follows that

$$\text{APY} = \frac{A(t)-a}{a} = \frac{a\left(1+\frac{r}{365}\right)^{365(1)} - a}{a} = \frac{a\left[\left(1+\frac{r}{365}\right)^{365} - 1\right]}{a} = \left(1+\frac{r}{365}\right)^{365} - 1;$$ Using the results from these two examples we would develop the function $I(n)$ for the APY of any account that compounds n times per year to be:

$$I(n) = \left(1+\frac{r}{n}\right)^{n} - 1$$

58. -
59. In an exponential decay function, the base of the exponent is a value between 0 and 1. Thus, for some number $b > 1,$ the exponential decay function can be written as $f(x) = a \cdot \left(\frac{1}{b}\right)^{x}$. Use this formula, along with the fact that $b = e^{n},$ to show that an exponential decay function takes the form $f(x) = a\left(e\right)^{-nx}$ for some positive number n.

    Let f be the exponential decay function $f(x) = a \cdot \left(\frac{1}{b}\right)^{x}$ such that $b > 1.$ Then for some number $n > 0,$ $f(x) = a \cdot \left(\frac{1}{b}\right)^{x} = a\left(b^{-1}\right)^{x} = a\left(\left(e^{n}\right)^{-1}\right)^{x} = a\left(e^{-n}\right)^{x} = a\left(e\right)^{-nx}.$

60. -

Real-World Applications

61. The fox population in a certain region has an annual growth rate of 9% per year. In the year 2012, there were $23{,}900$ fox counted in the area. What is the fox population predicted to be in the year 2020?

    Since the growth rate is a percentage we know that it is growing exponentially. In this exponential function, 23,900 represents the initial fox population, 0.09 represents the growth rate, and $1+0.09 = 1.09$ represents the growth factor. We can write this function as $f(x) = ab^{x} = 23{,}900\left(1.09\right)^{x}$, where x is the number of years that happen. Given the original year is 2012 and we are asked for the population in year 2020, we enter x as 8 years. Therefore $f(x) = ab^{x} = 23{,}900\left(1.09\right)^{8} = 47{,}622$ foxes.

62. -

63. In the year 1985, a house was valued at $\$110,000$. By the year 2005, the value had appreciated to $\$145,000$. What was the annual growth rate between 1985 and 2005? Assume that the value continued to grow by the same percentage. What was the value of the house in the year 2010?

Since this is an exponential growth problem we use the model $f(x)=ab^x$. We start with the given information that initial value $a=\$110,000$ and when $x=20$ years the value has grown to \$145,000. Plugging these numbers in we will solve for b

$$f(x)=ab^x$$

$$145,000=110,000b^{20} \quad \textit{divide by } 110,000$$

$$1.31\overline{18}=b^{20} \quad \textit{take the 20th root of both sides}$$

$$b=1.013908504$$

Using $b=1.013908504$ we can substitute that into the formula and find the value when $x=25$ years (since it started in 1985 and is going to 2010, that is 25 years)

$$f(25)=ab^x=110,000(1.013908504)^{25}=\$155,368.09$$

Therefore, the annual growth rate b, written as a percent and rounded to the hundredths of a percent is 1.39%; and the value in 2010 would be $\$155,368.09$

64. -

65. Jamal wants to save $\$54,000$ for a down payment on a home. How much will he need to invest in an account with 8.2% APR, compounding daily, in order to reach his goal in 5 years?

In exercise #33 we derived the formula for present value from the compound interest formula $P=A(t)\cdot\left(1+\frac{r}{n}\right)^{-nt}$ using this formula we solve for P when

$$A(t)=\$54000,\ r=8.2\%=.082,\ n=365\ \textit{daily, and}\ t=5\ \textit{years}$$

$$P=54000\cdot\left(1+\frac{0.082}{365}\right)^{-365(5)}=\$35,838.76$$

66. -

67. Alyssa opened a retirement account with 7.25% APR in the year 2000. Her initial deposit was $\$13,500$. How much will the account be worth in 2025 if interest compounds monthly? How much more would she make if interest compounded continuously?

For compounding an investment monthly we the interest formula $A(t)=P\left(1+\frac{r}{n}\right)^{nt}$ where

$$P=13,500,\ n=12 \text{ monthly},\ r=7.25\%=.0725 \text{ and } t=25 \text{ years}$$

$$A(t)=13,500\left(1+\frac{.0725}{12}\right)^{12(25)}=\$82,247.78$$

To find the amount compounded CONTINUOUSLY we use the formula $A(t)=Pe^{rt}$

$A(25)=13,000e^{.0725(25)}=\$82,697.53$, Taking the difference of the two amounts

$\$82,697.53-\$82,247.78=\$449.75$. Therefore $\$82,247.78;\ \$449.75$

68. -

Chapter 6
Exponential and Logarithmic Functions
6.2 Graphs of Exponential Functions

Section Exercises

Verbal

1. What is an asymptote? What role does the horizontal asymptote of an exponential function play in telling us about the end behavior of the graph?
   An asymptote is a line that the graph of a function approaches, as x either increases or decreases without bound or approaches from the left or right. The horizontal asymptote of an exponential function tells us the limit of the function's values as the independent variable gets either extremely large or extremely small.
2. -

Algebraic

3. The graph of $f(x)=3^x$ is reflected about the *y*-axis and stretched vertically by a factor of 4. What is the equation of the new function, $g(x)$? State its *y*-intercept, domain, and range.

   All exponential functions can be written in the form $f(x)=ab^{x+c}+d$. In general a will vertically stretch a graph when $|a|>1$ and it will vertically compress the graph when $|a|<1$. Inputting a negative sign in front of the function reflects the graph about the x-axis and inputting a negative in front of the x variable reflects the graph about the y-axis. The $+c$ being added to x in the exponent shifts the graph to the left when c is positive and to the right when c is negative. Finally d will shift the graph vertically, up when positive, down when negative.

   For this graph we are asked to reflect it about the y-axis, thus input a negative in front of the *x* variable and stretch it by a factor of 4, thus $b=4$ and $g(x)=4(3)^{-x}$;The y-intercept is found by inputting *x* = 0, therefore the y-intercept: $(0,\ 4)$; For domain we consider all *x* values we may input . There are no restrictions, therefore the Domain is $(-\infty,+\infty)$ all real numbers; For range, we consider the values that y involves. The y-values never are below the x-axis therefore the Range is $(0,+\infty)$ all real numbers greater than 0.
4. -

5. The graph of $f(x)=10^x$ is reflected about the *x*-axis and shifted upward 7 units. What is the equation of the new function, $g(x)$? State its *y*-intercept, domain, and range.

   For this graph we are asked to reflect it about the x-axis, thus input a negative in front of the function $=-10^x$, then to shift the graph up 7 units thus $d=7$, so our function would be $g(x)=-10^x+7$; The y-intercept is found by inputting x = 0, therefore the y-intercept: $(0,\ 6)$; For domain we consider all x values we may input . There are no restrictions, therefore the Domain is $(-\infty,+\infty)$ all real numbers; For the range, we consider the values that y outputs. The limit on our y-values before shifting it upward was the x-axis (it is asymptotic to it) therefore the asymptote is at $y=7$ and the Range is $(-\infty,7)$or all real numbers less than 7.

6. -

7. The graph of $f(x)=-\frac{1}{2}\left(\frac{1}{4}\right)^{x-2}+4$ is shifted left 2 units, stretched vertically by a factor of 4, reflected about the *x*-axis, and then shifted downward 4 units. What is the equation of the new function, $g(x)$? State its *y*-intercept, domain, and range.

   For this graph we to shift to the left , which makes $+c=+2$, therefore our exponent will be $x-2+2=x$. Stretching it vertically means that $a=4$, multiplying it by 4,we get $-\frac{1}{2}\text{g}4=-2$. Reflecting about the x-axis puts a negative in front of the function making our new $a=+2$. Finally shifting the graph down 4 units thus $d=-4$, so inputting that in our model our function would be $g(x)=2\left(\frac{1}{4}\right)^x$; The y-intercept is found by inputting x = 0, therefore the y-intercept: $(0\ ,\ 2)$; For domain we consider all x values we may input . There are no restrictions, therefore the Domain is all real numbers $(-\infty,+\infty)$; For the range, we consider the values that y outputs. The y-values never are below the x-axis therefore the Range is $(0,+\infty)$ all real numbers greater than 0.

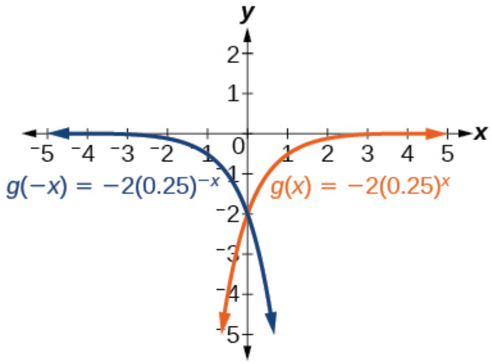
Graphical

For the following exercises, graph the function and its reflection about the *y*-axis on the same axes, and give the *y*-intercept.

8. -

9. $g(x) = -2(0.25)^x$

Reflection about the y-axis inputting a negative sign in front of the x, so all positive values of x would be negative and still plotting the y-values the same. the y-intercept remains $(0, -2)$



y-intercept: $(0, -2)$

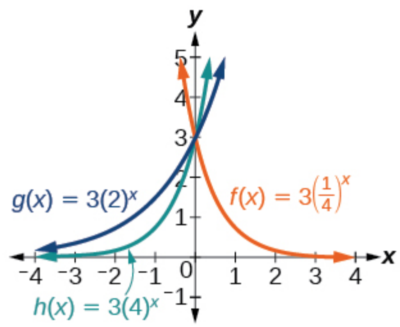
10. -

For the following exercises, graph each set of functions on the same axes.

11. $f(x) = 3\left(\frac{1}{4}\right)^x$, $g(x) = 3(2)^x$, and $h(x) = 3(4)^x$

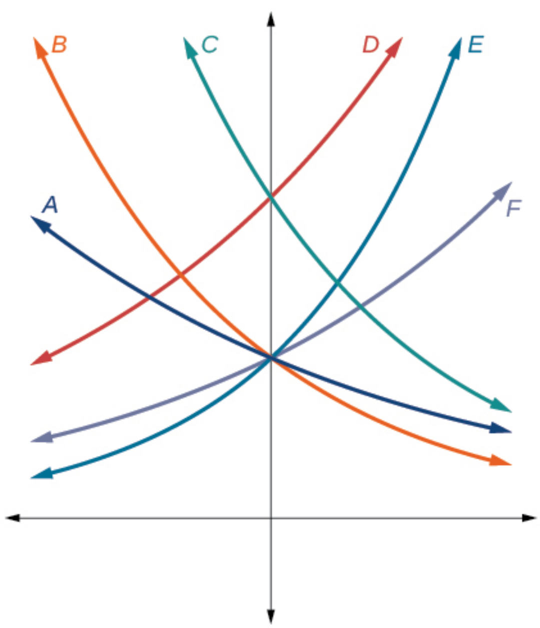
All exponential functions can be written in the form $f(x) = ab^{x+c} + d$. In general a will vertically stretch a graph when $|a| > 1$ and it will vertically compress the graph when $|a| < 1$. Inputting a negative sign in front of the function reflects the graph about the x-axis and inputting a negative in front of the x variable reflects the graph about the y-axis. The $+c$ being added to x in the exponent shifts the graph to the left when c is positive and to the right when c is negative. Finally d will shift the graph vertically, up when positive, down when negative.

For the graph of $f(x) = 3\left(\frac{1}{4}\right)^x$, $a = 3$, so the y-values will be multiplied by 3 , stretching it vertically. For the graph of $g(x) = 3(2)^x$, $a = 3$, so the y-values will be multiplied by 3 , stretching it vertically. For the graph of $h(x) = 3(4)^x$, $a = 3$, so the y-values will be multiplied by 3 , stretching it vertically.



12. -

For the following exercises, match each function with one of the graphs in



13. $f(x) = 2(0.69)^x$

This would be a choice between A, B or C as the *base* $b < 1$ means it is decreasing function and comparing with #15 and #18, #15 when x values are negative the y values are lower because of a the larger base and #18 would be higher y-values because of the multiplying factor of 4 having the same base as #13, therefore this has to be choice = B

14. -

15. $f(x) = 2(0.81)^x$

This would be a choice between A, B or C as the *base* $b < 1$ means it is decreasing function and comparing with #13 and #18, see #13 explanation this choice = A

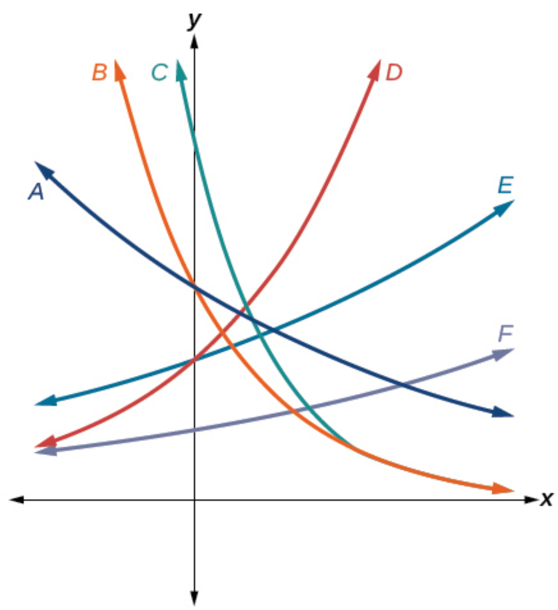
16. -

17. $f(x) = 2(1.59)^x$

See explanation for # 14 and # 16 which leaves this to be choice = E

18. -

For the following exercises, use the graphs shown. All have the form $f(x) = ab^x$.



19. Which graph has the largest value for b?

The graph which increases the fastest therefore choice D

20.

21. Which graph has the largest value for a?

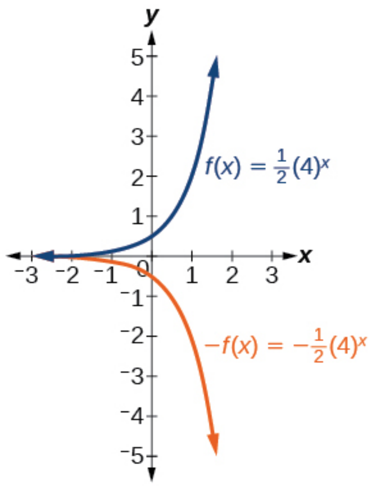
The graph with the highest y-intercept, choice C

22. -

For the following exercises, graph the function and its reflection about the *x*-axis on the same axes.

23. $f(x)=\frac{1}{2}(4)^x$

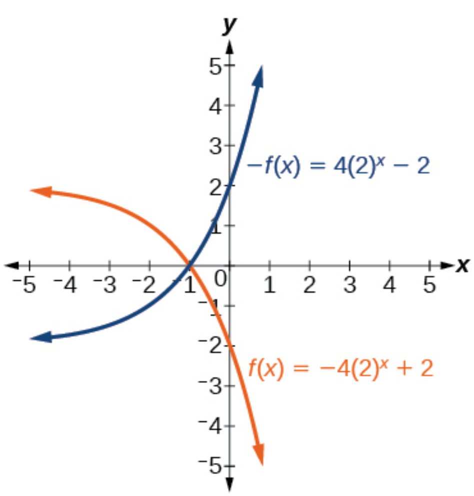
Inputting a negative sign in front of the function reflects the graph about the x-axis. For every x value you will be plotting the opposite y-value $g(x)=-\frac{1}{2}(4)^x$



24. -

25. $f(x)=-4(2)^x+2$

[Inputting a negative sign in front of the function reflects the graph about the x-axis. For every x value you will be plotting the opposite y-value $g(x)=-\left[-4(2)^x+2\right]=4(2)^x-2$

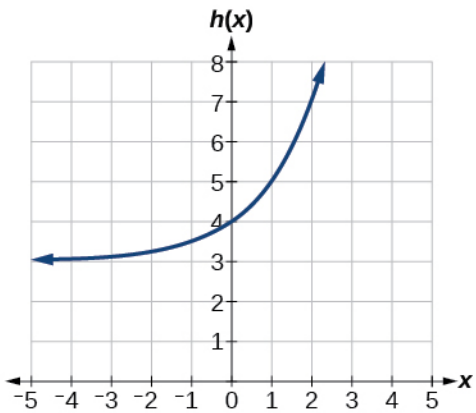


For the following exercises, graph the transformation of $f\left(x\right)=2^x$. Give the horizontal asymptote, the domain, and the range.

26. -

27. $h\left(x\right)=2^x+3$

We know that all basic exponential functions are asymptotic to the x-axis unless there is a vertical shift then that horizontal asymptote shifts accordingly. The graph has been shifted up 3, so our horizontal asymptote is now $y=3$. There are no restrictions on our x-values, therefore the Domain is $(-\infty,+\infty)$ or all real numbers and the graph has been shifted up 3 , therefore the Range is $(3,+\infty)$ all real numbers strictly greater than 3.



28. -

For the following exercises, describe the end behavior of the graphs of the functions.

29. $f\left(x\right)=-5\left(4\right)^x-1$

Since there is a negative in front of our function, this graph is reflected about the x- axis, putting the y-values below the x-axis. It is also shifted down 1 which means the horizontal asymptote will be at $y=-1$. It follows then that as $x\to\infty$, $f\left(x\right)\to-\infty$; and as $x\to-\infty$, $f\left(x\right)\to-1$

30. -

31. $f(x)=3(4)^{-x}+2$

Since the base $b>1$ this is an increasing function however, this has also reflected about the y-axis which changes it to a decreasing function. It is also shifted up 2 which means the horizontal asymptote will be at $y=2$. It follows then that as $x\to\infty$, $f(x)\to 2$; and as $x\to-\infty$, $f(x)\to\infty$

For the following exercises, start with the graph of $f(x)=4^x$. Then write a function that results from the given transformation.

32. -

33. Shift $f(x)$ 3 units downward

Shifting downward means $d=-3$, therefore $f(x)=4^x-3$

34. -

35. Shift $f(x)$ 5 units right

Shifting 5 units to the right means $c=-5$, therefore $f(x)=4^{x-5}$

36. -

37. Reflect $f(x)$ about the *y*-axis

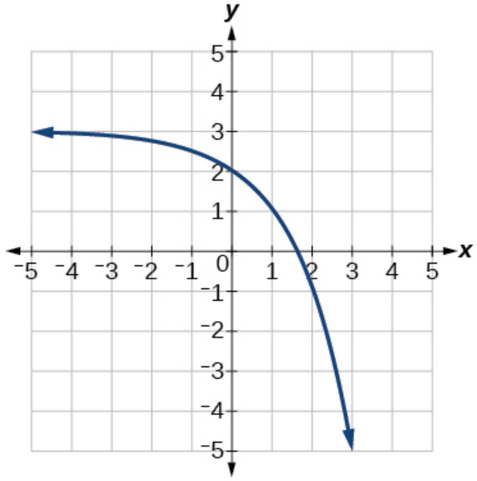
Inputting a negative sign in front of the x reflects the graph about the y -axis

$f(x)=4^{-x}$

For the following exercises, each graph is a transformation of $y=2^x$. Write an equation describing the transformation.

38. -

39.



All exponential functions can be written in the form $y = ab^{x+c} + d$

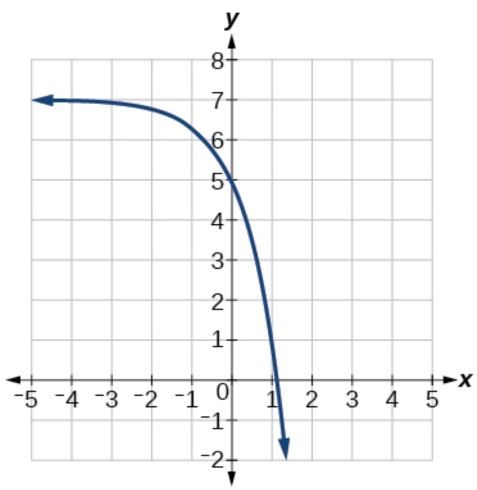
This graph has been reflected about the x-axis, thus a negative needs to be put in front of the function and we see that the horizontal asymptote is shifted up to $y = 3$, so it has shifted up 3 units making $d = +3$ Knowing that it has shifted up 3, puts our y-intercept at $(0,-1)$ prior to that shift up, which means that the multiplying factor $a = -1$.

$y = -2^x + 3$

40. -

For the following exercises, find an exponential equation for the graph.

41.



All exponential functions can be written in the form $y = ab^{x+c} + d$

This graph has been reflected about the x-axis, thus a negative needs to be put in front of the function and we see that the horizontal asymptote is shifted up to $y = 7$, so it has shifted up 7 units making $d = +7$ Knowing that it has shifted up 7, puts our y-intercept at $(0, -2)$ prior to that shift up, which means that the multiplying factor $a = -2$.

$y = -2(3)^x + 7$

42. -

Numeric

For the following exercises, evaluate the exponential functions for the indicated value of x.

43. $g(x) = \frac{1}{3}(7)^{x-2}$ for $g(6)$.

From the graph which is $f(x) = 7^{x-2}$ we obtain the y-value when x= 6,

$f(6) = 7^{6-2} = 7^4 = 2401$, from that we do the vertical compression by multiplying by $\frac{1}{3}$ which is $g(6) = \frac{1}{3}(2401) = 800.\overline{3}$

44. -

45. $h(x) = -\frac{1}{2}\left(\frac{1}{2}\right)^x + 6$ for $h(-7)$.

From the parent graph which is $f(x) = \left(\frac{1}{2}\right)^x$ we obtain the y-value when $x = -7$,

$f(-7) = \left(\frac{1}{2}\right)^{-7} = 128$, from that we do the vertical compressing and reflection about the x-axis by multiplying by $-\frac{1}{2}$ which is $g(-7) = -\frac{1}{2}(128) = -64$, then we shift it up 6, by adding 6, $-64 + 6 = -58$ therefore $h(-7) = -58$

Technology

For the following exercises, use a graphing calculator to approximate the solutions of the equation. Round to the nearest thousandth. $f(x) = ab^x + d$.

46. -

47. $116 = \frac{1}{4}\left(\frac{1}{8}\right)^x$

See directions in #47 and Input :

$Y_1 = (1 \div 4)(1 \div 8) \wedge (x)$ =

$Y_2 = 116$

Press **[WINDOW]**. Change y max to include 116. Continue to find the point of intersection. Your answer should be $x \approx -2.953$

48. -

49. $5 = 3\left(\frac{1}{2}\right)^{x-1} - 2$

See directions in #47 and Input :

$Y_1 = 3(1 \div 2) \wedge (x - 1) - 2$

$Y_2 = 5$

You may always hit ZOOM #6 ZSTD, which resets your window to zoom standard $x = -1 \text{ to } x = 10 \quad and \quad y = -10 \text{ to } y = 10$. Continue to find the point of intersection.

Your answer should be $x \approx -0.222$

50. -

Extensions

51. Explore and discuss the graphs of $F(x) = (b)^x$ and $G(x) = \left(\frac{1}{b}\right)^x$. Then make a conjecture about the relationship between the graphs of the functions b^x and $\left(\frac{1}{b}\right)^x$ for any real number $b > 0$.

Because if you were to write $\frac{1}{b}$ *as* b^{-1} it follows that $G(x) = \left(\frac{1}{b}\right)^x = (b^{-1})^x = b^{-x}$, this would then be the reflecting of the graph of $F(x) = (b)^x$ about the y-axis. thus The graph of $G(x) = \left(\frac{1}{b}\right)^x$ is the reflection about the *y*-axis of the graph of $F(x) = b^x$; For any real number $b > 0$ and function $f(x) = b^x$, the graph of $\left(\frac{1}{b}\right)^x$ is the reflection about the *y*-axis, $F(-x)$.

52. -

53. Explore and discuss the graphs of $f(x) = 4^x$, $g(x) = 4^{x-2}$, and $h(x) = \left(\frac{1}{16}\right)4^x$. Then make a conjecture about the relationship between the graphs of the functions b^x and $\left(\frac{1}{b^n}\right)b^x$ for any real number *n* and real number $b > 0$.

Because $g(x) = 4^{x-2} = 4^x g 4^{-2} = 4^x g\left(\frac{1}{16}\right)$ is the identical function $h(x) = \left(\frac{1}{16}\right)4^x = \left(\frac{1}{4^2}\right)4^x$, we see that since the graphs of $g(x)$ and $h(x)$ are the same and are a horizontal shift to the right of the graph of $f(x)$; For any real number *n*, real number $b > 0$, and function $f(x) = b^x$, the graph of $\left(\frac{1}{b^n}\right)b^x$ is the horizontal shift $f(x-n)$.

54. -

Chapter 6
Exponential and Logarithmic Functions
6.3 Logarithmic Functions

Section Exercises

Verbal

1. What is a base b logarithm? Discuss the meaning by interpreting each part of the equivalent equations $b^y = x$ and $\log_b x = y$ for $b > 0,\ b \neq 1.$

   A logarithm is an exponent. Specifically, it is the exponent to which a base b is raised to produce a given value. In the expressions given, the base b has the same value. The exponent, $y,$ in the expression b^y can also be written as the logarithm, $\log_b x,$ and the value of x is the result of raising b to the power of $y.$
2. -
3. How can the logarithmic equation $\log_b x = y$ be solved for x using the properties of exponents?

   Since the equation of a logarithm is equivalent to an exponential equation, the logarithm can be converted to the exponential equation $b^y = x,$ and then properties of exponents can be applied to solve for $x.$
4. -
5. Discuss the meaning of the natural logarithm. What is its relationship to a logarithm with base $b,$ and how does the notation differ?

   The natural logarithm is a special case of the logarithm with base b in that the natural log always has base $e.$ Rather than notating the natural logarithm as $\log_e(x),$ the notation used is $\ln(x).$

Algebraic

For the following exercises, rewrite each equation in exponential form.

6. -
7. $\log_a(b) = c$

   Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$, since the base = a and the exponent is c, we rewrite this $a^c = b$
8. -
9. $\log_x(64) = y$

   Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$, since the base = x and the exponent is y, we rewrite this $x^y = 64$
10. -

11. $\log_{15}(a) = b$

Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$, since the base = 15 and the exponent is b, we rewrite this $15^b = a$

12. -

13. $\log_{13}(142) = a$

Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$, since the base = 13 and the exponent is a, we rewrite this $13^a = 142$

14. -

15. $\ln(w) = n$

Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$, but since there is $\ln(w) = n$ this means the same thing as $\log_e w = n$ natural logarithm base which = e and the exponent is n, we rewrite this $e^n = w$

For the following exercises, rewrite each equation in logarithmic form.

16. -

17. $c^d = k$

Rewriting an exponential expression $base^{exponent} = N$ in logarithmic form is $\log_{base} N = exponent$. Since the $base = c$ and the exponent (which always is alone in the logarithmic equation), $exponent = d$, we rewrite this = $\log_c(k) = d$

18. -

19. $19^x = y$

Rewriting an exponential expression $base^{exponent} = N$ in logarithmic form is $\log_{base} N = exponent$. Since the $base = 19$ and the exponent (which always is alone in the logarithmic equation), $exponent = x$, we rewrite this $\log_{19} y = x$

20. -

21. $n^4 = 103$

Rewriting an exponential expression $base^{exponent} = N$ in logarithmic form is $\log_{base} N = exponent$. Since the $base = n$ and the exponent (which always is alone in the logarithmic equation), $exponent = 4$, we rewrite this $\log_n(103) = 4$

22. -

23. $y^x = \frac{39}{100}$

Rewriting an exponential expression $base^{exponent} = N$ in logarithmic form is

$\log_{base} N = exponent$. Since the $base = y$ and the exponent (which always is alone in the logarithmic equation), $exponent = x$, we rewrite this $\log_y \left(\frac{39}{100}\right) = x$

24.

25. $e^k = h$

Rewriting an exponential expression $base^{exponent} = N$ in logarithmic form is $\log_{base} N = exponent$. Since the $base = e$, we use $\ln$ which means $\log_e$. The exponent (which always is alone in the logarithmic equation), $exponent = k$, we rewrite this $\ln(h) = k$

For the following exercises, solve for x by converting the logarithmic equation to exponential form.

26. -

27. $\log_2(x) = -3$

Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$, since the base = 2 and the exponent is -3, we rewrite this $2^{-3} = x$, therefore $x = 2^{-3} = \frac{1}{8}$

28. -

29. $\log_3(x) = 3$

Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$, since the base = 3 and the exponent is 3, we rewrite this $3^3 = x$, therefore $x = 3^3 = 27$

30. -

31. $\log_9(x) = \frac{1}{2}$

Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$, since the base = 9 and the exponent is $\frac{1}{2}$, we rewrite this $9^{\frac{1}{2}} = x$, therefore $x = 9^{\frac{1}{2}} = \sqrt[2]{9^1} = 3$

32. -

33. $\log_6(x) = -3$

Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$, since the base = 6 and the exponent is -3, we rewrite this $6^{-3} = x$, therefore $x = 6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

34. -

35. $\ln(x) = 2$

Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$, since there is an ln written ,the base = e and the exponent is 2, we rewrite this $e^2 = x$, therefore $x = e^2$.
(Note: we leave this with e in our answer as it is the exact answer. To approximate it you may use the calculator and obtain $e^2 \approx 7.389056099$*)*

For the following exercises, use the definition of common and natural logarithms to simplify.

36. -

37. $10^{\log(32)}$

Let $\log(32) = x$, that means $10^x = 32$, so $10^{\log(32)} = 10^x = 32$

38. -

39. $e^{\ln(1.06)}$

Let $\ln(1.06) = x$, rewriting it exponentially yields $e^x = 1.06$, so $e^{\ln(1.06)} = e^x = 1.06$

40. -

41. $e^{\ln(10.125)} + 4$

Let $\ln(10.125) = x$, rewriting it exponentially yields $e^x = 10.125$, so
$e^{\ln(10.125)} = e^x = 10.125$, therefore $e^{\ln(10.125)} + 4 = 10.125 + 4 = 14.125$

Numeric

For the following exercises, evaluate the base b logarithmic expression without using a calculator.

42. -

43. $\log_6\left(\sqrt{6}\right)$

Let $\log_6\left(\sqrt{6}\right) = x$, then rewrite it in exponential form , since the base = 6 and the exponent is x, we rewrite this $6^x = \sqrt[2]{6^1} = 6^{\frac{1}{2}}$. Therefore x = $\frac{1}{2}$; $\log_6\left(\sqrt{6}\right) = \frac{1}{2}$

44. -

45. $6\log_8(4)$

Let $\log_8(4) = x$, then rewrite it in exponential form , since the base = 8 and the exponent is x, we rewrite this $8^x = 4$ rewriting each side with the a base of 2 we have

$8^x = 4$

$(2^3)^x = 2^2$

$2^{3x} = 2^2$

It follows that $3x = 2$ *so* $x = \frac{2}{3}$, Since $\log_8(4) = x = \frac{2}{3}$, evaluating

$6 \bullet \log_8(4) = 6 \bullet \left(\frac{2}{3}\right) = 4$

For the following exercises, evaluate the common logarithmic expression without using a calculator.

46. -
47. $\log(0.001)$

Let $\log(.001) = x$, that means $10^x = .001 = \frac{1}{1,000} = \frac{1}{10^3} = 10^{-3}$, that means $x = -3$, so $\log(.0001) = -3$.

48. -
49. $2\log(100^{-3})$

Let $\log(100^{-3}) = x$, then rewrite it in exponential form $b^y = x$, since the base $= 10$ and the exponent is x, we rewrite this $10^x = 100^{-3} = (10^2)^{-3} = 10^{-6}$. It follows that $x = -6$, therefore $\log(10^{-3}) = -6$. Evaluating $2\log(100^{-3}) = 2(-6) = -12$

For the following exercises, evaluate the natural logarithmic expression without using a calculator.

50. -
51. $\ln(1)$

Let $\ln(1) = x$, rewriting it exponentially yields $e^x = 1 = e^0$, since the bases are the same the exponents must be equal, therefore x must equal 0; $\ln(1) = 0$

52. -
53. $25\ln\left(e^{\frac{2}{5}}\right)$

Let $\ln(e^{\frac{2}{5}}) = x$, rewriting it exponentially yields $e^x = e^{\frac{2}{5}}$, since the bases are the same the exponents must be equal, therefore $x = \frac{2}{5}$; $25\ln\left(e^{\frac{2}{5}}\right) = 25\left(\frac{2}{5}\right) = 10$

Technology

For the following exercises, evaluate each expression using a calculator. Round to the nearest thousandth.

54. -
55. $\ln(15)$

    $\ln(15) = 2.708050201$ rounded to the thousandths place ≈ 2.708

56. -
57. $\log(\sqrt{2})$

    $\log(\sqrt{\ }(2)) = .1505149978$ rounded to the thousandths place ≈ 0.151

58. -

Extensions

59. Is $x = 0$ in the domain of the function $f(x) = \log(x)$? If so, what is the value of the function when $x = 0$? Verify the result.
    To verify we the end conclusion stated here, we input zero in for x and rewriting $N = \log(0)$ in exponential form we obtain $10^N = 0,$ Since there are no possible real solutions when raising 10 to the N power that can result in an answer of zero, zero must be eliminated from the domain. No, the function has no defined value for $x = 0$. Therefore, $x = 0$ is *not* the domain of the function $f(x) = \log(x)$.
60. -
61. Is there a number x such that $\ln x = 2$? If so, what is that number? Verify the result.
    Yes. Suppose there is a real number, x, such that $\ln x = 2$. Rewriting as an exponential equation gives $x = e^2$, which is a real number ($x = e^2 \approx 7.389056099$). To verify, let $x = e^2$. $\ln(x) = \ln 7.389056099 = 2.$ Then, by definition, $\ln(x) = \ln(e^2) = 2$.
62. -
63. Is the following true: $\dfrac{\ln(e^{1.725})}{\ln(1)} = 1.725$? Verify the result.

Since $\ln(1) = 0,$ we would have division by zero and this would be an undefined expression.

Therefore , no it is not true; The expression $\dfrac{\ln(e^{1.725})}{\ln(1)}$ is undefined.

Real-World Applications

64. -

65. Refer to the previous exercise. Suppose the light meter on a camera indicates an EI of $-2,$ and the desired exposure time is 16 seconds. What should the f-stop setting be?

Inserting the given numbers in their respective places in the formula, we will continue to solve for f ; $-2=\log_2\left(\frac{f^2}{16}\right)$. Rewriting this exponentially yields

$$2^{-2}=\frac{f^2}{16}$$

$$\frac{1}{4}=\frac{f^2}{16} \quad \text{multiply by } 16$$

$$16 \cdot \frac{1}{4}=16 \cdot \frac{f^2}{16}$$

$$4=f^2$$

$$\pm 2=f$$

We choose the positive value, therefore $f=2$

66. -

Chapter 6
Exponential and Logarithmic Functions
6.4 Graphs of Logarithmic Functions

Section Exercises
Verbal

1. The inverse of every logarithmic function is an exponential function and vice-versa. What does this tell us about the relationship between the coordinates of the points on the graphs of each?
   Since the functions are inverses, their graphs are mirror images about the line $y = x$. So for every point (a,b) on the graph of a logarithmic function, there is a corresponding point (b,a) on the graph of its inverse exponential function.
2. -
3. What type(s) of translation(s), if any, affect the domain of a logarithmic function?
   Shifting the function right or left and reflecting the function about the y-axis will affect its domain.
4. -
5. Does the graph of a general logarithmic function have a horizontal asymptote? Explain.
   No. A horizontal asymptote would suggest a limit on the range, and the range of any logarithmic function in general form is all real numbers.

Algebraic

For the following exercises, state the domain and range of the function.

6. -
7. $h(x) = \ln\left(\frac{1}{2} - x\right)$
   We must keep
   $$\frac{1}{2} - x > 0$$
   $$\frac{1}{2} - x + x > 0 + x$$
   $$\frac{1}{2} > x$$
   it follows that $x < \frac{1}{2}$, therefore the Domain is $\left(-\infty, \frac{1}{2}\right)$; The y-values then take on all values with no restriction, therefore the Range is $(-\infty, \infty)$
8. -

9. $h(x) = \ln(4x + 17) - 5$

We must keep

$$4x + 17 > 0$$
$$4x + 17 - 17 > 0 - 17$$
$$4x > -17$$
$$\frac{4x}{4} > \frac{-17}{4}$$

it follows that $x > \frac{-17}{4}$, therefore the Domain is $\left(-\frac{17}{4}, \infty\right)$; The y-values then take on all values with no restriction, therefore the Range is $(-\infty, \infty)$

10. -

For the following exercises, state the domain and the vertical asymptote of the function.

11. $f(x) = \log(x - 5)$

For the domain, we must keep

$$x - 5 > 0$$
$$x - 5 + 5 > 0 + 5$$
$$x > 5$$

it follows that $x > 5$, therefore the Domain is $(5, \infty)$; This graph shifts to the right 5 units, therefore the vertical asymptote which is normally occurs at $x = 0$, shifts to the right 5 units. The vertical asymptote will be at $x = 5$.

12. -

13. $f(x) = \log(3x + 1)$

For the domain, we must keep

$$3x + 1 > 0$$
$$3x + 1 - 1 > 0 - 1$$
$$3x > -1$$
$$\frac{3x}{3} > \frac{-1}{3}$$
$$x > \frac{-1}{3}$$

it follows that $x > \frac{-1}{3}$, therefore the Domain is $\left(-\frac{1}{3}, \infty\right)$; This graph shifts to the left $\frac{1}{3}$ units, therefore the vertical asymptote which is normally occurs at $x = 0$, shifts to the left $\frac{1}{3}$ units. The vertical asymptote will be at $x = -\frac{1}{3}$.

14. -

15. $g(x) = -\ln(3x+9) - 7$

For the domain, we must keep

$$3x + 9 > 0$$
$$3x + 9 - 9 > 0 - 9$$
$$3x > -9$$
$$\frac{3x}{3} > \frac{-9}{3}$$
$$x > -3$$

it follows that $x > -3$, therefore the Domain is $(-3, \infty)$; This graph reflects about the x-axis and shifts to the left 3 units, therefore the vertical asymptote which is normally occurs at $x = 0$, shifts to the left 3 units. The vertical asymptote will be at $x = -3$.

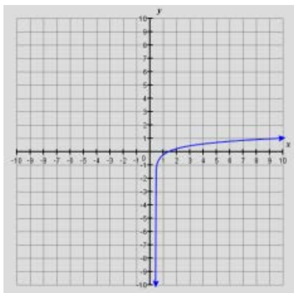
For the following exercises, state the domain, vertical asymptote, and end behavior of the function.

16. -

17. $f(x) = \log\left(x - \frac{3}{7}\right)$

For the domain, we must keep

$$x - \frac{3}{7} > 0$$
$$x - \frac{3}{7} + \frac{3}{7} > 0 + \frac{3}{7}$$
$$x > \frac{3}{7}$$



it follows that $x > \frac{3}{7}$, therefore the domain is $\left(\frac{3}{7}, \infty\right)$; This graph shifts to the right $\frac{3}{7}$ units, therefore the vertical asymptote which is normally occurs at $x = 0$, shifts to the right $\frac{3}{7}$ units. The vertical asymptote will be at $x = \frac{3}{7}$. Sketching this graph helps describe the end behavior.

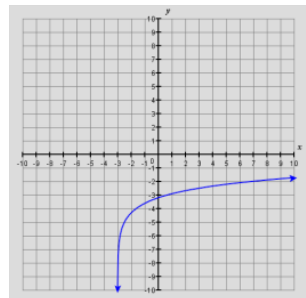
End behavior: as *x* approaches $\frac{3}{7}$ from the right, determine what *y*-value, $f(x)$, approaches. $x \to \left(\frac{3}{7}\right)^{+}$, $f(x) \to -\infty$ and as *x* approaches $+\infty$, determine what y-value, $f(x)$, approaches. As $x \to \infty, f(x) \to \infty$

18. -

19. $g(x) = \ln(2x+6) - 5$

For the domain, we must keep

$$2x + 6 > 0$$
$$2x + 6 - 6 > 0 - 6$$
$$2x > -6$$
$$\frac{2x}{2} > \frac{-6}{2}$$
$$x > -3$$



it follows that $x > -3$, therefore the domain is $(-3, \infty)$; This graph shifts to the left 3 units, therefore the vertical asymptote which is normally occurs at $x = 0$, shifts to the right -3 units. The vertical asymptote will be at $x = -3$. Sketching this graph helps describe the end behavior.

End behavior: as *x* approaches -3 from the right, determine what *y*-value, $f(x)$, approaches. $x \to -3^{+}$, $f(x) \to -\infty$ and as *x* approaches $+\infty$, determine what *y*-value, $f(x)$, approaches. As $x \to \infty, f(x) \to \infty$

20. -

For the following exercises, state the domain, range, and *x*- and *y*-intercepts, if they exist. If they do not exist, write DNE.

21. $h(x) = \log_4(x-1) + 1$

For the domain, we must keep

$$x - 1 > 0$$
$$x - 1 + 1 > 0 + 1$$
$$x > 1$$

it follows that $x > 1$, therefore the domain is $(1, \infty)$; The y-values then take on all values with no restriction, therefore the range is $(-\infty, \infty)$. This graph shifts to the right 1 unit, therefore the vertical asymptote which is normally occurs at $x = 0$, will be at $x = 1$.

To find the *x*-intercept we set y = 0:

$$0 = \log_4(x-1)+1$$
$$0-1 = \log_4(x-1)+1-1$$
$$-1 = \log_4(x-1) \quad \textit{rewrite exponentially}$$
$$4^{-1} = x-1$$
$$\frac{1}{4}+1 = x-1+1$$
$$\frac{5}{4} = x$$

Therefore the x-intercept is $\left(\frac{5}{4},\ 0\right)$;

To find the y-intercept we set x = 0:

$$y = \log_4(0-1)$$
$$y = \log_4(-1) \quad \textit{no real solution}$$
or rewrite exponentially
$$4^y = -1$$
no real solution

Therefore the *y*-intercept DNE.

22. -

23. $g(x) = \ln(-x) - 2$

For the domain, we must keep

$$-x > 0$$
$$\frac{-x}{-1} < \frac{0}{-1}$$
$$x < 0$$

it follows that $x < 0$, therefore the domain is $(-\infty,\ 0)$; The *y*-values then take on all values with no restriction, therefore the range is $(-\infty,\ \infty)$. This graph is not shifted, therefore the vertical asymptote is at $x = 0$.

To find the *x*-intercept we set *y* = 0:

$$0 = \log_e(-x)-2$$
$$0+2 = \log_e(-x)-2+2$$
$$2 = \log_e(-x) \quad \text{rewrite exponentially}$$
$$e^2 = -x$$
$$\frac{e^2}{-1} = \frac{-x}{-1}$$
$$-e^2 = x$$

Therefore the *x*-intercept is $(-e^2,\ 0)$;

To find the *y*-intercept we set *x* = 0:

$y = \log_e(0) - 2$
$y = \log_e(0)$ no real solution
or rewrite exponentially
$e^y = 0$
no real solution

Therefore the y-intercept DNE.

24. -

25. $h(x) = 3\ln(x) - 9$

For the domain, we must keep

$x > 0$

it follows that $x > 0$, therefore the Domain is $(0, \infty)$; The y-values then take on all values with no restriction, therefore the Range is $(-\infty, \infty)$. This graph does not shift, therefore the vertical asymptote remains at $x = 0$.

To find the *x*-intercept we set *y* = 0:

$$0 = 3\log_e(x) - 9$$
$$0 + 9 = 3\log_e(x) - 9 + 9$$
$$9 = 3\log_e(x)$$
$$\frac{9}{3} = \frac{3\log_e(x)}{3}$$
$$3 = \log_e(x) \quad \textit{rewrite exponentially}$$
$$e^3 = x$$

Therefore the x-intercept is $(e^3, 0)$;

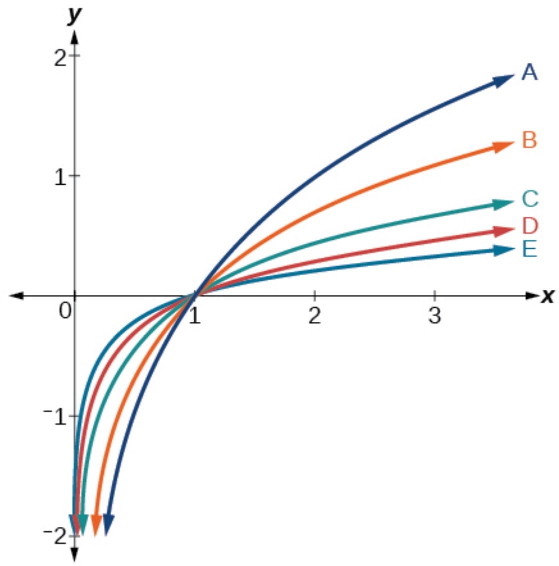
To find the y-intercept we set x = 0:

$y = 3\log_e(0) - 9$
$y = \log_e(0)$ no real solution
or rewrite exponentially
$e^y = 0$
no real solution

Therefore they-intercept DNE.

Graphical

For the following exercises, match each function with the letter corresponding to its graph.

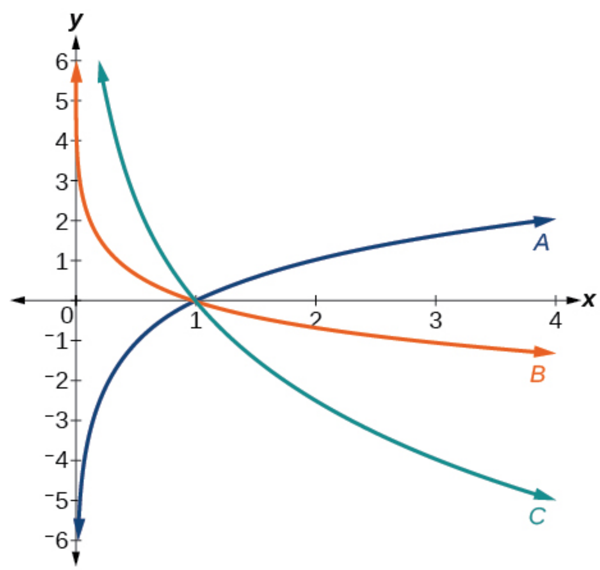


26. -
27. $f(x) = \ln(x)$

    Since this base is e and $e \approx 2.71828$ it would be the second lowest base to # 28, therefore it is choice = B
28. -
29. $h(x) = \log_5(x)$

    Since this base is 5 it would be the third lowest base to bases 2 and e, therefore it is choice = C
30. -

For the following exercises, match each function with the letter corresponding to its graph.



31. $f(x) = \log_{\frac{1}{3}}(x)$

Since the base $\frac{1}{3} < 1$, this graph will be reflected about the x-axis and since $\frac{1}{3} < \frac{3}{4}$ comparing it to the graph of #33, the smaller base will decrease slower, therefore this graph is choice = B

32. -

33. $h(x) = \log_{\frac{3}{4}}(x)$

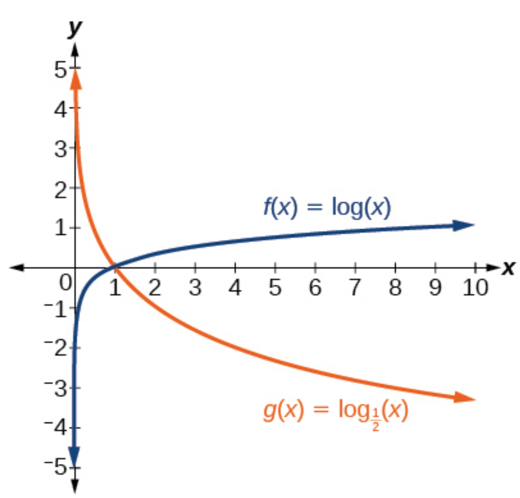
Since the base $\frac{3}{4} < 1$, this graph will be reflected about the x-axis and since $\frac{1}{3} < \frac{3}{4}$ comparing it to the graph of #31, this being the larger base will decrease faster, therefore this graph is choice = C

For the following exercises, sketch the graphs of each pair of functions on the same axis.

34. -

35. $f(x) = \log(x)$ and $g(x) = \log_{\frac{1}{2}}(x)$

From the first graph we have base $10 > 1$, which would have the normal logarithm model. The second graph with a base $\frac{1}{2} < 1$ will be reflected about the x-axis and decreasing at a faster rate than the reflected graph of the first which would be a base of $\frac{1}{10}$.



36. -

37. $f(x) = e^x$ and $g(x) = \ln(x)$

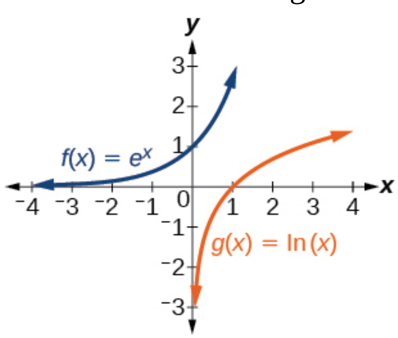
These graphs are simply inverses of one another. Plot the exponential function $f(x) = e^x$ and interchange the x and y values to plot the second graph. For example

$(0,1)$ *would be* $(1,0)$

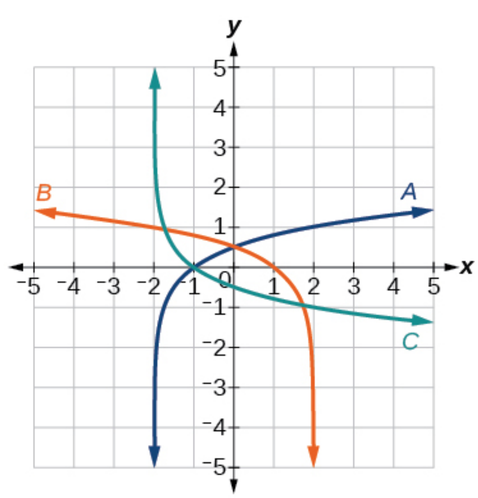
$(1,e)$ *would be* $(e,1)$

$(-1,\frac{1}{e})$ *would be* $(\frac{1}{e},-1)$

Also these would be mirror images of each other reflected about the x = y axis.



For the following exercises, match each function with the letter corresponding to its graph.



38. -

39. $g(x) = -\log_4(x+2)$

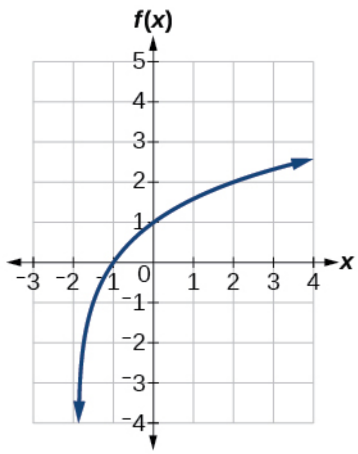
This should be the graph of $f(x) = \log_4(x)$ reflected about the x-axis axis making the x-intercept remain as $(1,0)$ and shifted left 2 units, making the x-intercept shift from $(1,0)$ *to* $(-1,0)$. Therefore choice = C

40. -

For the following exercises, sketch the graph of the indicated function.

41. $f(x) = \log_2(x+2)$

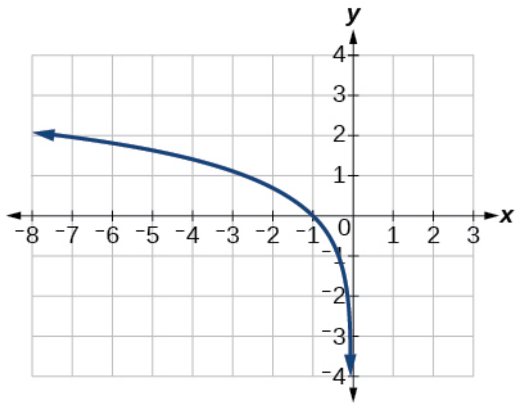
This should be the graph of $f(x) = \log_2(x)$ shifted left 2 units, making the x-intercept shift from $(1,0)$ *to* $(-1,0)$.



42. -

43. $f(x) = \ln(-x)$

This should be the graph of $f(x) = \ln(x)$ reflected about the y-axis, making the x-intercept shift from $(1,0)$ *to* $(-1,0)$.



44. -

45. $g(x)=\log(6-3x)+1$

For the domain, we must keep

$$6-3x>0$$
$$6-3x-6>0-6$$
$$-3x>-6$$
$$\frac{-3x}{-3}<\frac{-6}{-3}$$
$$x<2$$

it follows that since the domain is $(-\infty,\ 2)$; This graph shifts to the right 2 units, therefore the vertical asymptote which normally occurs at $x=0$, shifts to the right 2 units and is at $x=2$. It also has been shifted up 1 unit.

To find the *x*-intercept we set *y* = 0:

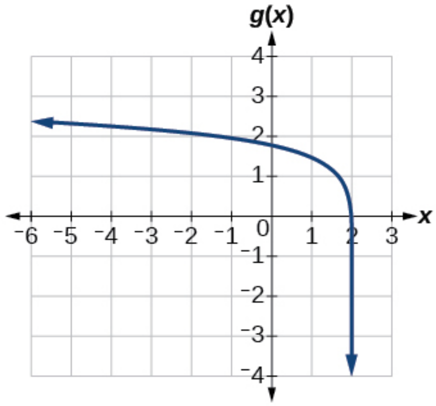
$$0=\log(6-3x)+1$$
$$0+-1=\log(6-3x)+1-1$$
$$-1=\log(6-3x) \qquad \textit{rewrite exponentially}$$
$$10^{-1}=6-3x$$
$$10^{-1}-6=6-3x-6$$
$$-5.9=-3x$$
$$\frac{-5.9}{-3}=\frac{-3x}{-3}$$
$$1.966\overline{6}=x$$
$$1.96\approx x$$

Therefore the *x*-intercept is $(1.96,\ 0)$;

To find the *y*-intercept we set *x* = 0:

$y = \log(6 - 3(0)) + 1$
$y = \log(6) + 1$
$y = .7781512504 + 1$
$y \approx 1.8$

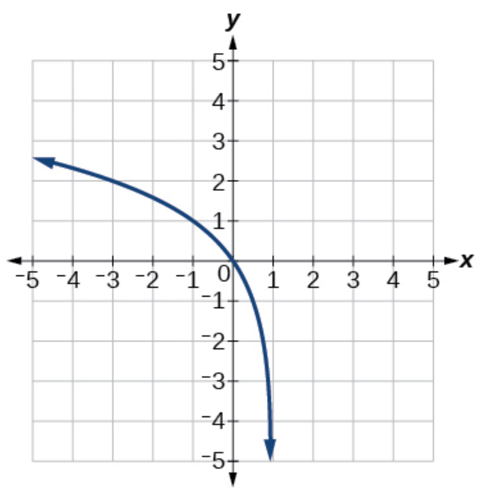
Therefore the y-intercept is $(0,\ 1.8)$



46. -

For the following exercises, write a logarithmic equation corresponding to the graph shown.

47. Use $y = \log_2(x)$ as the parent function.

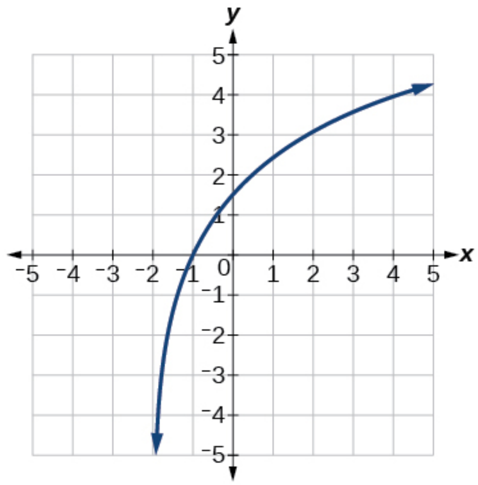


Using the model given in the text: $y = a\log_b(x + c) + d$

This graph has been reflected about the y-axis thus we put a negative sign in front of the x in the function. We also see that the x-intercept has shifted from $(-1, 0)$ *to* $(0, 0)$ which means shifting 1 unit to the right would make $+c$ in our model be $+c = -1$, therefore our equation would be $y = \log_2(-(x - 1))$

48. -

49. Use $f(x) = \log_4(x)$ as the parent function.



Using the model given in the text: $y = a\log_b(x+c)+d$

This graph has NOT been reflected about either axes. We also see that the x-intercept has shifted from $(1,0)$ *to* $(-1,0)$ which means shifting 2 units to the left would make $+c$ in our model be $+c = +2$. This graph has a vertical asymptote at $x = -2$. We know it has no vertical shift because it stayed on the x-axis therefore there may be a vertical stretch. Now to find a we substitute any coordinates we see on the graph such as $(0,1.5)$ into our model:

$$y = a\log_b(x+c)$$
$$1.5 = a\log_4(0+2)$$
$$1.5 = a\log_4(2) \qquad \log_4(2)\, is\, \frac{1}{2}$$
$$1.5 = a(.5)$$
$$\frac{1.5}{.5} = \frac{a(.5)}{.5}$$
$$3 = a$$

Therefore our equation would be $y = 3\log_4(x+2)$.

50. -

Technology

For the following exercises, use a graphing calculator to find approximation solutions to each equation.

51. $\log(x-1)+2=\ln(x-1)+2$

Enter each function into your graphing calculator

$\mathbf{Y_1}=\log(x-1)+2$

$\mathbf{Y_2}=\ln(x-1)+2$

2nd calculate "intersection" and round answer to the thousandths place $x=2$

52. -

53. $\ln(x-2)=-\ln(x+1)$

$\mathbf{Y_1}=\ln(x-2)$

$\mathbf{Y_2}=-\ln(x+1)$

2nd calculate "intersection" and round answer to the thousandths place $x\approx 2.303$

54. -

55. $\frac{1}{3}\log(1-x)=\log(x+1)+\frac{1}{3}$

$\mathbf{Y_1}=\frac{1}{3}\log(1-x)$

$\mathbf{Y_2}=\log(x+1)+\frac{1}{3}$

2nd calculate "intersection" and round answer to the thousandths place $x\approx -0.472$

Extensions

56. -

57. Explore and discuss the graphs of $f(x)=\log_{\frac{1}{2}}(x)$ and $g(x)=-\log_2(x)$. Make a conjecture based on the result.

The graphs of $f(x)=\log_{\frac{1}{2}}(x)$ and $g(x)=-\log_2(x)$ appear to be the same; Conjecture: for any positive base $b\neq 1$, $\log_b(x)=-\log_{\frac{1}{b}}(x)$.

58. -

59. What is the domain of the function $f(x)=\ln\left(\frac{x+2}{x-4}\right)$? Discuss the result.

Recall that the argument of a logarithmic function must be positive, so we determine where $\frac{x+2}{x-4}>0$. For the domain $\frac{x+2}{x-4}>0$, this is true when both $x+2>0$ and $x-4>0$, which would be intersection of the two sets $x>-2$ *and* $x>4$, which would be $x>4$. Of course it would also be true when both $x+2<0$ and $x-4<0$, which would be intersection of the two sets $x<-2$ *and* $x<4$, which would be $x<-2$. So the domain is restricted to $x<-2$ *or* $x>4$. Domain $(-\infty, -2)\cup(4, \infty)$.

From the graph of the function $f(x)=\frac{x+2}{x-4}$, note that the graph lies above the *x*-axis on the interval $(-\infty, -2)$ and again to the right of the vertical asymptote, that is $(4, \infty)$. Therefore, the domain is $(-\infty, -2)\cup(4, \infty)$.

60. -

Chapter 6
Exponential and Logarithmic Functions
6.5 Logarithmic Properties

Section Exercises

Verbal

1. How does the power rule for logarithms help when solving logarithms with the form $\log_b\left(\sqrt[n]{x}\right)$?

   Any root expression can be rewritten as an expression with a rational exponent so that the power rule can be applied, making the logarithm easier to calculate. Thus, $\log_b\left(\sqrt[n]{x}\right)$ can be written as $\log_b\left(x^{\frac{1}{n}}\right)=\frac{1}{n}\log_b(x)$.

2. -

Algebraic

For the following exercises, expand each logarithm as much as possible. Rewrite each expression as a sum, difference, or product of logs.

3. $\log_b(7x\cdot 2y)$

   $\log_b(7x\cdot 2y)=\log_b(7x)+\log_b(2y)$, then each term is the product rule again, where $\log_b(7\cdot x)=\log_b(7)+\log_b(x)$ and $\log_b(2\cdot y)=\log_b(2)+\log_b(y)$, So the final expansion is $\log_b(2)+\log_b(7)+\log_b(x)+\log_b(y)$

4. -

5. $\log_b\left(\frac{13}{17}\right)$

   The quotient rule for logs is subtraction for individual logs therefore $\log_b\left(\frac{13}{17}\right)=$ $\log_b(13)-\log_b(17)$

6. -

7. $\ln\left(\frac{1}{4^k}\right)$

   $\ln\left(\frac{1}{4^k}\right)=\ln 1-\ln(4^k)=0-\ln(4^k)=-\ln(4^k)$ then using the power rule the final expansion is $-k\ln(4)$

8. -

For the following exercises, condense to a single logarithm if possible.

9. $\ln(7)+\ln(x)+\ln(y)$

   Since addition is going on as individual logarithms, using the product rule in reverse, condensing to a single logarithm will result in multiplication, therefore

   $\ln(7)+\ln(x)+\ln(y)$ would be equal to $\ln(7 \cdot x \cdot y) = \ln(7xy)$

10. -

11. $\log_b(28)-\log_b(7)$

    Since subtraction is going on as individual logarithms, using the quotient rule in reverse, condensing to a single logarithm will result in division, therefore

    $\log_b(28)-\log_b(7)=\log_b\left(\frac{28}{7}\right) = \log_b(4)$

12. -

13. $-\log_b\left(\frac{1}{7}\right)$

    Rewriting this as individual logs because of division would be subtraction , thus

    $-\log_b\left(\frac{1}{7}\right)=-\left(\log_b(1)-\log_b(7)\right)=-\left(0-\log_b(7)\right) = \log_b(7)$.

14. -

For the following exercises, use the properties of logarithms to expand each logarithm as much as possible. Rewrite each expression as a sum, difference, or product of logs.

15. $\log\left(\frac{x^{15}y^{13}}{z^{19}}\right)$

    Applying the product rule to the numerator will result in addition and applying the quotient rule to the denominator will result in subtraction we have

    $\log\left(\frac{x^{15}y^{13}}{z^{19}}\right)=\log(x)^{15}+\log(y)^{13}-\log(z)^{19}$ next we apply the power rule and each of the exponents become coefficients, therefore the final expansion will be

    $15\log(x)+13\log(y)-19\log(z)$

16. -

17. $\log\left(\sqrt{x^3y^{-4}}\right)$

    Rewriting this and then applying the product rule we have

    $\log\left(\sqrt{x^3y^{-4}}\right)=\log\left(x^3 \cdot y^{-4}\right)^{\frac{1}{2}}=\log\left(x^{\frac{3}{2}} \cdot y^{-2}\right)=\log\left(x^{\frac{3}{2}}\right)+\log\left(y^{-2}\right)$. Next we apply the power rule and each of the exponents become coefficients, therefore the final expansion will be $= \frac{3}{2}\log(x) - 2\log(y)$.

18. -

19. $\log\left(x^2 y^3 \sqrt[3]{x^2 y^5}\right)$

Rewriting this, and simplifying we have

$$\log\left(x^2 y^3 \sqrt[3]{x^2 y^5}\right) = \log\left(x^2 y^3 g\left(x^2 y^5\right)^{\frac{1}{3}}\right) = \log\left(x^2 y^3 g\left(x^{\frac{2}{3}} y^{\frac{5}{3}}\right)\right) = \log\left(x^2 \cdot x^{\frac{2}{3}} \cdot y^3 \cdot y^{\frac{5}{3}}\right)$$

$$= \log\left(x^{2+\frac{2}{3}} \cdot y^{3+\frac{5}{3}}\right) = \log\left(x^{\frac{8}{3}} \cdot y^{\frac{14}{3}}\right)$$

Now we will apply the product rule and power rules $\log\left(x^{\frac{8}{3}} \cdot y^{\frac{14}{3}}\right) = \frac{8}{3}\log(x) + \frac{14}{3}\log(y)$.

For the following exercises, condense each expression to a single logarithm using the properties of logarithms.

20. -

21. $\ln(6x^9) - \ln(3x^2)$

Since subtraction is going on as individual logarithms, using the quotient rule in reverse, condensing to a single logarithm will result in division, therefore

$$\ln(6x^9) - \ln(3x^2) = \ln\left(\frac{6x^9}{3x^2}\right) = \ln(2x^{9-2}) = \ln(2x^7)$$

22. -

23. $\log(x) - \frac{1}{2}\log(y) + 3\log(z)$

Since there are coefficients on the front of the logarithms we apply the power rule first and place them as exponents $\log(x) - \frac{1}{2}\log(y) + 3\log(z) = \log(x) - \log\left(y^{\frac{1}{2}}\right) + \log(z^3)$ Next we group the first two terms and apply the quotient rule, then product rule and condensing to a single logarithm will result in

$$\left(\log(x) - \log\left(y^{\frac{1}{2}}\right)\right) + \log(z^3) = \left(\log\left(\frac{x}{y^{\frac{1}{2}}}\right)\right) + \log(z^3) = \left(\log\left(\frac{x}{y^{\frac{1}{2}}} g z^3\right)\right) = \log\left(\frac{xz^3}{\sqrt{y}}\right)$$

24. -

For the following exercises, rewrite each expression as an equivalent ratio of logs using the indicated base.

25. $\log_7(15)$ to base e

$$\log_b M = \frac{\log_n M}{\log_n b} = \frac{\log_e 15}{\log_e 7}$$, therefore $\log_7(15) = \frac{\ln(15)}{\ln(7)}$

26. -

For the following exercises, suppose $\log_5(6) = a$ and $\log_5(11) = b.$ Use the change-of-base formula along with properties of logarithms to rewrite each expression in terms of a and $b.$ Show the steps for solving.

27. $\log_{11}(5)$

Since we are given $\log_5(6) = a$ and $\log_5(11) = b.$ to use we need to change this expression with a base of 5 and realize that $\log_5(5) = 1$, then it follows that

$$\log_{11}(5) = \frac{\log_5(5)}{\log_5(11)} = \frac{1}{b}$$

28. -

29. $\log_{11}\left(\frac{6}{11}\right)$

Since we are given $\log_5(6) = a$ and $\log_5(11) = b.$ to use, first we need to change this expression with a base of 5. $\log_{11}\left(\frac{6}{11}\right) = \frac{\log_5\left(\frac{6}{11}\right)}{\log_5(11)}$. Next we need to think about rewriting the $\log_5\left(\frac{6}{11}\right)$ in terms of the given information $\log_5(11) = b.$ and $\log_5(6) = a$ Applying the quotient rule to this yields $\log_5\left(\frac{6}{11}\right) = \log_5(6) - \log_5(11) = a - b$, therefore it follows that

$$\log_{11}\left(\frac{6}{11}\right) = \frac{\log_5\left(\frac{6}{11}\right)}{\log_5(11)} = \frac{\log_5(6) - \log_5(11)}{\log_5(11)} = \frac{a-b}{b} \quad or \quad \frac{a}{b} - 1$$

Numeric

For the following exercises, use properties of logarithms to evaluate without using a calculator.

30. -

31. $6\log_8(2)+\dfrac{\log_8(64)}{3\log_8(4)}$

First we apply the power rule rewriting $6\log_8(2)=\log_8(2^6)$ and $3\log_8(4)=\log_8(4^3)$.

Recall rewriting each individual exponentially to solve it

1) $x=\log_8(2^6)$ *written exponentially is* $8^x=2^6$ *or* $2^{3x}=2^6$, *so* $x=2$, therefore $\log_8(2^6)=2$

2) $x=\log_8(4^3)$ *written exponentially is* $8^x=4^3$ *or* $2^{3x}=(2^2)^3$ *or* $2^{3x}=2^6$, $x=2$, therefore $\log_8(4^3)=2$

3) $x=\log_8(64)$ *written exponentially is* $8^x=64=8^2$, *so* $x=2$, therefore $\log_8(64)=2$

Entering these values into the expression yields $6\log_8(2)+\dfrac{\log_8(64)}{3\log_8(4)}=2+\dfrac{2}{2}=2+1=3.$

32. -

For the following exercises, use the change-of-base formula to evaluate each expression as a quotient of natural logs. Use a calculator to approximate each to five decimal places.

33. $\log_3(22)$

$$\log_3(22)=\frac{\log 22}{\log 3}=\log(22)\div\log(3)\approx 2.81359$$

34. -

35. $\log_6(5.38)$

$$\log_6(5.38)=\frac{\log 5.38}{\log 6}=\log(5.38)\div\log(6)\approx 0.93913$$

36. -

37. $\log_{\frac{1}{2}}(4.7)$

$$\log_{\frac{1}{2}}(4.7)=\frac{\log(4.7)}{\log\frac{1}{2}}=\log(4.7)\div\log(1\div 2)\approx -2.23266$$

Extensions

38. -

39. Use the quotient rule for logarithms to find all x values such that $\log_6(x+2)-\log_6(x-3)=1$. Show the steps for solving.

Using the quotient rule for logarithms we condense the left-hand side as a single logarithm:

$$\log_6(x+2) - \log_6(x-3) = 1$$
$$\log_6\left(\frac{x+2}{x-3}\right) = 1.$$

Rewriting as an exponential equation and solving for x:

$$6^1 = \frac{x+2}{x-3}$$
$$0 = \frac{x+2}{x-3} - 6$$
$$0 = \frac{x+2}{x-3} - \frac{6(x-3)}{(x-3)}$$
$$0 = \frac{x+2-6x+18}{x-3}$$
$$0 = \frac{x-4}{x-3}$$
$$x = 4$$

Checking, we find that $\log_6(4+2) - \log_6(4-3) = \log_6(6) - \log_6(1)$ is defined, so $x = 4$.

40. -

41. Prove that $\log_b(n) = \dfrac{1}{\log_n(b)}$ for any positive integers $b > 1$ and $n > 1$.

Let b and n be positive integers greater than 1. Then, by the change-of-base formula,

$$\log_b(n) = \frac{\log_n(n)}{\log_n(b)} = \frac{1}{\log_n(b)}.$$

42. –

Chapter 6
Exponential and Logarithmic Functions
6.6 Exponential and Logarithmic Equations

Section Exercises
Verbal

1. How can an exponential equation be solved?

   Determine first if the equation can be rewritten so that each side uses the same base. If so, the exponents can be set equal to each other. If the equation cannot be rewritten so that each side uses the same base, then apply the logarithm to each side and use properties of logarithms to solve.

2. -
3. When can the one-to-one property of logarithms be used to solve an equation? When can it not be used?

   The one-to-one property can be used if both sides of the equation can be rewritten as a single logarithm with the same base. If so, the arguments can be set equal to each other, and the resulting equation can be solved algebraically. The one-to-one property cannot be used when each side of the equation cannot be rewritten as a single logarithm with the same base.

Algebraic

For the following exercises, use like bases to solve the exponential equation.

4. -
5. $64 \cdot 4^{3x} = 16$

   Find the same common base between 64, 4, and 16, which would be 4. We want to rewrite each side with the base of 4, using all the correct exponent rules. Once we have a single base expression on each side and the base is the same, we set the exponents equal and solve for *x*.

   $64 \cdot 4^{3x} = 16$

   $4^3 \cdot 4^{3x} = 4^2$ Rewrite each side as a power with base 4.

   $4^{3+3x} = 4^2$ Product exponent rule on left side, add exponents.

   $3 + 3x = 2$ Apply the one-to-one property of exponents.

   $3x = -1$ Subtract 3 from both sides.

   $x = \frac{-1}{3}$ Divide by 3.

6. -

7. $2^{-3n} \cdot \frac{1}{4} = 2^{n+2}$

Find the same common base between 2 and 4 which would be 2. We want to rewrite each side with the base of 2, using all the correct exponent rules. Once we have a single base expression on each side and the base is the same, we set the exponents equal and solve for *x*.

$2^{-3n} \cdot \frac{1}{4} = 2^{n+2}$

$2^{-3n} \cdot \frac{1}{2^2} = 2^{n+2}$ Rewrite each side as a power with base 2.

$2^{-3n} \cdot 2^{-2} = 2^{n+2}$ Rewrite $\frac{1}{2^2}$ on left side as a negative exponent.

$2^{-3n+-2} = 2^{n+2}$ Product exponent rule on left side, add exponents

$-3n - 2 = n + 2$ Apply the one-to-one property of exponents.

$-4n - 2 = 2$ Subtract n from both sides.

$-4n = 4$ Add 2 to both sides.

$n = -1$ Divide by -4.

8. -

9. $\frac{36^{3b}}{36^{2b}} = 216^{2-b}$

Find the same common base between 36 and 216, which would be 6. We want to rewrite each side with the base of 6, using all the correct exponent rules. Once we have a single base expression on each side and the base is the same, we set the exponents equal and solve for *b*.

$$\frac{36^{3b}}{36^{2b}} = 216^{2-b}$$

$$\frac{\left(6^2\right)^{3b}}{\left(6^2\right)^{2b}} = \left(6^3\right)^{2-b}$$ Rewrite each side as a power with base 6.

$$\frac{6^{6b}}{6^{4b}} = 6^{6-3b}$$ Apply power to power, multiply the exponents, rule

$$6^{6b-4b} = 6^{6-3b}$$ Quotient exponent rule on left side, subtract exponents

$$6^{2b} = 6^{6-3b}$$

$$2b = 6 - 3b$$ Apply the one-to-one property of exponents.

$$5b = 6$$ Add 3b to both sides.

$$b = \frac{6}{5}$$ Divide by 5.

10. -

For the following exercises, use logarithms to solve.

11. $9^{x-10} = 1$

The terms of this exponential equation cannot be rewritten with a common base. In these cases, we solve by taking the logarithm of each side. Recall, since $\log(a) = \log(b)$ is equivalent to $a = b$, we may apply logarithms with the same base on both sides of an exponential equation.

$$9^{x-10} = 1$$ There is no easy way to get the powers to have the same base.

$$\ln 9^{x-10} = \ln 1$$ Take ln of both sides.

$$(x-10)\ln 9 = 0$$ Use laws of logs.

$$x\ln 9 - 10\ln 9 = 0$$ Use the distributive law.

$$x\ln 9 = 10\ln 9$$ Get terms containing x on one side, terms without x on the other.

$$\frac{x\ln 9}{\ln 9} = \frac{10\ln 9}{\ln 9}$$ Divide by $\ln 9$.

$$x = 10$$

12. -

13. $e^{r+10} - 10 = -42$

We want the e^{r+10} alone on one side before we begin.

$e^{r+10} - 10 = -42$ Add 10 to both sides

$e^{r+10} = -32$ Take the ln of both sides

$\ln e^{r+10} = \ln(-32)$

We can stop here because $\ln(-32)$ is not defined, therefore there is no solution

14. -

15. $-8 \cdot 10^{p+7} - 7 = -24$

We want the 10^{9a} alone on one side before we begin.

$-8 \cdot 10^{p+7} - 7 = -24$ Add 7 to both sides.

$-8 \cdot 10^{p+7} = -17$ Divide both sides by $-8 \cdot$

$$10^{p+7} = \frac{-17}{-8}$$

$$10^{p+7} = \frac{17}{8}$$

$\log 10^{p+7} = \log\left(\frac{17}{8}\right)$ Since it's in base 10, take $\log_{10}$ of both sides.

$(p+7)\log 10 = \log\left(\frac{17}{8}\right)$ Use laws of logs. Remember log10 =1

$$(p+7)(1) = \log\left(\frac{17}{8}\right)$$

$p+7 = \log\left(\frac{17}{8}\right)$ Subtract 7 from both sides

$$p = \log\left(\frac{17}{8}\right) - 7$$

16. -

17. $-5e^{9x-8} - 8 = -62$

We want the e^{9x-8} alone on one side before we begin.

$$-5e^{9x-8}-8=-62$$ Add 8 to both sides

$$-5e^{9x-8}=-54$$ Divide both sides by -5.

$$e^{9x-8}=\frac{-54}{-5}$$

$$e^{9x-8}=\frac{54}{5}$$

$$\ln e^{9x-8}=\ln\frac{54}{5}$$ Take ln of both sides.

$$(9x-8)\ln e=\ln\left(\frac{54}{5}\right)$$ Use laws of logs. Remember ln e=1

$$(9x-8)(1)=\ln\left(\frac{54}{5}\right)$$

$$9x-8=\ln\left(\frac{54}{5}\right)$$ Add 8 to both sides

$$9x=\ln\left(\frac{54}{5}\right)+8$$

$$x=\frac{\ln\left(\frac{54}{5}\right)+8}{9}$$ Divide by 9

18. -

19. $2^{x+1}=5^{2x-1}$

$$2^{x+1}=5^{2x-1}$$ There is no easy way to get the powers to have the same base.

$$\ln 2^{x+1}=\ln 5^{2x-1}$$ Take ln of both sides.

$$(x+1)\ln(2)=(2x-1)\ln(5)$$ Use laws of logs.

$$x\ln(2)+\ln(2)=2x\ln(5)-\ln(5)$$ Use the distributive law.

$$x\ln(2)-2x\ln(5)=-\ln(5)-\ln(2)$$ Get terms containing x on one side, terms without x on the other.

$$x(\ln(2)-2\ln(5))=-\ln(5)-\ln(2)$$ On the left hand side, factor out an x.

$$x=\frac{-\ln(5)-\ln(2)}{\ln(2)-2\ln(5)}$$ Divide both sidesby $(\ln(2)-2\ln(5))$

$$x=\frac{-\ln(2)-\ln(5)}{\ln(2)-2\ln(5)}$$

20. -

21. $7e^{8x+8} - 5 = -95.$

We want the e^{8x+8} alone on one side before we begin.

$7e^{8x+8} - 5 = -95.$ Add 5 to both sides

$7e^{8x+8} = -90$ Divide both sides by 7.

$$e^{8x+8} = \frac{-90}{7}$$

$\ln e^{8x+8} = \ln\left(\frac{-90}{7}\right)$ Take ln of both sides.

We can stop here because $\ln\left(\frac{-90}{7}\right)$ is not defined, therefore there is no solution

22. -

23. $4e^{3x+3} - 7 = 53.$

We want the e^{3x+3} alone on one side before we begin.

$4e^{3x+3} - 7 = 53.$ Add 7 to both sides

$4e^{3x+3} = 60$ Divide both sides by 4.

$$e^{3x+3} = \frac{60}{4}$$

$$e^{3x+3} = 15$$

$\ln e^{3x+3} = \ln(15)$ Take ln of both sides.

$(3x+3)\ln e = \ln(15)$ Use laws of logs. Remember ln e =1

$$(3x+3)(1) = \ln(15)$$

$3x + 3 = \ln(15)$ Subtract 3 from both sides

$$3x = \ln(15) - 3$$

$x = \frac{\ln(15) - 3}{3}$ Divide by 3

24. -

25. $3^{2x+1} = 7^{x-2}$

$3^{2x+1} = 7^{x-2}$ There is no easy way to get the powers to have the same base.

$\ln 3^{2x+1} = \ln 7^{x-2}$ Take ln of both sides.

$(2x+1)\ln(3) = (x-2)\ln(7)$ Use laws of logs.

$2x\ln(3) + \ln(3) = x\ln(7) - 2\ln(7)$ Use the distributive law.

$2x\ln(3) - x\ln(7) = -2\ln(7) - \ln(3)$ Get terms containing x on one side, terms without x on th

$x(2\ln(3) - \ln(7)) = -2\ln(7) - \ln(3)$ On the left hand side, factor out an x.

$x = \dfrac{-2\ln(7) - \ln(3)}{2\ln(3) - \ln(7)}$ Divide both sidesby $(2\ln(3) - \ln(7))$

26. -
27. $3e^{3-3x} + 6 = -31.$

We want the e^{3-3x} alone on one side before we begin.

$3e^{3-3x} + 6 = -31.$ Subtract 6 from both sides

$3e^{3-3x} = -37$ Divide both sides by 3.

$e^{3-3x} = \dfrac{-37}{3}$

$\ln e^{3-3x} = \ln\left(\dfrac{-37}{3}\right)$ Take ln of both sides.

We can stop here because $\ln\left(\dfrac{-37}{3}\right)$ is not defined, therefore there is no solution

For the following exercises, use the definition of a logarithm to rewrite the equation as an exponential equation.

28. -
29. $\log_{324}(18) = \dfrac{1}{2}$

$\log_{base}(S) = \exp onent$ if and only if $base^{exponent} = S$.

In this problem, the base is 324 and the exponent is $\dfrac{1}{2}$, therefore we rewrite this equation $324^{\frac{1}{2}} = 18$

For the following exercises, use the definition of a logarithm to solve the equation.

30. –

31. $-8\log_9 x = 16$

We want the $\log_9 x$ alone on one side before we begin.

$-8\log_9 x = 16$ Divide both sides by -8

$\log_9 x = -2$ Rewrite exponentially

$9^{-2} = x$

Therefore $x = \frac{1}{81}$

32. -

33. $2\log(8n+4)+6=10$

We want the $\log(8n+4)$ alone on one side before we begin.

$2\log(8n+4)+6=10$ Subtract 6 from both sides

$2\log(8n+4)=4$ Divide both sides by 2

$\log(8n+4)=2$

$10^2 = 8n+4$ Rewrite exponentially

$100-4=8n$ Subtract 4 from both sides

$96=8n$ Divide both sides by 8

$12=n$

Substituting $n=12$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution.

Therefore $n=12$

34. -

For the following exercises, use the one-to-one property of logarithms to solve.

35. $\ln(10-3x)=\ln(-4x)$

$\ln(10-3x)=\ln(-4x)$

$10-3x=-4x$ Use the one-to-one property of the logarithm.

$10=-1x$ Add 3x to both sides

$-10=x$ Divide by -1

Substituting $x=-10$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution.

Therefore $x=-10$

36. -

37. $\log(x+3)-\log(x)=\log(74)$

We have to rewrite each side as a single logarithm first

$$\log(x+3)-\log(x)=\log(74)$$

$$\log\left(\frac{x+3}{x}\right)=\log(74)$$ Use the one-to-one property of the logarithm

$$\left(\frac{x+3}{x}\right)=(74)$$ Multiply both sides by x

$$x+3=74x$$ Subtract x from both sides

$$3=73x$$ Divide by 73

$$\frac{3}{73}=x$$

$$\textit{Therefore}\quad x=\frac{3}{73}$$

Substituting $x=\frac{3}{73}$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution.

Therefore $x=\frac{3}{73}$

38. -

39. $\log_4(6-m)=\log_4 3m$

$$\log_4(6-m)=\log_4 3m$$

$$6-m=3m$$ Use the one-to-one property of the logarithm

$$6=4m$$ Add m to both sides

$$\frac{6}{4}=m$$ Divide by 4

$$\frac{3}{2}=m$$

$$\textit{Therefore}\quad m=\frac{3}{2}$$

Substituting $m=\frac{3}{2}$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution.

Therefore $m=\frac{3}{2}$

40. -

41. $\log_9(2n^2-14n)=\log_9(-45+n^2)$

$\log_9\left(2n^2-14n\right)=\log_9\left(-45+n^2\right)$

$2n^2-14n=-45+n^2$ Use the one-to-one property of the logarithm.

$n^2-14n+45=0$ Get zero on one side before factoring.

$(n-9)(n-5)=0$ Factor using FOIL.

$n-9=0$ or $n-5=0$ If a product is zero, one of the factors must be zero.

$x=9$ or $x=5$ Solve for *x*.

Substituting 5 into the original logarithmic functions we see:

$\log_9\left(2(5)^2-14(5)\right)=\log_9\left(-45+(5)^2\right)$

$\log_9\left(-20\right)=\log_9\left(-20\right)$

Because the argument of the logarithm functions is not positive, 5 is not a solution.

Substituting 9 into the original logarithmic functions we see:

$\log_9\left(2(9)^2-14(9)\right)=\log_9\left(-45+(9)^2\right)$

$\log_9\left(36\right)=\log_9\left(36\right)$

Because the argument of the logarithm functions is positive, 9 is a solution

Therefore $n=9$

42. –

For the following exercises, solve each equation for *x*.

43. $\log\left(x+12\right)=\log\left(x\right)+\log\left(12\right)$

We have to rewrite each side as a single logarithm first

$\log\left(x+12\right)=\log\left(x\right)+\log\left(12\right)$

$\log\left(x+12\right)=\log\left(x\cdot 12\right)$

$\log\left(x+12\right)=\log\left(12x\right)$ Use the one-to-one property of the logarithm

$x+12=12x$ Subtract x from both sides

$12=11x$ Divide by 11

$\frac{12}{11}=x$

Substituting $x=\frac{12}{11}$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution.

Therefore $x=\frac{12}{11}$.

44. -

45. $\log_2(7x+6)=3$

The logarithm is alone on one side, so we can rewrite this in exponential form

$\log_2(7x+6)=3$ rewrite exponentially

$2^3=7x+6$ subtract 6

$8=7x+6$

$2=7x$ divide by 7

$\frac{2}{7}=x$

$x=\frac{2}{7}$

Substituting $x=\frac{2}{7}$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution. Therefore $x=\frac{2}{7}$.

46. -

47. $\log_8(x+6)-\log_8(x)=\log_8(58)$

We have to rewrite each side as a single logarithm first

$\log_8(x+6)-\log_8(x)=\log_8(58)$

$\log_8\left(\frac{x+6}{x}\right)=\log_8(58)$ Use the one-to-one property of the logarithm

$\left(\frac{x+6}{x}\right)=(58)$ Multiply both sides by x

$x+6=58x$ Subtract x from both sides

$6=57x$ Divide by 57

$\frac{6}{57}=x$

$\frac{2}{19}=x$

Substituting $x=\frac{2}{19}$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution. Therefore $x=\frac{2}{19}$.

48. -

49. $\log_3(3x)-\log_3(6)=\log_3(77)$

We have to rewrite each side as a single logarithm first

$$\log_3(3x) - \log_3(6) = \log_3(77)$$

$$\log_3\left(\frac{3x}{6}\right) = \log_3(77) \quad \text{Use the one-to-one property of the logarithm}$$

$$\left(\frac{1x}{2}\right) = (77) \quad \text{Multiply both sides by 2}$$

$$x = 77(2)$$

$$x = 154$$

Substituting $x = 154$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution. Therefore $x = 154$.

Graphical

For the following exercises, solve the equation for x, if there is a solution. Then graph both sides of the equation, and observe the point of intersection (if it exists) to verify the solution.

50. -

51. $\log_3(x) + 3 = 2$

First we want the logarithm alone on one side, so we can rewrite this in exponential form

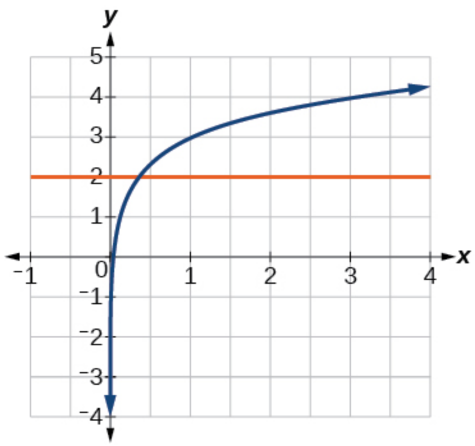
$$\log_3(x) + 3 = 2 \quad \text{Subtract 3 from both sides}$$

$$\log_3(x) = -1 \quad \text{rewrite exponentially}$$

$$3^{-1} = x$$

$$x = \frac{1}{3}$$

Substituting $x = \frac{1}{3}$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution. Therefore $x = \frac{1}{3}$.



52. -

53. $\ln(x-5)=1$

Since the logarithm is alone on one side already we may rewrite it exponentially;

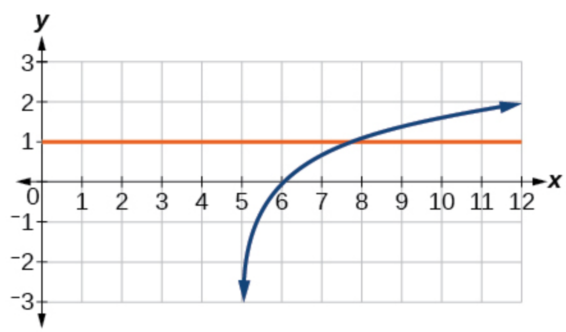
$\ln(x-5)=1$ rewrite exponentially

$e^1 = x-5$ add 5

$e+5=x$

$x=e+5\approx 7.7$

Substituting $x=e+5\approx 7.7$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution. Therefore $x=e+5\approx 7.7$.



54. -

55. $-7+\log_3(4-x)=-6$

First we want the logarithm alone on one side, so we can rewrite this in exponential form

$-7+\log_3(4-x)=-6$ Add 7

$\log_3(4-x)=1$

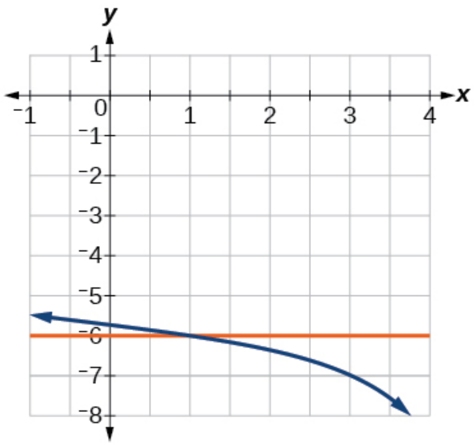
$3^1=4-x$ Rewrite exponentially, subtract 4

$-1=-x$ Divide by -1

$1=x$

$x=1$

Substituting $x=1$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution. Therefore $x=1$.



56. -

57. $\log(4-2x)=\log(-4x)$

Since there are logs on both sides we immediately can set their arguments equal

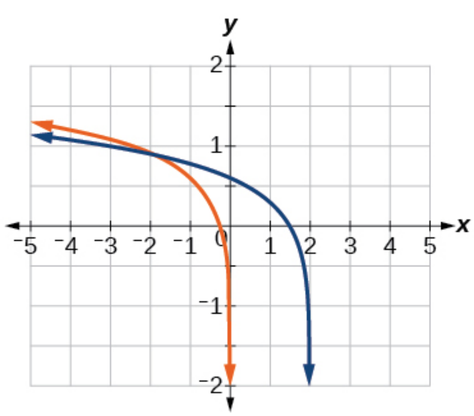
$\log(4-2x)=\log(-4x)$

$4-2x=-4x$ Use the one-to-one property of the logarithm.

$2x=-4$ Add 4x and Subtract 4 from both sides

$x=-2$ Divide by 2

Substituting $x=-2$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution. Therefore $x=-2$



58. -

59. $\ln(2x+9)=\ln(-5x)$

Since there are logs on both sides we immediately can set their arguments equal

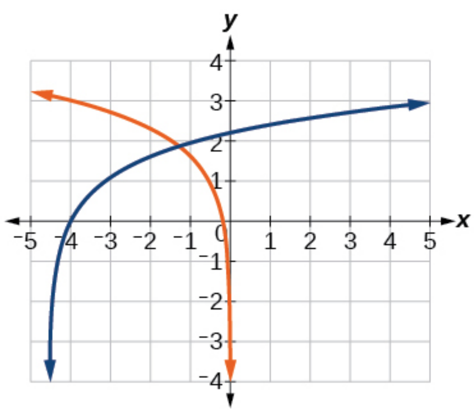
$\ln(2x+9)=\ln(-5x)$

$2x+9=-5x$ Use the one-to-one property of the logarithm.

$7x=-9$ Add 5x and Subtract 9 from both sides

$x=\frac{-9}{7}$ Divide by 7

Substituting $x=-\frac{9}{7}\approx -1.3$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution. $x=-\frac{9}{7}\approx -1.3$



60. -

61. $\log(x^2+13)=\log(7x+3)$

Since there are logs on both sides we immediately can set their arguments equal

$\log(x^2+13)=\log(7x+3)$ Use the one-to-one property of the logarithm

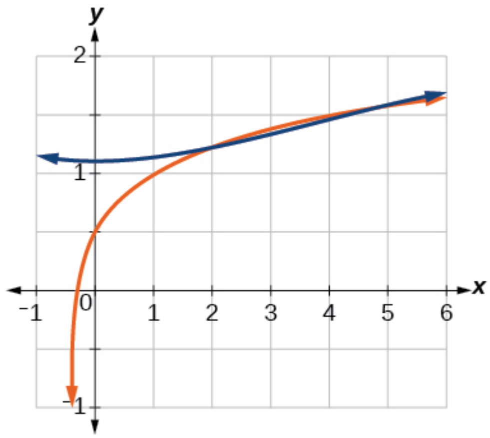
$x^2+13=7x+3$ Subtract 7x and 3 from both sides

$x^2-7x+10=0$ Factor into two binomials

$(x-5)(x-2)=0$

$x=5$ *or* $x=2$

Substituting $x=2$ or $x=5$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore they are both solutions. Therefore the solutions are $x=2$ and $x=5$.



62. -

63. $\ln(x) - \ln(x+3) = \ln(6)$

We have to rewrite each side as a single logarithm first

$$\ln(x) - \ln(x+3) = \ln(6)$$

$$\ln\left(\frac{x}{x+3}\right) = \ln(6) \quad \text{Use the one-to-one property of the logarithm}$$

$$\frac{x}{x+3} = 6 \quad \text{Multiply both sides by x+3}$$

$$x = 6(x+3)$$

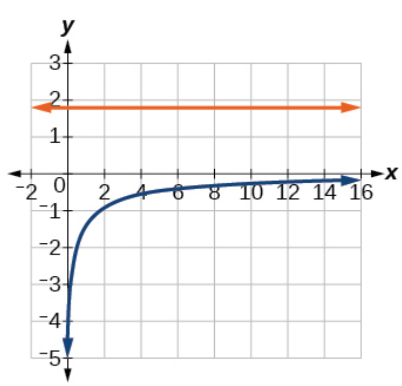
$$x = 6x + 18 \quad \text{Subtract 6x from both sides}$$

$$-5x = 18$$

$$x = \frac{18}{-5} \quad \text{Divide by } -5$$

$$x = -3.6$$

There is a solution for x , however when $x = -3.6$ is substituted into the original equation because the argument of the logarithm functions is not positive, there is no solution .



For the following exercises, solve for the indicated value, and graph the situation showing the solution point.

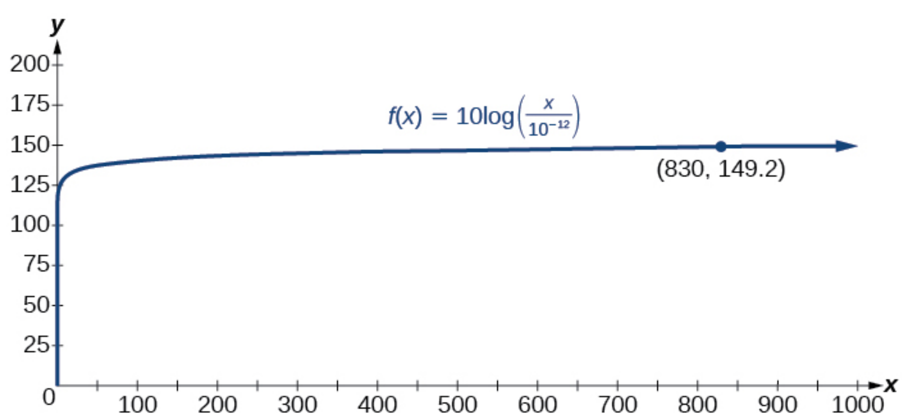
64. -

65. The formula for measuring sound intensity in decibels D is defined by the equation $D = 10\log\left(\frac{I}{I_0}\right)$, where I is the intensity of the sound in watts per square meter and $I_0 = 10^{-12}$ is the lowest level of sound that the average person can hear. How many decibels are emitted from a jet plane with a sound intensity of $8.3 \cdot 10^2$ watts per square meter?

Substituting the given information into

$$D = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{8.3\cdot 10^2}{10^{-12}}\right) = 10\log(8.3\cdot 10^{14})$$
$$= 10\left(\log\left(10^{14}\right) + \log\left(8.3\right)\right)$$
$$= 10\left(14\log\left(10\right) + \log\left(8.3\right)\right)$$
$$= 10\left(14 + \log\left(8.3\right)\right)$$
$$= 10\left(14 + 0.9191\right)$$
$$\approx 149.2$$

Therefore about 149.2 decibels.



66. -

Technology

For the following exercises, solve each equation by rewriting the exponential expression using the indicated logarithm. Then use a calculator to approximate x to 3 decimal places.

67. $1000(1.03)^t = 5000$ using the common log.

$1000(1.03)^t = 5000$

$\frac{1000(1.03)^t}{1000} = \frac{5000}{1000}$ *get* $(1.03)^t$ alone on one side

$(1.03)^t = 5$ take log of both sides to solve for t

$\log(1.03)^t = \log 5$ recall power rule of logs on left side

$t\log(1.03) = \log 5$ Divide by $\log(1.03)$

$t = \frac{\log(5)}{\log(1.03)}$

$t = \log(5) \div \log(1.03) = 54.44868501 \approx 54.449$

68. -

69. $3(1.04)^{3t} = 8$ using the common log

$3(1.04)^{3t} = 8$

$\frac{3(1.04)^{3t}}{3} = \frac{8}{3}$ *get* $(1.04)^{3t}$ alone on one side

$(1.04)^{3t} = \frac{8}{3}$ take log of both sides to solve for t

$\log(1.04)^{3t} = \log\left(\frac{8}{3}\right)$ recall power rule of logs on left side

$3t\log(1.04) = \log\left(\frac{8}{3}\right)$ Divide by $3\log(1.04)$

$t = \frac{\log\left(\frac{8}{3}\right)}{3\log(1.04)}$

$t = \log(8 \div 3) \div (3\log(1.04)) = 8.3359801 \approx 8.336$

70. -

71. $50e^{-0.12t} = 10$ using the natural log

$50e^{-0.12t} = 10$ $\quad$ *get* $e^{-0.12t}$ alone on one side

$\frac{50e^{-0.12t}}{50} = \frac{10}{50}$ $\quad$ $20,000 \div 1650$ hit math Frac to get a fraction instead of a repeating decimal

$e^{-0.12t} = \frac{1}{5}$ $\quad$ take ln of both sides to solve for t

$\ln\left(e^{-0.12t}\right) = \ln\left(\frac{1}{5}\right)$ $\quad$ recall power rule of logs on leftt side and ln e = 1

$-0.12t\ln(e) = \ln\left(\frac{1}{5}\right)$

$-0.12t = \ln\left(\frac{1}{5}\right)$ $\quad$ Divide by -0.12

$t = \frac{\ln\left(\frac{1}{5}\right)}{-0.12}$

$t = \ln(1 \div 5) \div -0.12 = 13.4119826 \approx 13.412$

For the following exercises, use a calculator to solve the equation. Unless indicated otherwise, round all answers to the nearest ten-thousandth.

72. -

73. $\ln(3) + \ln(4.4x + 6.8) = 2$

First we want the logarithm alone on one side, so we can rewrite this in exponential form

$\ln(3) + \ln(4.4x + 6.8) = 2$

$\ln(3(4.4x + 6.8)) = 2$ $\quad$ Adding individual logs is product rule as single log

$\ln(13.2x + 20.4) = 2$

$e^2 = 13.2x + 20.4$ $\quad$ Rewrite exponentially, subtract 20.4

$e^2 - 20.4 = 13.2x$

$\frac{e^2 - 20.4}{13.2} = x$ $\quad$ Divide by 13.2

$x = (e \wedge (2) - 20.4) \div 13.2 = -0.9856775683 \approx -.9857$

$x \approx -0.9857$

74. -

75. Atmospheric pressure P in pounds per square inch is represented by the formula $P = 14.7e^{-0.21x}$, where x is the number of miles above sea level. To the nearest foot, how high is the peak of a mountain with an atmospheric pressure of 8.369 pounds per square inch? (*Hint*: there are 5280 feet in a mile)

We are being asked to solve for x when $P = 8.369$

$$P = 14.7e^{-0.21x}$$

$$8.369 = 14.7e^{-0.21x} \quad \textit{get } e^{-0.21x} \text{ alone on one side}$$

$$\frac{8.369}{14.7} = e^{-0.21x}$$

$$.56913197279 = e^{-0.21x} \quad \text{take ln of both sides to solve for x}$$

$$\ln(.56913197279) = \ln\left(e^{-0.21x}\right) \quad \text{recall power rule of logs on right side and ln e = 1} = \text{ about}$$

$$\ln(.56913197279) = -0.21x\ln(e)$$

$$\ln(.56913197279) = -0.21x \quad \text{Divide by } -0.21$$

$$\frac{\ln(.56913197279)}{-0.21} = x$$

$$x = \ln(.56913197279) \div -0.21 = 2.682443289 \quad \textit{miles}$$

Since 1 mile is 5280 feet, we take $2.682443289(5280) = 14163.30057$. Therefore The peak of this mountain would be about $14{,}163$ feet above sea level

76. -

Extensions

77. Use the definition of a logarithm along with the one-to-one property of logarithms to prove that $b^{\log_b x} = x.$

Let $b^{\log_b x} = n.$ We show that $n = x.$ When rewriting $base^{\text{exp}onent} = n$ as a logarithm it would be $\log_{base} n = \text{exponent}$. Therefore it follows if I write $b^{\log_b x} = n$ as a logarithm it would be $\log_b n = \log_b x.$ By the one-to-one property of logarithms, $n = x.$ Therefore, $b^{\log_b x} = x.$

78. -

79. Recall the compound interest formula $A = a\left(1+\frac{r}{k}\right)^{kt}$. Use the definition of a logarithm along with properties of logarithms to solve the formula for time t.

$A = a\left(1+\frac{r}{k}\right)^{kt}$. $\quad$ *get* $\left(1+\frac{r}{k}\right)^{kt}$ alone on one side

$\frac{A}{a} = \left(1+\frac{r}{k}\right)^{kt}$ $\quad$ take ln of both sides to solve for t

$\ln\left(\frac{A}{a}\right) = \ln\left(\left(1+\frac{r}{k}\right)^{kt}\right)$ $\quad$ recall power rule of logs on right side and ln e = 1

$\ln\left(\frac{A}{a}\right) = kt\ln\left(1+\frac{r}{k}\right)$ $\quad$ Divide by $k\ln\left(1+\frac{r}{k}\right)$

$$\frac{\ln\left(\frac{A}{a}\right)}{k\ln\left(1+\frac{r}{k}\right)} = t$$

$$t = \frac{\ln\left(\frac{A}{a}\right)}{k\ln\left(1+\frac{r}{k}\right)} = \frac{\ln\left(\frac{A}{a}\right)}{\ln\left(\left(1+\frac{r}{k}\right)^{k}\right)}$$

80. -

Chapter 6
Exponential and Logarithmic Functions
6.7 Exponential and Logarithmic Models

Section Exercises

Verbal

1. With what kind of exponential model would *half-life* be associated? What role does half-life play in these models?
   Half-life is a measure of decay and is thus associated with exponential decay models. The half-life of a substance or quantity is the amount of time it takes for half of the initial amount of that substance or quantity to decay.
2. -
3. With what kind of exponential model would *doubling time* be associated? What role does doubling time play in these models?
   Doubling time is a measure of growth and is thus associated with exponential growth models. The doubling time of a substance or quantity is the amount of time it takes for the initial amount of that substance or quantity to double in size.
4. -
5. What is an order of magnitude? Why are orders of magnitude useful? Give an example to explain.
   An order of magnitude is the nearest power of ten by which a quantity exponentially grows. It is also an approximate position on a logarithmic scale; Sample response: Orders of magnitude are useful when making comparisons between numbers that differ by a great amount. For example, the mass of Saturn is 95 times greater than the mass of Earth. This is the same as saying that the mass of Saturn is about 10^2 times, or *2 orders of magnitude* greater, than the mass of Earth.

Numeric

6. -

For the following exercises, use the logistic growth model $f(x)=\dfrac{150}{1+8e^{-2x}}$.

7. Find and interpret $f(0)$. Round to the nearest tenth.
   $f(x)=\dfrac{150}{1+8e^{-2x}} \Rightarrow f(0)=\dfrac{150}{1+8e^{-2(0)}}=\dfrac{150}{1+8e^{0}}=\dfrac{150}{1+8}=\dfrac{150}{9}\approx 16.7 \quad f(0)\approx 16.7;$ $f(0)$ represents the amount initially (when t = 0) present is about 16.7 units.
8. -
9. Find the carrying capacity.
   Since the equation is in the form $f(x)=\dfrac{c}{1+ae^{-bx}}$, *given* $f(x)=\dfrac{150}{1+8e^{-2x}}$, $c=150$, which is the carrying capacity $=$ 150

10. -

11. Determine whether the data from the table could best be represented as a function that is linear, exponential, or logarithmic. Then write a formula for a model that represents the data.

x	–2	–1	0	1	2	3	4	5
***f*(*x*)**	0.694	0.833	1	1.2	1.44	1.728	2.074	2.488

If you take any f(x) value and divide it by the preceding one, $.833 \div .694 = 1 \div .833 = 1.2 \div 1$ tells us that 1.2 is the ratio increase which makes this model exponential with a base of 1.2. Therefore exponential; $f(x) = 1.2^x$

12. -

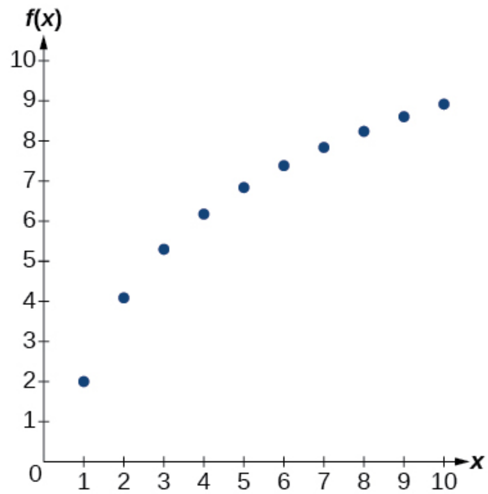
Technology

For the following exercises, enter the data from each table into a graphing calculator and graph the resulting scatter plots. Determine whether the data from the table could represent a function that is linear, exponential, or logarithmic.

13.

x	1	2	3	4	5	6	7	8	9	10
***f*(*x*)**	2	4.079	5.296	6.159	6.828	7.375	7.838	8.238	8.592	8.908

Using a graphing calculator, you can plot individual points using STAT Plot. Press 2^{nd} STAT PLOT, which is above Y=. Highlight 1:Plot1…On, Enter, Choose what type of mark you'd like , enter. To enter coordinates hit the STAT button, 1:Edit, enter. It brings L1 and L2 and L3. Enter the x values down column L1 and then the corresponding y-values in L2 column. Once they are all in exit that screen and hit GRAPH. It should plot the points for you. See below. Because of the nature of this curve (being concave down), it is logarithmic.

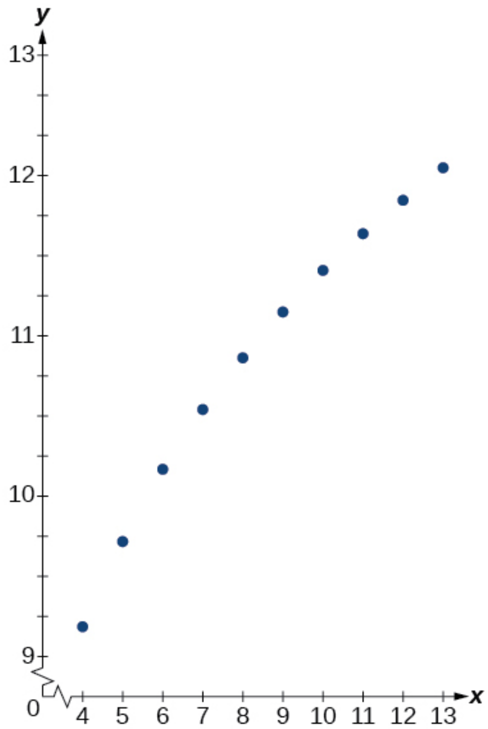


14. -

15.

x	4	5	6	7	8	9	10	11	12	13
$f(x)$	9.429	9.972	10.415	10.79	11.115	11.401	11.657	11.889	12.101	12.295

Using a graphing calculator, you can plot individual points using STAT Plot. Press 2nd STAT PLOT, which is above Y=. Highlight 1:Plot1…On, Enter, Choose what type of mark you'd like , enter. To enter coordinates hit the STAT button, 1:Edit, enter. It brings L1 and L2 and L3. Enter the x values down column L1 and then the corresponding y-values in L2 column. Once they are all in exit that screen and hit GRAPH. It should plot the points for you. See below. Because of the nature of this curve (being concave down), it is logarithmic.



16. -

For the following exercises, use a graphing calculator and this scenario: the population of a fish farm in t years is modeled by the equation $P(t) = \frac{1000}{1+9e^{-0.6t}}$.

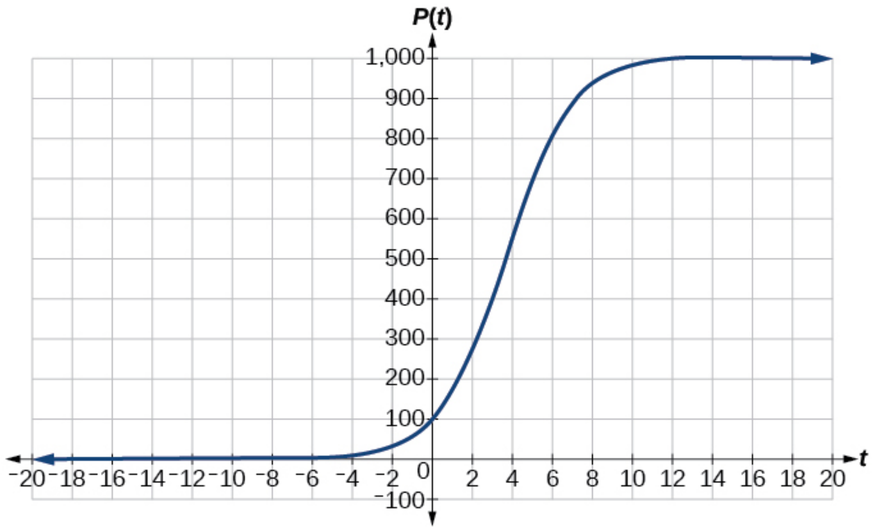
Graph the function.

Since we know that the highest y value will be 1000 (the carrying capacity) , we should make our window to include that. A nice window to show the graph would be

$x\min = -20, x\max 20, xscl = 2, y\min = -100, y\max = 1100, yscl = 100$

$Enter \quad Y_1 = 1000 \div (1+9e \wedge (-0.6x)), graph$

17.



18. -
19. To the nearest tenth, what is the doubling time for the fish population?

Doubling time follows the model given in the Chapter

$A = A_0 e^{kt}$, doubling time is $t = \dfrac{\ln 2}{k}$, where k>0. Therefore since

$k = -\ln b = -\ln .6$, $\quad t = \dfrac{\ln 2}{-\ln 0.6} = 1.3569 \approx 1.4$ which is about 1.4 years

20. -
21. To the nearest tenth, how long will it take for the population to reach 900?

We use the original model and solve for t, plugging in 900 for the population size

$$P(t) = \frac{1000}{1+9e^{-0.6t}}$$
$$900 = \frac{1000}{1+9e^{-0.6t}}$$
$$900\left(1+9e^{-0.6t}\right) = 1000 \quad \text{multiply both sides by } 1+9e^{-0.6t}$$
$$900 + 8100e^{-0.6t} = 1000 \quad \text{distribute, subtract 900}$$
$$8100e^{-0.6t} = 100 \quad \text{divide by 8100}$$
$$e^{-0.6t} = \frac{1}{81} \quad \text{take the ln of both sides}$$
$$\ln e^{-0.6t} = \ln\left(\frac{1}{81}\right) \quad \text{log properties and lne = 1}$$
$$-.6t \ln e = \ln\left(\frac{1}{81}\right)$$
$$-.6t = \ln\left(\frac{1}{81}\right) \quad \text{divide by } -.6$$
$$t = \frac{\ln\left(\frac{1}{81}\right)}{-.6} = (\ln(1 \div 81)) \div (-.6) = 7.324081924 \approx 7.3$$

It would take about 7.3 years.

22. -

Extensions

23. A substance has a half-life of 2.045 minutes. If the initial amount of the substance was 132.8 grams, how many half-lives will have passed before the substance decays to 8.3 grams? What is the total time of decay?
If we take 132.8 and repeatedly divide it by 2, we see it takes 4 cycles of the half life to reach a size of 8.3. Therefore since the half-life is 2.045 minutes we multiply $4 * 2.045 = 8.18$.
Four half-lives; 8.18 minutes

24. -

25. Recall the formula for calculating the magnitude of an earthquake, $M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$. Show each step for solving this equation algebraically for the seismic moment S.

$$M = \frac{2}{3}\log\left(\frac{S}{S_0}\right) \quad \text{multiply both sides by } \frac{3}{2}$$

$$\frac{3}{2}M = \log\left(\frac{S}{S_0}\right) \quad \text{rewrite this exponentially, base 10}$$

$$10^{\frac{3M}{2}} = \frac{S}{S_0} \quad \text{multiply by } S_0$$

$$S_0 10^{\frac{3M}{2}} = S$$

$$S = S_0 10^{\frac{3M}{2}}$$

26. -

27. Prove that $b^x = e^{x\ln(b)}$ for positive $b \neq 1$.
Let $y = b^x$ for some non-negative real number b such that $b \neq 1$. Then,

$$\ln(y) = \ln(b^x)$$
$$\ln(y) = x\ln(b)$$
$$e^{\ln(y)} = e^{x\ln(b)}$$
$$y = e^{x\ln(b)}$$

Real-World Applications

For the following exercises, use this scenario: A doctor prescribes 125 milligrams of a therapeutic drug that decays by about 30% each hour.

28. -
29. Write an exponential model representing the amount of the drug remaining in the patient's system after t hours. Then use the formula to find the amount of the drug that would remain in the patient's system after 3 hours. Round to the nearest milligram.
Using the information we found $y = ae^{(\ln b)x} = 125e^{(\ln .7)x}$. So the exponential model would be $A = 125e^{(-0.3567t)}$; Using this model we will plug in t = 3 hours.
$f(3) = 125e^{(\ln .7)3} = 42.875 \approx 43mg$ the amount of drug remaining is $A \approx 43$ mg
30. -

For the following exercises, use this scenario: A tumor is injected with 0.5 grams of Iodine-125, which has a decay rate of 1.15% per day.

31. To the nearest day, how long will it take for half of the Iodine-125 to decay?
Since this has a decay rate of 1.1%, that means that 100% - 1.15% = 98.85% is remaining. We are being asked for the half-life, so since $k = \ln b = \ln .9985$, the half-life is $t = \frac{-\ln 2}{k} = \frac{-\ln 2}{\ln(.9885)} = 59.92642602 \approx 60$. About 60 days
32. -

33. A scientist begins with 250 grams of a radioactive substance. After 225 minutes, the sample has decayed to 32 grams. Rounding to five significant digits, write an exponential equation representing this situation. To the nearest minute, what is the half-life of this substance?

Since the initial amount $A_0 = 250g$ and after 225 minutes it decays to 32 grams we need to find k, so plugging in the given information we see:

$A_0 = A_0 e^{kt}$

$32 = 250e^{k(225\min)}$ divide both sides by 250 to get e^{kt} *alone*

$0.128 = e^{225k}$ take the ln of both sides and use power rule on rigft side

$\ln 0.128 = 225k \ln e$ ln e = 1

$\ln 0.128 = 225k$ divide both sides by 225

$$\frac{\ln 0.128}{225} = k$$

$k = -.0091365556 \approx -.00914$

Therefore $f(t) = 250e^{-0.00914t}$

To find the half-life, $t = \frac{-\ln 2}{k} = \frac{-\ln 2}{-0.00914} = 75.8366 \approx 76.$

$f(t) = 250e^{(-0.00914t)}$; The half-life is about 76 minutes

34. -

35. The half-life of Erbium-165 is 10.4 hours. What is the hourly decay rate? Express the decimal result to four significant digits and the percentage to two significant digits.

Since $t = \frac{-\ln 2}{k}$, it follows that $k = \frac{-\ln 2}{t} = \frac{-\ln 2}{10.4} = -0.0666487674 \approx -0.06665.$

$r \approx -0.0667$, So the hourly decay rate is about 6.67%

36. -

37. A research student is working with a culture of bacteria that doubles in size every twenty minutes. The initial population count was 1350 bacteria. Rounding to five significant digits, write an exponential equation representing this situation. To the nearest whole number, what is the population size after 3 hours?

Using the given information we will solve for k, the growth constant and then we will use the model, to find f(3). Doubling the initial amount is $2A_0$.

$$A_0 = A_0 e^{kt}$$

$2A_0 = A_0 e^{k(20)}$ divide both sides by A_0

$2 = e^{20k}$ take the ln of both sides and use power rule on rigft side

$\ln 2 = 20k \ln e$ ln e = 1

$\ln 2 = 20k$ divide both sides by 20

$$\frac{\ln 2}{20} = k$$

$$k \approx .034657359$$

Therefore $f(t) = 1350e^{0.034657359t}$

Plugging 3 hours into this we must change it to minutes, therefore t = 3 hours means 60*3=180 minutes. Inputting this into this model yields

$f(180) = 1350e^{0.034657359(180)} \approx 691199.9965$. Thus the model, rounding to five significant digits is $f(t) = 1350e^{(0.03466t)}$; and after 3 hours, which is 180 minutes : $P(180) \approx 691{,}200$

For the following exercises, use this scenario: A biologist recorded a count of 360 bacteria present in a culture after 5 minutes and 1000 bacteria present after 20 minutes.

38. -
39. Rounding to six significant digits, write an exponential equation representing this situation. To the nearest minute, how long did it take the population to double?

    The equation was shown in the previous, rounding the rate to six significant digits is

    $f(t) = 256e^{(0.068110t)}$; doubling time is $t = \dfrac{\ln 2}{k} = \dfrac{\ln 2}{.0681100832} = 10.1768 \approx 10$, to the nearest minute is about 10 minutes

For the following exercises, use this scenario: A pot of boiling soup with an internal temperature of $100°$ Fahrenheit was taken off the stove to cool in a $69°F$ room. After fifteen minutes, the internal temperature of the soup was $95°F$.

40. -
41. To the nearest minute, how long will it take the soup to cool to $80°F$?

    Now we use our model and plug in T(t) = 70 and solve for t

$$T(t)=31e^{(-0.011726t)}+69$$
$$80=31e^{(-0.011726t)}+69$$
$$\frac{11}{31}=\frac{31e^{(-0.011726t)}}{31}$$
$$\frac{11}{31}=e^{-0.011726t}$$ Take the ln of both sides, power rule on right side and ln e = 1
$$\ln\left(\frac{11}{31}\right)=-0.011726t$$ divide by -0.011726
$$\frac{\ln\left(\frac{11}{31}\right)}{-0.011726}=t$$
$$t=88.3585137$$

It will cool to 80 degrees in about 88 minutes.

42. -

For the following exercises, use this scenario: A turkey is taken out of the oven with an internal temperature of $165°$ Fahrenheit and is allowed to cool in a 75℉ room. After half an hour, the internal temperature of the turkey is 145℉.

43. Write a formula that models this situation.

The model formula we use is $T(t)=Ae^{kt}+T_s$ and we know $T_s=75°$ and $A=165-75=90$, therefore to find k, we plug in the fact that when $t=30$ min, $T(30)=145$

$$T(t)=Ae^{kt}+T_s$$
$$145=90^{k(30)}+75$$ Subtract 75 and then divide by 90
$$\frac{70}{90}=\frac{90e^{k(30)}}{90}$$
$$\frac{7}{9}=e^{30k}$$ Take the ln of both sides, power rule on right side and ln e = 1
$$\ln\left(\frac{7}{9}\right)=30k$$ divide by 30
$$\frac{\ln\left(\frac{7}{9}\right)}{30}=k$$
$$k\approx-0.0083771476$$

Therefore our model would be: $T(t)=90e^{(-0.008377t)}+75$, where t is in minutes.

44. -

45. To the nearest minute, how long will it take the turkey to cool to $110°F$?

To solve this we use our model and T(t)=110, and solve for t.

$$T(t) = 90e^{(-0.008377t)} + 75$$

$$110 = 90e^{(-0.008377t)} + 75 \quad \text{subtract 75, divide by 90}$$

$$\frac{35}{90} = \frac{90e^{(-0.008377t)}}{90}$$

$$\frac{7}{18} = e^{-0.008377t} \quad \text{Take the ln of both sides, power rule on right side and ln e = 1}$$

$$\ln\left(\frac{7}{18}\right) = -0.008377t \quad \text{divide by } -0.008377t$$

$$\frac{\ln\left(\frac{7}{18}\right)}{-0.008377t} = t$$

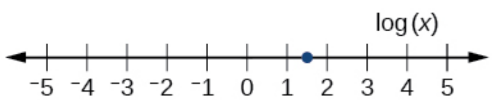
$$t = 112.7446$$

for the turkey to cool to 110 degrees it will take about 113 minutes.

For the following exercises, find the value of the number shown on each logarithmic scale. Round all answers to the nearest thousandth.

46. -

47.



We solve the equation where $\log(x) = 1.5$. To solve this we rewrite it exponentially.

$$\log_{10} x = 1.5$$

$$x = 10^{1.5}$$

$$x \approx 31.623$$

48. -

49. Recall the formula for calculating the magnitude of an earthquake, $M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$. One earthquake has magnitude 3.9 on the MMS scale. If a second earthquake has 750 times

as much energy as the first, find the magnitude of the second quake. Round to the nearest hundredth.

$$M_1 = 3.9 = \frac{2}{3}\log\left(\frac{S_1}{S_0}\right)$$

$$M_2 = \frac{2}{3}\log\left(\frac{S_2}{S_0}\right) = \frac{2}{3}\log\left(\frac{750S_1}{S_0}\right)$$

$$M_2 = \frac{2}{3}\log\left(\frac{750S_1}{S_0}\right) = \frac{2}{3}\log\left(750\cdot\frac{S_1}{S_0}\right) = \frac{2}{3}\log(750) + \frac{2}{3}\log\left(\frac{S_1}{S_0}\right)$$

$$M_2 = \frac{2}{3}\log(750) + \frac{2}{3}\log\left(\frac{S_1}{S_0}\right) = 1.9167 + 3.9 = 5.8167 \approx 5.82$$

For the following exercises, use this scenario: The equation $N(t) = \frac{500}{1+49e^{-0.7t}}$ models the number of people in a town who have heard a rumor after *t* days.

50. -

51. To the nearest whole number, how many people will have heard the rumor after 3 days? To solve this we plug in t = 3.

$$N(3) = \frac{500}{1+49e^{-0.7(3)}} = 500 \div (1+49 * e \wedge (-0.7*3)) = 71.42484729 \approx 71 \cdot \quad N(3) \approx 71$$

52. -

53. A doctor injects a patient with 13 milligrams of radioactive dye that decays exponentially. After 12 minutes, there are 4.75 milligrams of dye remaining in the patient's system. Which is an appropriate model for this situation?

A. $f(t) = 13(0.0805)^t$

B. $f(t) = 13e^{0.9195t}$

C. $f(t) = 13e^{(-0.0839t)}$

D. $f(t) = \frac{4.75}{1+13e^{-0.83925t}}$

To answer this we must find k, the rate by plugging the given information in

$$A(t) = A_0 e^{kt}$$

$4.75 = 13e^{k(12)}$ divide both sides by 13 to get e^{kt} *alone*

$.3653846154 = e^{12k}$ take the ln of both sides and use power rule on rigft side

$\ln .3653846154 = 12k \ln e$ ln e = 1

$\ln .3653846154 = 12k$ divide both sides by 5730

$$\frac{\ln .3653846154}{12} = k$$

$$k \approx -0.0839003949$$

Therefore $f(t) = 13e^{-0.0839003949t}$

Choice C

Chapter 6
Exponential and Logarithmic Functions
6.8 Fitting Exponential Models to Data

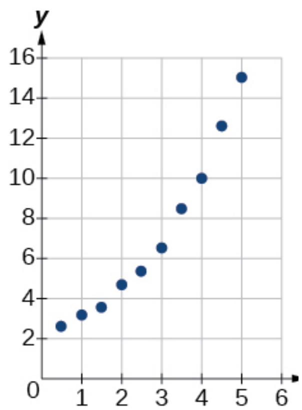
Section Exercises

Verbal

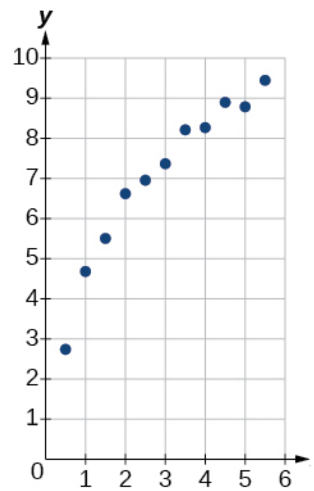
1. What situations are best modeled by a logistic equation? Give an example, and state a case for why the example is a good fit.
   Logistic models are best used for situations that have limited values. For example, populations cannot grow indefinitely since resources such as food, water, and space are limited, so a logistic model best describes populations.
2. -
3. What is regression analysis? Describe the process of performing regression analysis on a graphing utility.
   Regression analysis is the process of finding an equation that best fits a given set of data points. To perform a regression analysis on a graphing utility, first list the given points using the STAT then EDIT menu. Next graph the scatter plot using the STAT PLOT feature. The shape of the data points on the scatter graph can help determine which regression feature to use. Once this is determined, select the appropriate regression analysis command from the STAT then CALC menu.
4. -
5. What does the *y*-intercept on the graph of a logistic equation correspond to for a population modeled by that equation?
   The *y*-intercept on the graph of a logistic equation corresponds to the initial population for the population model.

Graphical

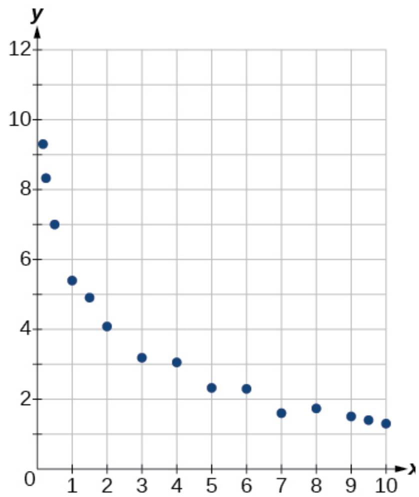
For the following exercises, match each scatterplot with the function of best fit.



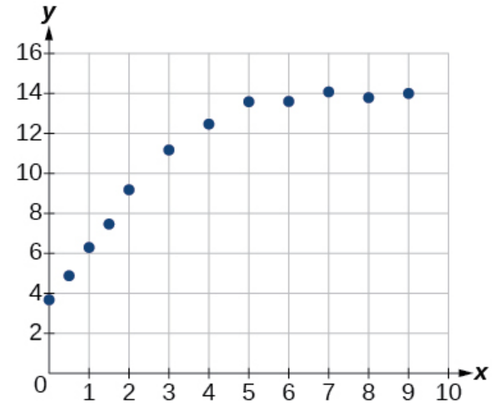
(a)



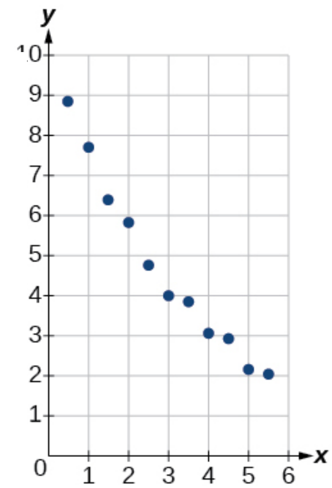
(b)



(c)



(d)



(e)

6. -
7. $y = 5.598 - 1.912\ln(x)$

   Because this is a logarithmic function but with a negative sign in front of the function which reflects it about the x-axis its curve would then be either choice C or E, then we look at the point (1,5.598) should be on the plots, therefore function Choice = C
8. -
9. $y = 4.607 + 2.733\ln(x)$

   Because this is a logarithmic function it would be between Choice B or D, and since the point (1, 4.607) should be on the graph it would be Choice = B
10. -

Numeric

11. To the nearest whole number, what is the initial value of a population modeled by the logistic equation $P(t) = \dfrac{175}{1 + 6.995e^{-0.68t}}$? What is the carrying capacity?

    The initial value means when t = 0, thus $P(0) = \dfrac{175}{1 + 6.995e^{-0.68(0)}} = \dfrac{175}{7.995} = 21.88868043$, rounded to the nearest whole number $P(0) = 22$; The carrying capacity is the numerator of the logistic growth model expression , thus it = 175 .
12. -
13. A logarithmic model is given by the equation $h(p) = 67.682 - 5.792\ln\left(p\right)$. To the nearest hundredth, for what value of p does $h(p) = 62$?

    We will solve the given function for p:

$$62 = 67.682 - 5.792\ln(p)$$
$$62 - 67.682 = -5.792\ln(p)$$
$$-5.682 = -5.792\ln(p) \quad \text{divide by } -5.792$$
$$0.9810082873 = \ln(p) \quad \text{rewrite exponentially}$$
$$e^{0.9810082873} = p$$
$$2.667144134 = p$$
$$p \approx 2.67$$

14. -
15. What is the *y*-intercept on the graph of the logistic model given in Exercise 14?
    to find the y-intercept we plug x = 0 into the function
    $P(0) = \dfrac{90}{1+5e^{-0.42(0)}} = \dfrac{90}{6} = 15$. Therefore the *y*-intercept is $(0,\ 15)$

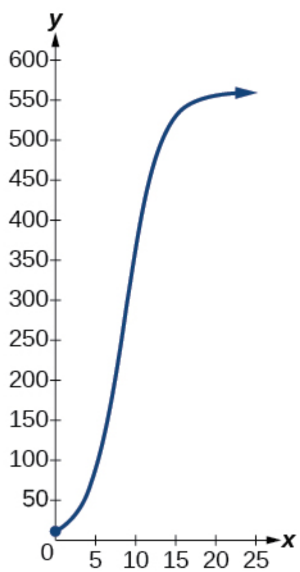
Technology

For the following exercises, use this scenario: The population P of a koi pond over x months is modeled by the function $P(x) = \dfrac{68}{1+16e^{-0.28x}}$.

16. -
17. What was the initial population of koi?
    Algebraically, the initial value means when t = 0, thus $P(0) = \dfrac{68}{1+16e^{-0.28(0)}}. = \dfrac{68}{17} = 4$,
    Using the graph in your graphing calculator hit 2nd calc, for calculate, then 1:value, enter, your display will be x=, input 0 (zero), enter. Read the display at the bottom x=0 y =4.
    answer: 4 koi
18. -
19. How many months will it take before there are 20 koi in the pond?
    Enter a second function into the graph $Y_2 = 20$. Then find the point of intersection of these two graphs. Recall , 2nd CALC, 5:intersection, move cursor to the point of intersection and hit enter 3 times. Read the display at the bottom x= 6.7754285 y =20. Therefore it will take about 6.8 months.
20. -

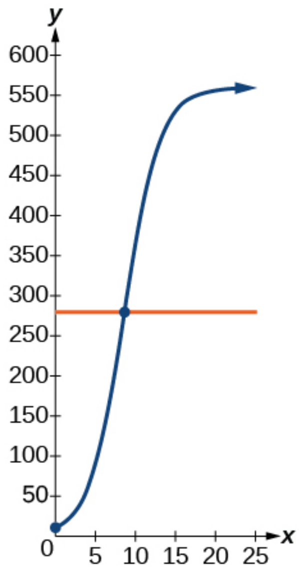
For the following exercises, use this scenario: The population P of an endangered species habitat for wolves is modeled by the function $P(x) = \frac{558}{1+54.8e^{-0.462x}}$, where x is given in years.

21. Graph the population model to show the population over a span of 10 years.
    In to your graphing calculator enter the following for Y1
    $Y_1 = 558 \div \left(1+54.8e \wedge \left(-.462x\right)\right)$. You want your y-values to go to 558 and your data is in terms of years, so 10 years for the x-values to reach, a nice window to show the graph would be $x\min = 0, x\max = 20, y\min = 0, y\max = 600$, scl50



22. -
23. How many wolves will the habitat have after 3 years?
    Using the graph in your graphing calculator hit 2nd calc, for calculate, then 1:value, enter, your display will be x=, input 3, enter. Read the display at the bottom x= 3 y =37.948771.
    $P(3) \approx 37.9$, so about 38 wolves
24. -
25. Use the intersect feature to approximate the number of years it will take before the population of the habitat reaches half its carrying capacity.
    Half its carrying capacity would be one half of 558, which is 279. Enter a second function into the graph $Y_2 = 279$. Then find the point of intersection of these two

graphs. Recall , 2nd CALC, 5:intersection, move cursor to the point of intersection and hit enter 3 times. Read the display at the bottom x=8.6659961 y = 279
It will take approximately 8.7 years for the population to reach half capacity.



For the following exercises, use the table of data shown.

x	1	2	3	4	5	6
$f(x)$	1125	1495	2310	3294	4650	6361

26. -

27. Use the regression feature to find an exponential function that best fits the data in the table.
To Find the equation that models the data. Select "ExpReg" from the STAT then CALC menu. Use the values returned for a and b to record the model, $y = ab^x$.
Your screen will display

$y = a * b \wedge x$

$a = 776.6824292$

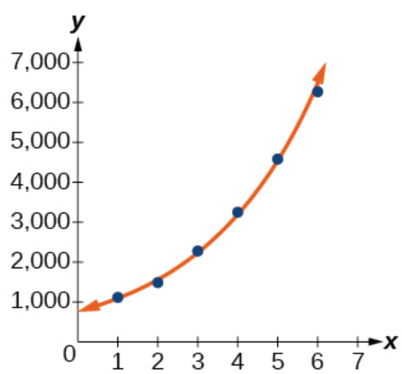
$b = 1.426021377$

Therefore the exponential function that best fits the data in the table is

$f(x) = 776.682(1.426)^x$

28. -
29. Graph the exponential equation on the scatter diagram.

    Input $Y_1 = 776.682e \wedge (0.3549x)$.



30. -

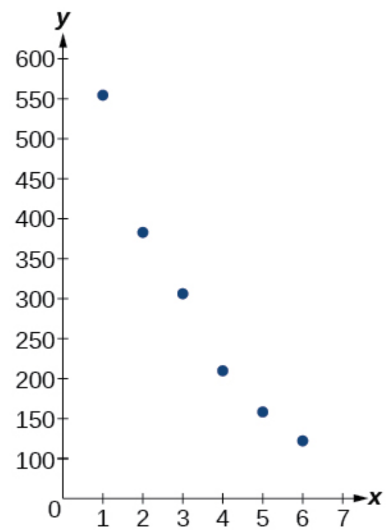
For the following exercises, use the table of data shown.

x	1	2	3	4	5	6
***f*(*x*)**	555	383	307	210	158	122

31. Use a graphing calculator to create a scatter diagram of the data.

    Using a graphing calculator, you can plot individual points using STAT Plot. Press 2nd STAT PLOT, which is above Y=. Highlight 1:Plot1…On, Enter, Choose what type of mark you'd like , enter. To enter coordinates hit the

STAT button, 1:Edit, enter. It brings L1 and L2 and L3. Enter the x values down column L1 and then the corresponding y-values in L2 column. Once they are all in exit that screen and hit GRAPH. It should plot the points for you. You have to go to the y= menu and make sure you highlight Plot1 above the Y1 symbol. Also Use ZOOM [9] to adjust axes to fit the data.



32. -

33. Write the exponential function as an exponential equation with base e.

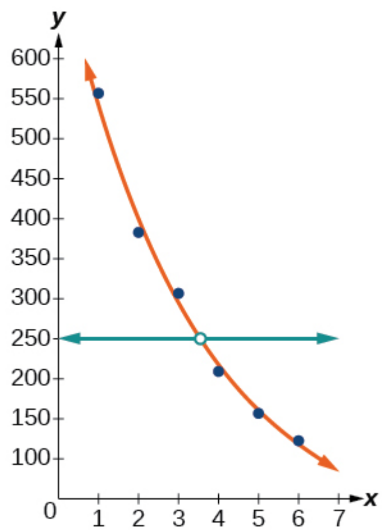
Since $y = ae^{(\ln b)x} = 731.92e^{(\ln 0.738)x} = 731.92e^{-.303811454x}$. Therefore

$f(x) = 731.92e^{-0.3038x}$

34. -

35. Use the intersect feature to find the value of x for which $f(x) = 250$.

Enter a second function into the graph $Y_2 = 250$. Then find the point of intersection of these two graphs. Recall , 2nd CALC, 5:intersection, move cursor to the point of intersection and hit enter 3 times. Read the display at the bottom x= 3.5359128 y =250. Therefore when $f(x) = 250$, $x \approx 3.5$.



For the following exercises, use the table of data shown.

x	1	2	3	4	5	6
***f*(*x*)**	5.1	6.3	7.3	7.7	8.1	8.6

36. -
37. Use the LOGarithm option of the REGression feature to find a logarithmic function of the form $y = a + b\ln(x)$ that best fits the data in the table.

    To Find the logarithmic equation that models the data. Select "LnReg" from the STAT then CALC menu. Use the values returned for *a* and *b* to record the model, $y = a + b\ln(x)$

    Your screen will display

    $y = a + b\ln x$

    $a = 5.062758549$

    $b = 1.933874889$

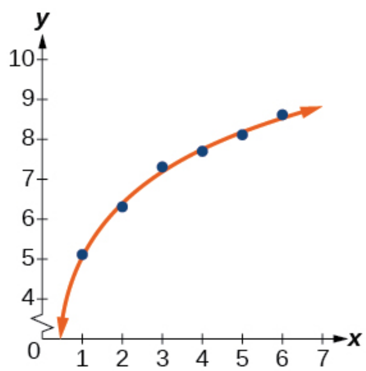
    Therefore the logarithmic function that best fits the data in the table is

    $f(x) = 5.063 + 1.934\ln(x)$

38. -

39. Graph the logarithmic equation on the scatter diagram.

You have done this in the previous exercise, enter $Y_1 = 5.063 + 1.934\ln(x)$



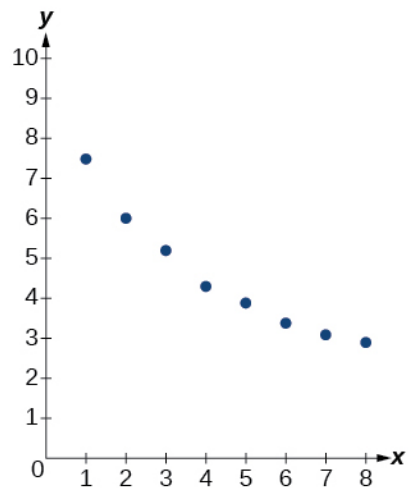
40. -

For the following exercises, use the table of data shown:

x	1	2	3	4	5	6	7	8
***f*(*x*)**	7.5	6	5.2	4.3	3.9	3.4	3.1	2.9

41. Use a graphing calculator to create a scatter diagram of the data.

So Using a graphing calculator, you can plot individual points using STAT Plot. Press 2nd STAT PLOT, which is above Y=. Highlight 1:Plot1…On, Enter, Choose what type of mark you'd like , enter. To enter coordinates hit the STAT button, 1:Edit, enter. It brings L1 and L2 and L3. Enter the x values down column L1 and then the corresponding y-values in L2 column. Once they are all in exit that screen and hit GRAPH. It should plot the points for you. You have to go to the y= menu and make sure you highlight Plot1 above the Y1 symbol. Also Use ZOOM [9] to adjust axes to fit the data.

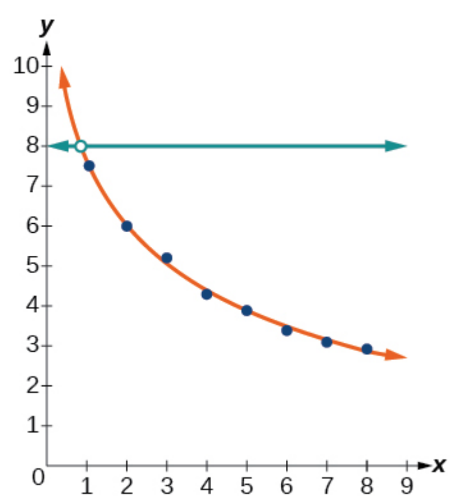


42. -
43. Use the logarithmic function to find the value of the function when $x = 10$.

    Input $Y_1 = 7.544 - 2.268\ln(x)$, and 2nd CALC value x= , input 10,enter. Your display will say ERROR. This is because we used ZOOMSTAT and x= 10 is NOT in the domain of our graph. You need to go to the window and change xmax to 10, then it will read your value for you. Display x = 10 y = 2.321737, therefore $f(10) \approx 2.3$.
44. -

45. Use the intersect feature to find the value of x for which $f(x) = 8$.

    Enter a second function into the graph $Y_2 = 8$. Then find the point of intersection of these two graphs. Recall , 2nd CALC, 5:intersection, move cursor to the point of intersection and hit enter 3 times. Read the display at the bottom x= .81786483 y =8. When $f(x) = 8$, $x \approx 0.82$.



For the following exercises, use the table of data shown.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	8.7	12.3	15.4	18.5	20.7	22.5	23.3	24	24.6	24.8

46. -

47. Use the LOGISTIC regression option to find a logistic growth model of the form $y = \frac{c}{1+ae^{-bx}}$ that best fits the data in the table.

To find the LOGISTIC equation that models the data. Select "B:Logistic" from the STAT then CALC menu. Use the values returned for a, b and c to record the model,

$y = \frac{c}{1+ae^{-bx}}$

Your screen will display

$y = c/(1+ae\wedge(-bx))$

$a = 3.181871105$

$b = .5453339819$

$c = 25.08133836$

Therefore the LOGISTIC function that best fits the data in the table is

$$f(x) = \frac{25.081}{1+3.182e^{-0.545x}}$$

48. -

49. To the nearest whole number, what is the predicted carrying capacity of the model?

The numerator of the model is the carrying capacity therefore it is about 25 .

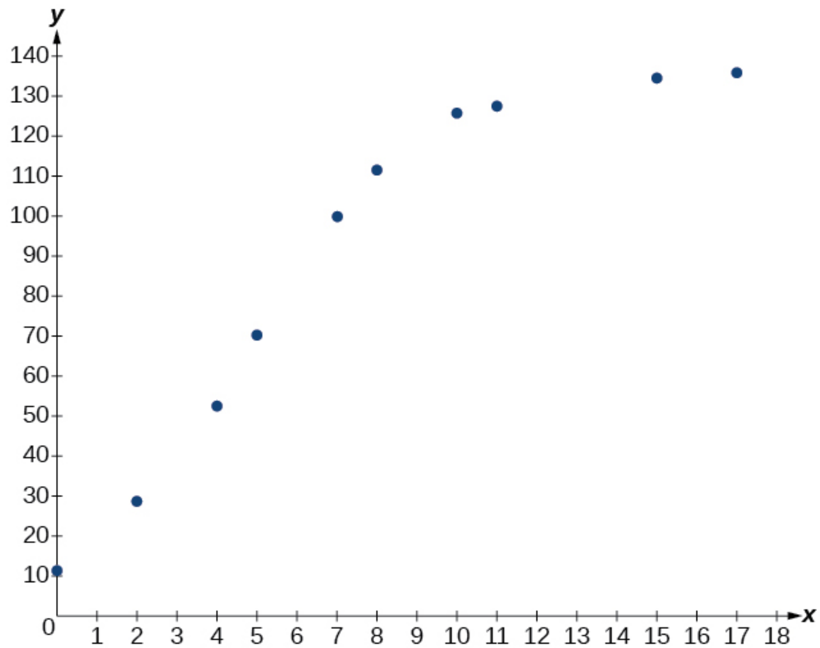
50. -

For the following exercises, use the table of data shown.

x	0	2	4	5	7	8	10	11	15	17
$f(x)$	12	28.6	52.8	70.3	99.9	112.5	125.8	127.9	135.1	135.9

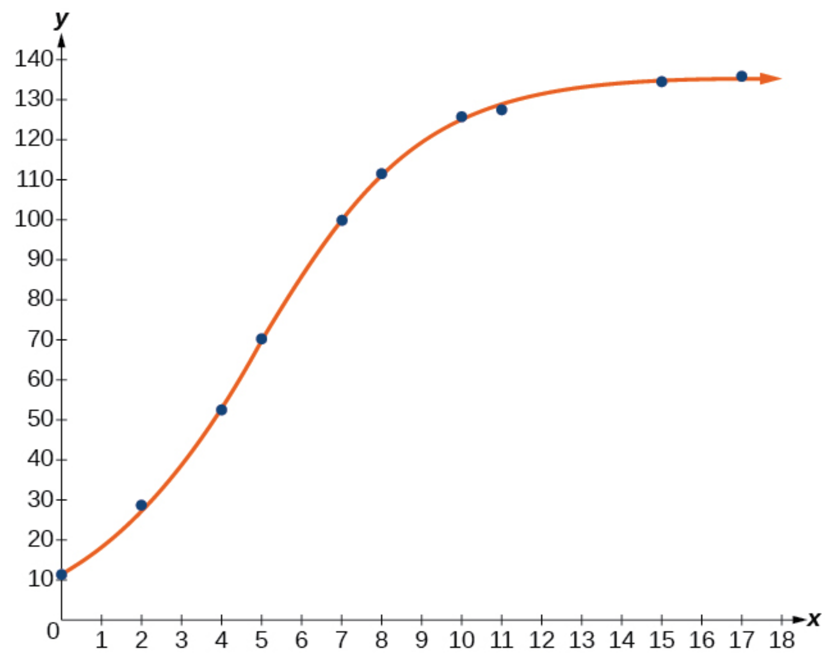
51. Use a graphing calculator to create a scatter diagram of the data.

Using a graphing calculator, you can plot individual points using STAT Plot. Press 2nd STAT PLOT, which is above Y=. Highlight 1:Plot1…On, Enter, Choose what type of mark you'd like , enter. To enter coordinates hit the STAT button, 1:Edit, enter. It brings L1 and L2 and L3. Enter the x values down column L1 and then the corresponding y-values in L2 column. Once they are all in exit that screen and hit GRAPH. It should plot the points for you. You have to go to the y= menu and make sure you highlight Plot1 above the Y1 symbol. Also Use ZOOM [9] to adjust axes to fit the data.

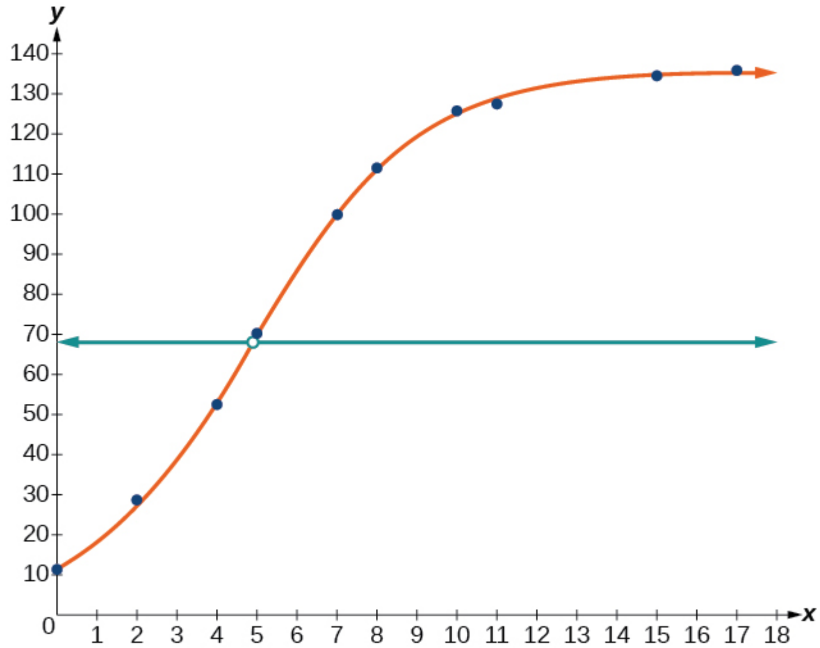


52. -

53. Graph the logistic equation on the scatter diagram.
Input $Y_1 = 136.068 \div \left(1 + 10.324e \wedge \left(-0.480x\right)\right)$



54. -
55. Use the intersect feature to find the value of x for which the model reaches half its carrying capacity.
Half its carrying capacity would be one half of 136, which is 68. Enter a second function into the graph $Y_2 = 68$. Then find the point of intersection of these two graphs. Recall , 2nd CALC, 5:intersection, move cursor to the point of intersection and hit enter 3 times. Read the display at the bottom x=4.8613995 y = 68
When $f(x) = 68,\ x \approx 4.9.$



Extensions

56. -

57. Use a graphing utility to find an exponential regression formula $f(x)$ and a logarithmic regression formula $g(x)$ for the points $(1.5,\ 1.5)$ and $(8.5,\ 8.5)$. Round all numbers to 6 decimal places. Graph the points and both formulas along with the line $y = x$ on the same axis. Make a conjecture about the relationship of the regression formulas.

    To find an exponential regression formula $f(x)$ exponential regression formula: We input the two points x values and y values in for STAT L1 and L2, hit “ExpReg” in the StatCalc menu and obtain a and b putting them in the form $y = ab^x$.

    $f(x) = 1.034341(1.281204)^x$;

    To find a logarithmic function of the form $y = a + b\ln(x)$ that best fits the data in the table. Select “LnReg” from the STAT then CALC menu. Use the values returned for *a* and *b* to record the model to obtain the logarithmic regression formula:
    $g(x) = 4.035510\ln(x) - 0.136259$;

    The regression curves are symmetrical about $y = x$, so it appears that they are inverse functions.

58. -

59. Find the inverse function $f^{-1}(x)$ for the logistic function $f(x) = \frac{c}{1+ae^{-bx}}$. Show all steps.

Let $y = \frac{c}{1+ae^{-bx}}$. Solving for x we get

$$y = \frac{c}{1+ae^{-bx}}$$

$$1+ae^{-bx} = \frac{c}{y}$$

$$ae^{-bx} = \frac{c}{y} - 1$$

$$e^{-bx} = \frac{\frac{c}{y}-1}{a}$$

Taking the natural log of each side:

$$\ln\left(e^{-bx}\right) = \ln\left(\frac{\frac{c}{y}-1}{a}\right)$$

$$-bx = \ln\left(\frac{\frac{c}{y}-1}{a}\right)$$ writing right hand sides as individual logs, division is subtraction

$$-bx = \ln\left(\frac{c}{y}-1\right) - \ln(a)$$ multiply by $\frac{-1}{b}$

$$x = \frac{-\ln\left(\frac{c}{y}-1\right) + \ln(a)}{b}$$

$$x = \frac{\ln(a) - \ln\left(\frac{c}{y}-1\right)}{b}$$

Therefore, $f^{-1}(x) = \frac{\ln(a) - \ln\left(\frac{c}{x}-1\right)}{b}$

60. -

Chapter 6 Review Exercises

Section 6.1

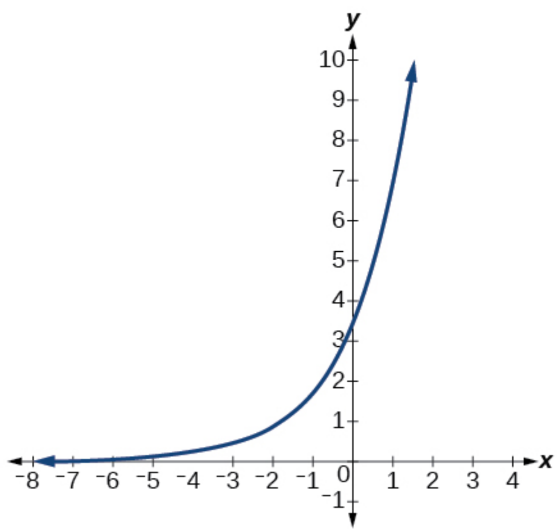
1. Determine whether the function $y = 156(0.825)^t$ represents exponential growth, exponential decay, or neither. Explain
   If the base <1, this base=.825, it is exponential decay; The growth factor, 0.825, is between 0 and 1.
2. -
3. Find an exponential equation that passes through the points $(2,\ 2.25)$ and $(5,\ 60.75)$.
   Because we don't have the initial value (when x = 0) we substitute both points into an equation of the form $f(x) = ab^x$, and then solve the system for a and b. Substituting $(2,\ 2.25)$ gives a first equation of $2.25 = ab^2$ substituting $(5,\ 60.75)$.gives a second equation of $60.75 = ab^5$. Solving the first equation for a in terms of b we divide both sides by b^2 : $\frac{2.25}{b^2} = \frac{ab^2}{b^2}$, therefore $a = \frac{2.25}{b^2}$, now we substitute this into the second equation for a:
   $$\begin{aligned} 60.75 &= ab^5 \\ 60.75 &= \left(\frac{2.25}{b^2}\right)b^5 \\ 60.75 &= 2.25b^3 \\ \frac{60.75}{2.25} &= b^3 \\ 27 &= b^3 \\ 3 &= b \end{aligned}$$
   Input b = 3, then solve for a $a = \frac{2.25}{b^2}$ so $a = \frac{2.25}{b^2} = \frac{2.25}{9} = .25$. Since $a = .25$ and b = 3, then $f(x) = ab^x$ is $y = 0.25(3)^x$
4. -
5. A retirement account is opened with an initial deposit of $8,500 and earns 8.12% interest compounded monthly. What will the account be worth in 20 years?
   for this we use the formula for compound interest, monthly n = 12
   $$A(t) = P\left(1+\frac{r}{n}\right)^{nt} = 8500\left(1+\frac{0.0812}{12}\right)^{12(20)} = 42888.1812 \quad = \quad \$42,888.18$$
6. -

7. Does the equation $y = 2.294e^{-0.654t}$ represent continuous growth, continuous decay, or neither? Explain.
   It is exponential therefore is continuous decay; the growth rate is negative.
8. -

Section 6.2

9. Graph the function $f(x) = 3.5(2)^x$. State the domain and range and give the *y*-intercept.

   The y-intercept is found by inputting x = 0, therefore the y-intercept: $(0,\ 3.5)$; For domain we consider all x values we may input . There are no restrictions, therefore the Domain is $(-\infty, +\infty)$ all real numbers; For the range, we consider the values that y outputs. The limit on our y-values is the x-axis (it is asymptotic to it) therefore the asymptote is at $y = 0$ and the Range is $(0, +\infty)$ all real numbers strictly greater than zero; *y*-intercept: $(0,\ 3.5)$.



10. -

11. The graph of $f(x) = 6.5^x$ is reflected about the *y*-axis and stretched vertically by a factor of 7. What is the equation of the new function, $g(x)$? State its *y*-intercept, domain, and range.

For this graph we are asked to reflect it about the y-axis, thus input a negative in front of the x in the exponent thus $=6.5^{-x}$, then stretched vertically means to multiply it by a factor of 7. $g(x)=7(6.5)^{-x}$; The y-intercept is found by inputting x = 0, $g(0)=7(6.5)^{0}=7$ therefore the y-intercept is $(0,\ 7)$; For domain we consider all x values we may input . There are no restrictions, therefore the Domain is $(-\infty,+\infty)$ all real numbers; For the range, we consider the values that y outputs. The limit on our y-values is the x-axis (it is asymptotic to it) therefore the asymptote is at $y=0$ and the Range is $(0,+\infty)$

$g(x)=7(6.5)^{-x}$; y-intercept: $(0,\ 7)$; Domain: all real numbers; Range: all real numbers greater than 0.

12. -

Section 6.3

13. Rewrite $\log_{17}(4913)=x$ as an equivalent exponential equation.

Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent}=N$; $17^{x}=4913$

14. -

15. Rewrite $a^{-\frac{2}{5}}=b$ as an equivalent logarithmic equation.

Rewriting an exponential expression $base^{exponent}=N$ in logarithmic form is $\log_{base} N = exponent$. Since the $base=a$ and the exponent (which always is alone in the logarithmic equation), $exponent=\frac{-2}{5}$, we rewrite this $\log_a b=-\frac{2}{5}$

16. -

17. Solve for x by converting the logarithmic equation $\log_{64}(x)=\frac{1}{3}$ to exponential form.

Rewriting an exponential expression $base^{exponent}=N$ in logarithmic form is $\log_{base} N = exponent$ yields $x=64^{\frac{1}{3}}=\sqrt[3]{64}=4$

18. -

19. Evaluate $\log(0.000001)$ without using a calculator.

Let $\log(0.000001) = x,$ that means $10^x = 0.000001 = \dfrac{1}{1{,}000{,}000} = \dfrac{1}{10^6} = 10^{-6}$, that means

$x = -6$, so $\log(0.000001) = -6$

20. -

21. Evaluate $\ln\left(e^{-0.8648}\right)$ without using a calculator

$\ln\left(e^{-0.8648}\right) = -0.8648\ln(e) = -0.8648(1) = -0.8648$

22. -

Section 6.4

23. Graph the function $g(x) = \log(7x+21) - 4$.

We must keep

$$7x + 21 > 0$$
$$7x + 21 - 21 > 0 - 21$$
$$7x > -21$$
$$\frac{7x}{7} > \frac{-21}{7}$$
$$x > -3$$

it follows that $x > -3$, therefore this graph shifts to the left 3 units, therefore the vertical asymptote which is normally occurs at $x = 0$, will be at $x = -3$.

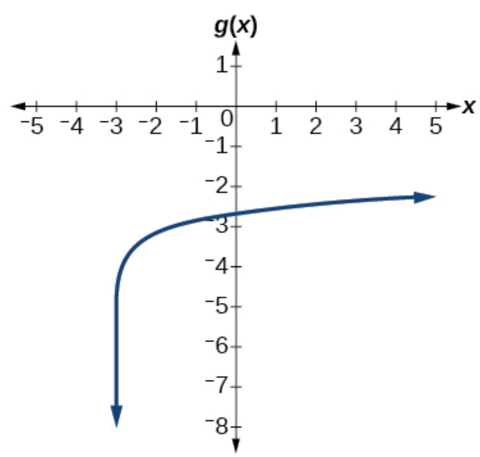
To find the x-intercept we set y = 0:

$$0 = \log(7x+21) - 4$$
$$0 + 4 = \log(7x+21) - 4 + 4$$
$$4 = \log(7x+21) \quad \textit{rewrite exponentially}$$
$$10^4 = 7x + 21$$
$$9979 = 7x$$
$$1425 \approx x$$

Therefore the x-intercept would be $(1425,\ 0)$;

To find the y-intercept we set x = 0:

$$y = \log(7(0) + 21) - 4.$$
$$y = \log(21) - 4.$$
$$y \approx -2.68$$



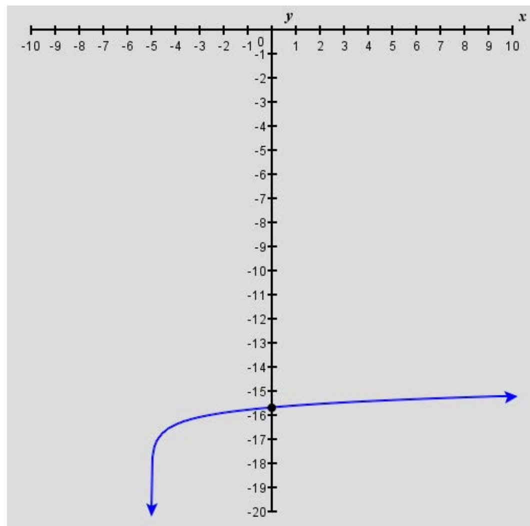
24. -

25. State the domain, vertical asymptote, and end behavior of the function $g(x) = \ln(4x+20) - 17$.

For the domain, we must keep

$$4x+20>0$$
$$4x+20-20>0-20$$
$$4x>-20$$
$$\frac{4x}{4}>\frac{-20}{4}$$
$$x>-5$$

it follows that $x>-5$, therefore the Domain is $(-5,\infty)$; This graph shifts to the left 5 units, therefore the vertical asymptote which is normally occurs at $x=0$, shifts to the left 5 units. The vertical asymptote will be at $x=-5$. Sketching this graph helps describe the end behavior. This is also translated down 17.



Domain: $x>-5$; Vertical asymptote: $x=-5$; End behavior: as x approaches -5 from the right the function values approach negative infinity and as x approaches positive infinity, f(x) increases without bound . Therefore as $x \to -5^+$, $f(x) \to -\infty$ and as $x \to \infty$, $f(x) \to \infty$.

Section 6.5

26. -

27. Rewrite $\log_8(x)+\log_8(5)+\log_8(y)+\log_8(13)$ in compact form.

Since addition is going on as individual logarithms, using the product rule in reverse, condensing to a single logarithm will result in multiplication, therefore

$$\log_8(x)+\log_8(5)+\log_8(y)+\log_8(13)=\log_8(x \bullet 5 \bullet y \bullet 13)=\log_8(65xy)$$

28. -

29. Rewrite $\ln(z)-\ln(x)-\ln(y)$ in compact form.

Since subtraction is going on as individual logarithms, using the quotient rule in reverse, condensing to a single logarithm will result in division, therefore grouping the first two terms we get $\left(\ln(z)-\ln(x)\right)-\ln(y) = \ln\left(\dfrac{\frac{z}{x}}{y}\right)$ then, since $\dfrac{\frac{z}{x}}{y}=\dfrac{z}{x}\div\dfrac{y}{1}=\dfrac{z}{x}\bullet\dfrac{1}{y}=\dfrac{z}{xy}$ our final answer is $\ln\left(\dfrac{\frac{z}{x}}{y}\right)=\ln\left(\dfrac{z}{xy}\right)$

30. -

31. Rewrite $-\log_y\left(\dfrac{1}{12}\right)$ as a single logarithm.

$$-\log_y\left(\frac{1}{12}\right)=-\log_y\left(12^{-1}\right)=-1\left(-1\log_y(12)\right)= \log_y(12)$$

32. -

33. Use properties of logarithms to expand $\ln\left(2b\sqrt{\dfrac{b+1}{b-1}}\right)$.

Rewriting this, and simplifying the numerator we have

$\ln\left(2b\sqrt{\dfrac{b+1}{b-1}}\right).=\ln\left(2b\bullet\left(\dfrac{b+1}{b-1}\right)^{\frac{1}{2}}\right)=\ln\left(2\bullet b\bullet\dfrac{(b+1)^{\frac{1}{2}}}{(b-1)^{\frac{1}{2}}}\right)=\ln\left(\dfrac{2\bullet b\bullet(b+1)^{\frac{1}{2}}}{(b-1)^{\frac{1}{2}}}\right)$. Now we will apply the product, quotient rule and power rules

$$\ln\left(\frac{2\bullet b\bullet(b+1)^{\frac{1}{2}}}{(b-1)^{\frac{1}{2}}}\right)=\ln(2)+\ln(b)+\ln(b+1)^{\frac{1}{2}}-\ln(b-1)^{\frac{1}{2}}=\ln(2)+\ln(b)+\frac{1}{2}\ln(b+1)-\frac{1}{2}\ln(b-1)=$$

$$\ln(2)+\ln(b)+\frac{\ln(b+1)-\ln(b-1)}{2}$$

34. -

35. Condense the expression $3\log_7 v+6\log_7 w-\dfrac{\log_7 u}{3}$ to a single logarithm.

Since there are coefficients on the front of the logarithms we apply the power rule first (the last term has $\dfrac{1}{3}$ as a coefficient) and place them as exponents

$3\log_7 v + 6\log_7 w - \frac{\log_7 u}{3} = \log_7\left(v^3\right) + \log_7\left(w^6\right) + \log_7\left(u^{\frac{1}{3}}\right)$. Since addition is going on as individual logarithms and subtraction of the last term, using the product & quotient rule in reverse, condensing to a single logarithm will result in multiplication then division, therefore

$$\log_7\left(v^3\right)+\log_7\left(w^6\right)+\log_7\left(u^{\frac{1}{3}}\right)=\log_7\left(\frac{v^4 \bullet w^6}{u^{\frac{1}{3}}}\right)= \log_7\left(\frac{v^3 w^6}{\sqrt[3]{u}}\right)$$

36. -

37. Rewrite $5^{12x-17} = 125$ as a logarithm. Then apply the change of base formula to solve for x using the common log. Round to the nearest thousandth.

$$5^{12x-17} = 125$$
$$\log_5 125 = 12x - 17$$
$$\frac{\log 125}{\log 5} = 12x - 17$$
$$\frac{\frac{\log 125}{\log 5} + 17}{12} = \frac{12x}{12}$$
$$\frac{\frac{\log 125}{\log 5} + 17}{12} = x$$
$$x = \frac{\frac{\log(125)}{\log(5)} + 17}{12} = \frac{5}{3}$$

Section 6.6

38. -

39. Solve $\frac{125}{\left(\frac{1}{625}\right)^{-x-3}} = 5^3$ by rewriting each side with a common base.

Find the same common base between 125, 625 and 5 which would be 5. We want to rewrite each side with the base of 5, using all the correct exponent rules. Once we have a single base expression on each side and the base is the same, we set the exponents equal and solve for x

$$\frac{125}{\left(\frac{1}{625}\right)^{-x-3}} = 5^3$$

$$\frac{5^3}{\left(5^{-4}\right)^{(-x-3)}} = 5^3 \quad \text{Rewrite each side as a power with base 5.}$$

$$\frac{5^3}{5^{4x+12}} = 5^3 \quad \text{subtract exponents on left side .}$$

$$5^{3-(4x+12)} = 5^3$$

$$5^{3-4x-12} = 5^3$$

$$-4x - 9 = 3 \quad \text{Apply the one-to-one property of exponents.}$$

$$-4x = 12 \quad \text{Subtract 12 from both sides.}$$

$$x = -3 \quad \text{Divide by -4.}$$

$x = -3$

40. -

41. Use logarithms to find the exact solution for $3e^{6n-2} + 1 = -60$. If there is no solution, write *no solution*.

We want the e^{6n-2} alone on one side before we begin.

$$3e^{6n-2} + 1 = -60. \quad \text{Subtract 1 from both sides}$$

$$3e^{6n-2} = -61 \quad \text{Divide both sides by 3.}$$

$$e^{6n-2} = \frac{-61}{3}$$

$$\ln e^{6n-2} = \ln\left(\frac{-61}{3}\right) \quad \text{Take ln of both sides.}$$

We can stop here because $\ln\left(\frac{-61}{3}\right)$ is not defined, therefore there is no solution

42. -

43. Find the exact solution for $2e^{5x-2} - 9 = -56$. If there is no solution, write *no solution*.

We want the e^{5x-2} alone on one side before we begin.

$2e^{5x-2} - 9 = -56$ Add 9 to both sides

$2e^{5x-2} = -47$ Divide both sides by 2.

$e^{5x-2} = \dfrac{-47}{2}$

$\ln e^{5x-2} = \ln\left(\dfrac{-47}{2}\right)$ Take ln of both sides.

We can stop here because $\ln\left(\dfrac{-47}{2}\right)$ is not defined, therefore there is no solution

44. -

45. Find the exact solution for $e^{2x} - e^{x} - 110 = 0$. If there is no solution, write *no solution*.

Since this is a quadratic form of an exponential equation , we plan to use factoring to solve the problem. We always get zero on one side of the equation, because zero has the unique property that when a product is zero, one or both of the factors must be zero.

$e^{2x} - e^{x} - 110 = 0.$ Get one side of the equation equal to zero.

$(e^{x} + 10)(e^{x} - 11) = 0$ Factor by the FOIL method.

$e^{x} + 10 = 0$ or $e^{x} - 11 = 0$ If a product is zero, then one factor must be zero.

$e^{x} = -10$ or $e^{x} = 11$ Isolate the exponentials.

$e^{x} = 11$ Reject the equation in which the power equals a negative number.

$x = \ln(11)$ Solve the equation in which the power equals a positive number.

$x = \ln(11)$

46. -

47. Use the definition of a logarithm to find the exact solution for $9 + 6\ln(a+3) = 33$.

We want the $\ln(a+3)$ alone on one side before we begin.

$9 + 6\ln(a+3) = 33$ Subtract 9 from both sides

$6\ln(a+3) = 24$ Divide both sides by 6

$\ln(a+3) = 4$

$e^{4} = a + 3$ Rewrite exponentially

$e^{4} - 3 = a$ Subtract 3 from both sides

Substituting $e^{4} - 3 = a$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore this is a solution.

$a = e^{4} - 3$

48. -

49. Use the one-to-one property of logarithms to find an exact solution for $\ln(5)+\ln(5x^2-5)=\ln(56)$. If there is no solution, write *no solution*.

We have to rewrite each side as a single logarithm first

$$\ln(5)+\ln(5x^2-5)=\ln(56)$$
$$\ln(5g(5x^2-5))=\ln(56)$$
$$25x^2-25=56 \quad \text{Use the one-to-one property of the logarithm}$$
$$25x^2=81 \quad \text{Add 25 to both sides}$$
$$25x^2=81 \quad \text{Divide by 25}$$
$$x^2=\frac{81}{25} \quad \text{Take the square root of both sides}$$
$$x=\pm\frac{9}{5}$$

Substituting both $x=\pm\frac{9}{5}$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore each one is a solution.

$x=\pm\frac{9}{5}$

50. -

51. The population of a city is modeled by the equation $P(t)=256{,}114e^{0.25t}$ where t is measured in years. If the city continues to grow at this rate, how many years will it take for the population to reach one million?

We are being asked to solve for t when $P(t)=1{,}000{,}000$

$$P(t)=256{,}114e^{0.25t}$$
$$1{,}000{,}000=256{,}114e^{0.25t} \quad \textit{get } e^{0.25t} \text{ alone on one side}$$
$$\frac{1{,}000{,}000}{256{,}114}=e^{0.25t}$$
$$3.904511272=e^{0.25t} \quad \text{take ln of both sides to solve for x}$$
$$\ln(3.904511272)=\ln(e^{0.25t}) \quad \text{recall power rule of logs on right side and ln e = 1}$$
$$\ln(3.904511272)=0.25t\ln(e)$$
$$\ln(3.904511272)=0.25t \quad \text{Divide by 0.25}$$
$$\frac{\ln(3.904511272)}{0.25t}=x$$
$$x=\ln(3.904511272)\div 0.25=5.448530484$$

It will take about 5.45 years

52. -

53. Find the inverse function f^{-1} for the logarithmic function $f(x)=0.25\cdot\log_2(x^3+1)$.

$y=0.25\cdot\log_2\left(x^3+1\right)$ We solve for x so we want to get the logarithm is alone on one side, so we can rewrite this in exponential form

$y=0.25\cdot\log_2\left(x^3+1\right)$ get log alone on right side, divide by 0.25

$\frac{y}{0.25}=\log_2\left(x^3+1\right)$ $\quad \frac{1}{.25}=1\div.25=4$

$4y=\log_2\left(x^3+1\right)$ rewrite exponentially

$2^{4y}=x^3+1$ subtract 1

$2^{4y}-1=x^3$ take the cube root of both sides

$\sqrt[3]{2^{4y}-1}=x$

Where there is the y we place *x*, so the inverse is $f^{-1}\left(x\right)=\sqrt[3]{2^{4x}-1}$

Section 6.7

For the following exercises, use this scenario: A doctor prescribes 300 milligrams of a therapeutic drug that decays by about 17% each hour.

54. -
55. Write an exponential model representing the amount of the drug remaining in the patient's system after t hours. Then use the formula to find the amount of the drug that would remain in the patient's system after 24 hours. Round to the nearest hundredth of a gram.

    Referring to the previous problem we found $y=ab^x=300(.83)^x$, now we plug in t = 24 hours $f(24)=300(.83)^{24}=3.427642126$

    $f(t)=300\left(0.83\right)^t;\ f(24)\approx 3.43g$

For the following exercises, use this scenario: An oven brick with a temperature of 350° Fahrenheit was taken off the stove to cool in a 71℉ room. After fifteen minutes, the temperature of the brick was 175℉.

56. -

57. How many minutes will it take the brick to cool to 85℉?

    To solve this we use our model and T(t)=85, and solve for t.

$$T(t) = 279e^{(-0.065788t)} + 71,$$

$$85 = T(t) = 279e^{(-0.065788t)} + 71, \quad \text{subtract 71, divide by 279}$$

$$\frac{14}{279} = \frac{279e^{(-0.065788t)}}{279}$$

$$\frac{7}{18} = e^{-0.065788t} \quad \text{Take the ln of both sides, power rule on right side and ln e = 1}$$

$$\ln\left(\frac{14}{279}\right) = -0.065788t \quad \text{divide by } -0.065788$$

$$\frac{\ln\left(\frac{14}{279}\right)}{-0.065788} = t$$

$$t = 45.48172581$$

The brick will cool to 85 degrees in about 45 minutes

For the following exercises, use this scenario: The equation $N(t) = \frac{1200}{1+199e^{-0.625t}}$ models the number of people in a school who have heard a rumor after t days.

58. -

59. To the nearest tenth, how many days will it be before the rumor spreads to half the carrying capacity?

Half the carrying capacity is half of 1200 = 600, so we will solve for t

$$600 = \frac{1200}{1+199e^{-0.625t}}$$

$$600\left(1+199e^{-0.625t}\right) = 1200 \quad \text{divide by 600}$$

$$1+199e^{-0.625t} = 2$$

$$199e^{-0.625t} = 1$$

$$e^{-0.625t} = \frac{1}{199} \quad \text{take ln of both sides , use exponents rules}$$

$$\ln\left(\frac{1}{199}\right) = -0.625t \quad \text{divide by } -0.625$$

$$\frac{\ln\left(\frac{1}{199}\right)}{-0.625} = t$$

$$t = 8.46928772$$

In about 8.5 days.

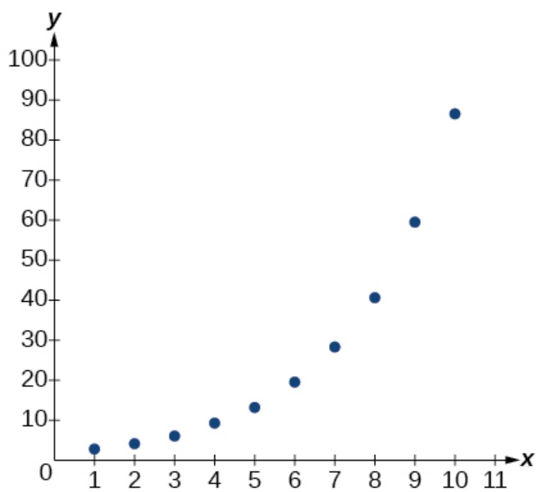
60. –

61.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	3.05	4.42	6.4	9.28	13.46	19.52	28.3	41.04	59.5	86.28

Using a graphing calculator, you can plot individual points using STAT Plot. Press 2nd STAT PLOT, which is above Y=. Highlight 1:Plot1…On, Enter, Choose what type of mark you'd like , enter. To enter coordinates hit the STAT button, 1:Edit, enter. It brings L1 and L2 and L3. Enter the x values down column L1 and then the corresponding y-values in L2 column. Once they are all in exit that screen and hit GRAPH. It should plot the points for you. You have to go to the y= menu and make sure you highlight Plot1 above the Y1 symbol. Also Use ZOOM [9] to adjust axes to fit the data.

Because of the concave up curve it is exponential.



62. -

63. Find a formula for an exponential equation that goes through the points $(-2,\ 100)$ and $(0,\ 4)$. Then express the formula as an equivalent equation with base *e*.

We are given the point (0,4) and since

$$f(x) = ab^x$$
$$4 = ab^0$$
$$4 = a$$

We solve for b by plugging in the given point (-2,100)

$100 = 4(b)^{-2}$

$25 = b^{-2}$ divide by 4

$25 = \frac{1}{b^2}$

$25b^2 = 1$

$b^2 = \frac{1}{25}$

$b = \pm\frac{1}{5} = .2$ choose the positive number

$y = 4(0.2)^x$;to change it to base e $y = ab^x = 4(0.2)^x = 4e^{(\ln .2)x}$ so $y = 4e^{-1.609438x}$

Section 6.8

64. -

65. The population of a culture of bacteria is modeled by the logistic equation $P(t) = \frac{14,250}{1+29e^{-0.62t}}$, where t is in days. To the nearest tenth, how many days will it take the culture to reach 75% of its carrying capacity?

75% of the carrying capacity would be $.75(14,2500) = 10,687.5$. We will solve the equation for t when it's equal to 10,687.5:

$$10,687.5 = \frac{14,250}{1+29e^{-0.62t}},$$

$$10,687.5\left(1+29e^{-0.62t}\right) = 14,250 \quad \text{divide by } 10,687.5$$

$$1+29e^{-0.62t} = \frac{4}{3}$$

$$29e^{-0.62t} = \frac{1}{3} \quad \text{divide by 29}$$

$$e^{-0.62t} = \frac{1}{87} \quad \text{take ln of both sides , use exponents rules}$$

$$\ln\left(\frac{1}{87}\right) = -0.62t \quad \text{divide by } -0.62$$

$$\frac{\ln\left(\frac{1}{87}\right)}{-0.62} = t$$

$$t = 7.203077611$$

So it will be in about 7.2 days .

For the following exercises, use a graphing utility to create a scatter diagram of the data given in the table. Observe the shape of the scatter diagram to determine whether the data is best described by an exponential, logarithmic, or logistic model. Then use the appropriate regression feature to find an equation that models the data. When necessary, round values to five decimal places.

66. -

67.

x	0.15	0.25	0.5	0.75	1	1.5	2	2.25	2.75	3	3.5
***f*(*x*)**	36.21	28.88	24.39	18.28	16.5	12.99	9.91	8.57	7.23	5.99	4.81

Using a graphing calculator, you can plot individual points using STAT Plot. Press 2nd STAT PLOT, which is above Y=. Highlight 1:Plot1…On, Enter, Choose what type of mark you'd like, enter. To enter coordinates hit the STAT button, 1:Edit, enter. It brings L1 and L2 and L3. Enter the x values down column L1 and then the corresponding y-values in L2 column. Once they are all in exit that screen and hit GRAPH. It should plot the points for you. You have to go to the y= menu and make sure you highlight Plot1 above the Y1 symbol. Also Use ZOOM [9] to adjust axes to fit the data.

Because it appear the data is asymptotic to the y-axis as it gets close to x=0, we would choose the logarithmic curve.

To find the logarithmic equation that models the data, select "LnReg" from the STAT then CALC menu. Use the values returned for *a* and *b* to record the model, $y = a + b\ln(x)$

Your screen will display

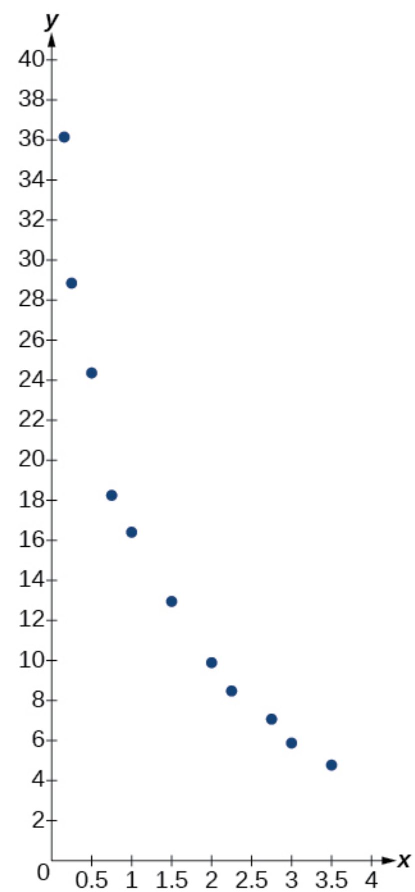
$y = a + b\ln x$

$a = 16.68718406$

$b = -9.718602513$

Therefore the logarithmic function that best fits the data in the table is

$y = 16.68718 - 9.71860\ln(x)$



68. -

Chapter 6 Practice Test

1. The population of a pod of bottlenose dolphins is modeled by the function $A(t) = 8(1.17)^t$, where t is given in years. To the nearest whole number, what will the pod population be after 3 years?

Since $A(t) = 8(1.17)^t$, we want to find $A(3) = 8(1.17)^3 = 12.812904 \approx 13$.

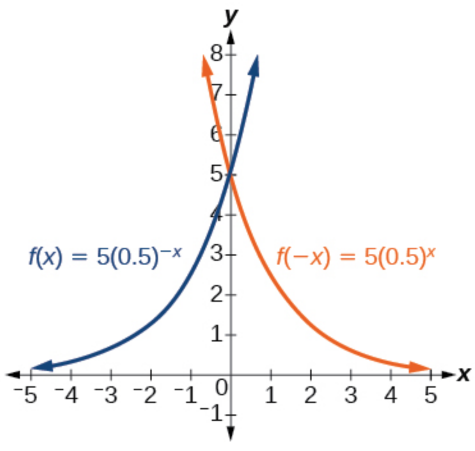
About 13 dolphins.

2. -
3. Drew wants to save $2,500 to go to the next World Cup. To the nearest dollar, how much will he need to invest in an account now with 6.25% APR, compounding daily, in order to reach his goal in 4 years?

   For this we use the Present Value formula derived from the compound interest formula, compounded daily n= 365

   $$P = A(t)\cdot\left(1+\frac{r}{n}\right)^{-nt} = 2500\cdot\left(1+\frac{0.0625}{365}\right)^{-365(4)} = 1947.045627$$. He will have to invest approximately $\$1947$.
4. -
5. Graph the function $f(x) = 5(0.5)^{-x}$ and its reflection across the *y*-axis on the same axes, and give the *y*-intercept.

   To graph this reflected about the y-axis puts a negative in front of the – x in the exponent. This would make the new function $f(x) = 5(0.5)^{-(-x)} = 5(0.5)^x$. The y-intercept is still $(0,\ 5)$.



6. -
7. Rewrite $\log_{8.5}(614.125) = a$ as an equivalent exponential equation.

   Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$;

   $8.5^a = 614.125$
8. -

9. Solve for x by converting the logarithmic equation $\log_{\frac{1}{7}}(x) = 2$ to exponential form.

   Rewriting $\log_{base} N = exponent$ in exponential form is $base^{exponent} = N$ yields

   $x = \left(\frac{1}{7}\right)^2 = \frac{1}{49}$
10. -
11. Evaluate $\ln(0.716)$ using a calculator. Round to the nearest thousandth.

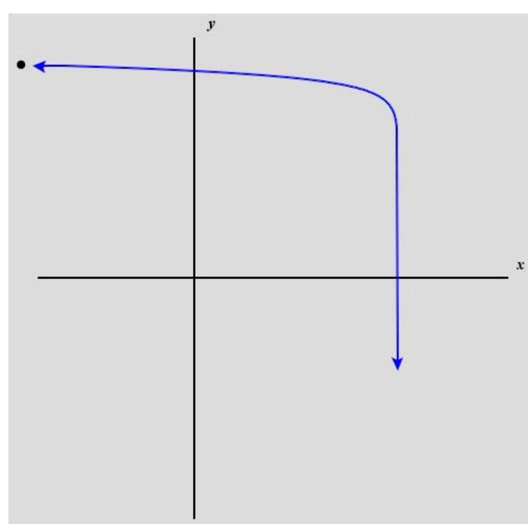
    $\ln(0.716) \approx -0.334$
12. -
13. State the domain, vertical asymptote, and end behavior of the function $f(x) = \log_5(39-13x)+7$.

    For the Domain we must keep

$$39 - 13x > 0$$
$$39 - 13x - 39 > 0 - 39$$
$$-13x > -39$$
$$\frac{-13x}{-13} < \frac{-39}{-13}$$
$$x < 3$$

it follows that $x < 3$, therefore the Domain is $(-\infty, 3)$;

This graph shifts to the right 3 units, therefore the vertical asymptote which is normally occurs at $x = 0$, shifts to the right 3 units. The vertical asymptote will be $x = 3$; Sketching this graph helps describe the end behavior.



As x approaches 3 from the left the end behavior: as $x \to 3^-$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to \infty$

14. -

15. Rewrite $\log_t(96) - \log_t(8)$ in compact form.

    Since subtraction is going on as individual logarithms, using the quotient rule in reverse, condensing to a single logarithm will result in division

    $$\log_t(96) - \log_t(8) = \log_t\left(\frac{96}{8}\right) = \log_t(12)$$

16. -

17. Use properties of logarithm to expand $\ln\left(y^3 z^2 \cdot \sqrt[3]{x-4}\right)$.

    apply the product, and power rules

    $$\ln\left(y^3 \cdot z^2 \cdot \sqrt[3]{x-4}\right) = \ln\left(y^3 \bullet z^2 \bullet (x-4)^{\frac{1}{3}}\right) = \ln\left(y^3\right) + \ln\left(z^2\right) + \ln(x-4)^{\frac{1}{3}} = 3\ln(y) + 2\ln(z) + \frac{1}{3}\ln(x-4) =$$

    $$3\ln(y) + 2\ln(z) + \frac{\ln(x-4)}{3}$$

18. -

19. Rewrite $16^{3x-5} = 1000$ as a logarithm. Then apply the change of base formula to solve for x using the natural log. Round to the nearest thousandth.

    $$16^{3x-5} = 1000$$
    $$\log_{16} 1000 = 3x - 5$$
    $$\frac{\ln 1000}{\ln 16} = 3x - 5$$
    $$\frac{\frac{\ln 1000}{\ln 16} + 5}{3} = \frac{3x}{3}$$
    $$\frac{\frac{\ln 1000}{\ln 16} + 5}{3} = x$$
    $$x = \frac{\frac{\ln(1000)}{\ln(16)} + 5}{3} \approx 2.497$$

20. -

21. Use logarithms to find the exact solution for $-9e^{10a-8} - 5 = -41$. If there is no solution, write *no solution*.

We want the e^{10a-8} alone on one side before we begin.

$-9e^{10a-8}-5=-41$ Add 5 to both sides

$-9e^{10a-8}=-36$

$e^{10a-8}=4$ Divide both sides by -9.

$\ln e^{10a-8}=\ln(4)$ Take ln of both sides.

$(10a-8)\ln e=\ln(4)$ Use laws of logs. Remember ln e = 1

$(10a-8)(1)=\ln(4)$

$10a-8=\ln(4)$ Add 8 and Divide both sides by 10

$$a=\frac{\ln(4)+8}{10}$$

22. -
23. Find the exact solution for $-5e^{-4x-1}-4=64$. If there is no solution, write *no solution*.

We want the e^{-4x-1} alone on one side before we begin.

$-5e^{-4x-1}-4=64$ Add 4 to both sides

$-5e^{-4x-1}=60$ Divide both sides by -5.

$e^{-4x-1}=\frac{60}{-5}$

$\ln\left(e^{-4x-1}\right)=\ln(-12)$ Take ln of both sides.

We can stop here because $\ln(-12)$ is not defined, therefore there is No solution

24. -
25. Find the exact solution for $e^{2x}-e^{x}-72=0$. If there is no solution, write *no solution*.

Since this is a quadratic form of an exponential equation, we plan to use factoring to solve the problem. We always get zero on one side of the equation, because zero has the unique property that when a product is zero, one or both of the factors must be zero.

$e^{2x} - e^{x} - 72 = 0$ Get one side of the equation equal to zero.

$(e^{x} + 8)(e^{x} - 9) = 0$ Factor by the FOIL method.

$e^{x} + 8 = 0$ or $e^{x} - 9 = 0$ If a product is zero, then one factor must be zero.

$e^{x} = -8$ or $e^{x} = 9$ Isolate the exponentials.

$e^{x} = 9$ Reject the equation in which the power equals a negative number.

$x = \ln(9)$ Solve the equation in which the power equals a positive number.

26. -

27. Use the one-to-one property of logarithms to find an exact solution for $\log\left(4x^2 - 10\right) + \log(3) = \log(51)$ If there is no solution, write *no solution*.

We have to rewrite each side as a single logarithm first

$\log\left(4x^2 - 10\right) + \log(3) = \log(51)$

$\log\left(\left(4x^2 - 10\right) g3\right) = \ln(51)$

$12x^2 - 30 = 51$ Use the one-to-one property of the logarithm

$12x^2 = 81$ Divide by 12

$x^2 = \frac{81}{12}$

$x^2 = \frac{27}{4}$ Take the square root of both sides

$x = \pm\frac{\sqrt{27}}{2} = \pm\frac{3\sqrt{3}}{2}$

Substituting both $x = \pm\frac{3\sqrt{3}}{2}$ into the original logarithmic functions we see the argument of the logarithm functions is positive therefore each one is a solution.

$x = \pm\frac{3\sqrt{3}}{2}$

28. -

29. A radiation safety officer is working with 112 grams of a radioactive substance. After 17 days, the sample has decayed to 80 grams. Rounding to five significant digits, write an exponential equation representing this situation. To the nearest day, what is the half-life of this substance?

Since the initial amount $A_0 = 112g$ and after 17 days, it decays to 80 grams we need to find k, so plugging in the given information we see:

$$A_0 = A_0 e^{kt}$$

$$80 = 112e^{k(17days)}$$ divide both sides by 112 to get e^{kt} *alone*

$$0.7142857143 = e^{17k}$$ take the ln of both sides and use power rule on rigft side

$$\ln 0.7142857143 = 17k \ln e$$ ln e = 1

$$\ln 0.7142857143 = 17k$$ divide both sides by 17

$$\frac{\ln 0.7142857143}{17} = k$$

$$k = -.0197924845 \approx -.01979$$

Therefore $f(t) = 112e^{-.019792t}$

To find the half-life, $t = \frac{-\ln 2}{k} = \frac{-\ln 2}{-.01979} = 35.0215835 \approx 35$ days

30. -

31. A bottle of soda with a temperature of $71°$ Fahrenheit was taken off a shelf and placed in a refrigerator with an internal temperature of $35°F$. After ten minutes, the internal temperature of the soda was $63°F$. Use Newton's Law of Cooling to write a formula that models this situation. To the nearest degree, what will the temperature of the soda be after one hour?

The model formula we use is $T(t) = Ae^{kt} + T_s$ and we know $T_s = 35°$ and $A = 71 - 35 = 36$, therefore to find k, we plug in the fact that when $t = 10\text{ min}, T(10) = 63$

$$T(t) = Ae^{kt} + T_s$$

$63 = 36e^{k(10)} + 35$ Subtract 35 and then divide by 36

$$\frac{28}{36} = \frac{36e^{k(10)}}{36}$$

$\frac{7}{9} = e^{10k}$ Take the ln of both sides, power rule on right side and ln e = 1

$\ln\left(\frac{7}{9}\right) = 10k$ divide by 10

$$\frac{\ln\left(\frac{7}{9}\right)}{10} = k$$

$$k \approx -0.0251314428$$

Therefore our model would be: $T(t) = 36e^{-0.025131t} + 35$,

Using our model to find the temperature after 1 hours we will plug in t= 60 minutes

$$T(60) = 36e^{-0.025131(60)} + 35 = 42.96979633 \approx 43°F$$

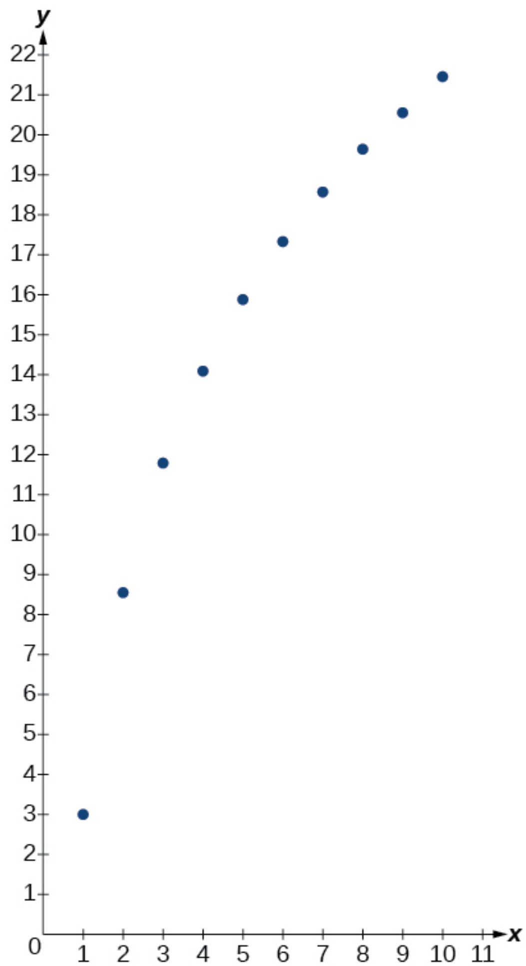
32. -

33. Enter the data from the table below into a graphing calculator and graph the resulting scatter plot. Determine whether the data from the table would likely represent a function that is linear, exponential, or logarithmic.

x	1	2	3	4	5	6	7	8	9	10
***f*(*x*)**	3	8.55	11.79	14.09	15.88	17.33	18.57	19.64	20.58	21.42

So Using a graphing calculator, you can plot individual points using STAT Plot. Press 2nd STAT PLOT, which is above Y=. Highlight 1:Plot1…On, Enter, Choose what type of mark you'd like , enter. To enter coordinates hit the STAT button, 1:Edit, enter. It brings L1 and L2 and L3. Enter the x values down column L1 and then the corresponding y-values in L2 column. Once they are all in exit that screen and hit GRAPH. It should plot the points for you. You have to go to the y= menu and make sure you highlight Plot1 above the Y1 symbol. Also Use ZOOM [9] to adjust axes to fit the data.

This is a logarithmic curve.



34. -

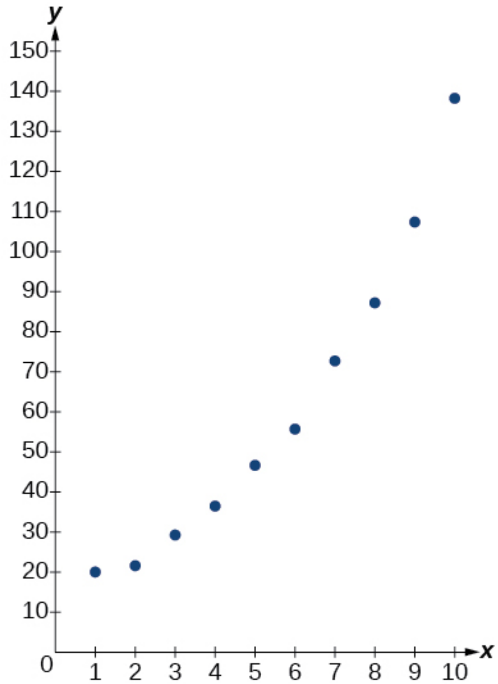
35.

x	1	2	3	4	5	6	7	8	9	10
***f*(*x*)**	20	21.6	29.2	36.4	46.6	55.7	72.6	87.1	107.2	138.1

So Using a graphing calculator, you can plot individual points using STAT Plot. Press 2nd STAT PLOT, which is above Y=. Highlight 1:Plot1…On, Enter, Choose what type of mark you'd like , enter. To enter coordinates hit the STAT button, 1:Edit, enter. It brings L1 and L2 and L3. Enter the x values down column L1 and then the corresponding y-values in L2 column. Once they are all in exit that screen and hit GRAPH. It should plot the points for you. You have to go to the

y= menu and make sure you highlight Plot1 above the Y1 symbol. Also Use ZOOM [9] to adjust axes to fit the data.
This is an exponential curve.

To Find the equation that models the data. Select "ExpReg" from the STAT then CALC menu. Use the values returned for a and b to record the model, $y = ab^x$.
Your screen will display
$y = a*b \wedge x$
$a = 15.10062202$
$b = 1.246214498$
Therefore the exponential function that best fits the data in the table is
$y = 15.10062(1.24621)^x$



36. -

37.

x	0	0.5	1	1.5	2	3	4	5	6	7	8
f(x)	2.2	2.9	3.9	4.8	6.4	9.3	12.3	15	16.2	17.3	17.9

This has the shape of a logistic curve. Therefore we will use:
To find the LOGISTIC equation that models the data. Select "B:Logistic" from the STAT then CALC menu. Use the values returned for a, b and c to record the model,

$$y = \frac{c}{1 + ae^{-bx}}$$

Your screen will display

$y = c/(1 + ae \wedge (-bx))$

$a = 7.546440174$

$b = .6837504736$

$c = 18.41658808$

Therefore the LOGISTIC function that best fits the data in the table is

$$y = \frac{18.41659}{1 + 7.54644e^{-0.68375x}}$$