

For a circle  $S = \rho\theta$  Or  $\rho = S/\theta$  For an arc  $\rho = dS/d\theta$

We already saw  $dS = dx \sqrt{1 + (dy/dx)^2}$

$$\rho = \frac{dx}{d\theta} \sqrt{1 + (dy/dx)^2}$$

What is  $dx/d\theta$ ?  
Well it is  $1 / d\theta/dx$ !

Recall  $\tan\theta = \frac{dy}{dx}$  So  $\theta = \arctan\left(\frac{dy}{dx}\right)$

$$\frac{d\theta}{dx} = \frac{1}{1 + (dy/dx)^2} \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{1}{1 + (dy/dx)^2} \frac{d^2y}{dx^2}$$

Finally 
$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

