

ABSOLUTE DEPENDENT MOTION ANALYSIS OF TWO PARTICLES

Today's Objectives:

Students will be able to:

1. Relate the positions, velocities, and accelerations of particles undergoing dependent motion.



In-Class Activities:

- Check homework
- Reading Quiz
- Applications
- Define Dependent Motion
- Develop Position, Velocity, and Acceleration Relationships
- Concept Quiz
- Group Problem Solving
- Attention Quiz



READING QUIZ

1. When particles are interconnected by a cable, the motions of the particles are _____.
 - A) always independent
 - B) always dependent
 - C) dependent, but not always
 - D) None of the above.
2. If the motion of one particle is dependent on that of another particle, each coordinate axis of the particles _____.
 - A) should be directed along the path of motion
 - B) can be directed anywhere
 - C) should have the same origin
 - D) None of the above.



APPLICATIONS



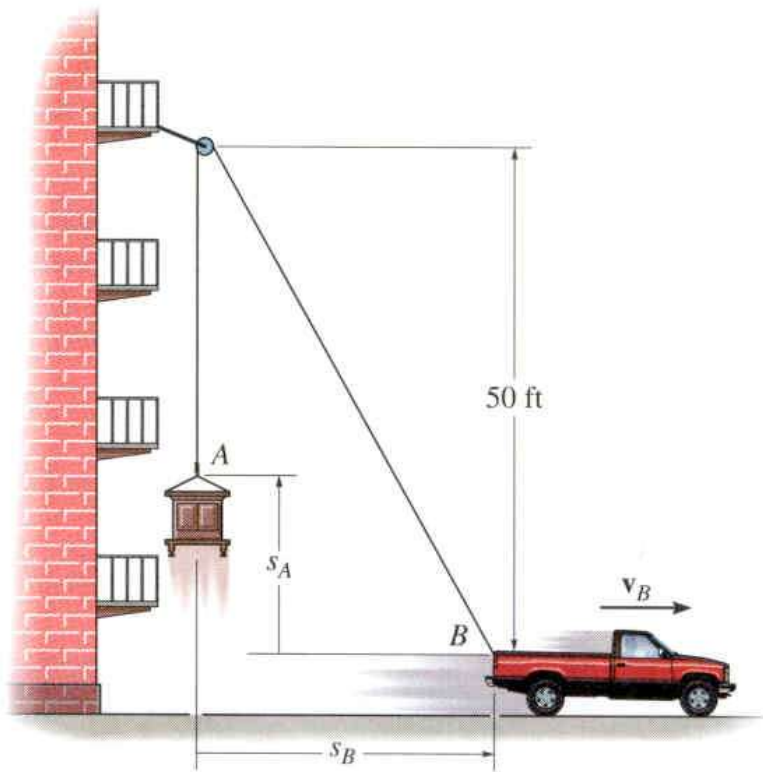
The cable and pulley system shown here can be used to modify the speed of block B relative to the speed of the motor. It is important to relate the various motions in order to determine the power requirements for the motor and the tension in the cable.

If the speed of the cable coming onto the motor pulley is known, how can we determine the speed of block B?



APPLICATIONS

(continued)



Rope and pulley arrangements are often used to assist in lifting heavy objects. The total lifting force required from the truck depends on the acceleration of the cabinet.

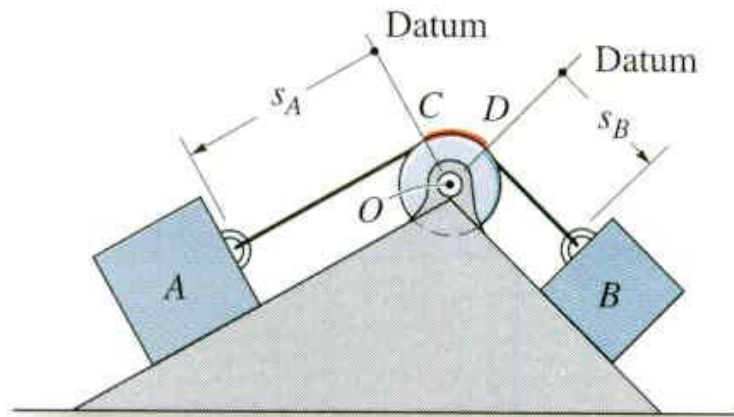
How can we determine the acceleration and velocity of the cabinet if the acceleration of the truck is known?



DEPENDENT MOTION

(Section 12.9)

In many kinematics problems, the motion of one object will **depend** on the motion of another object.



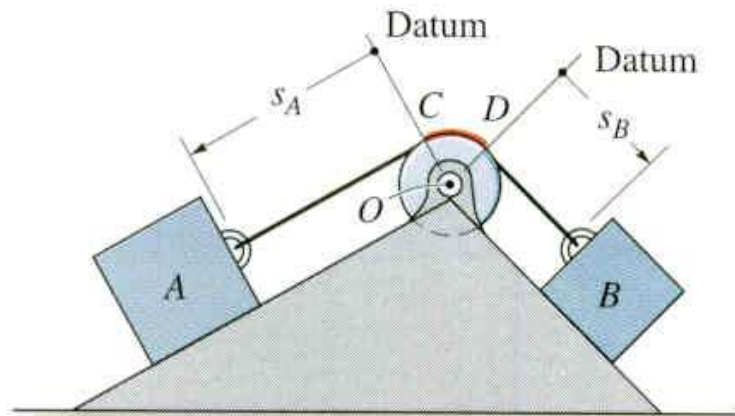
The blocks in this figure are connected by an **inextensible cord** wrapped around a pulley. If block A moves downward along the inclined plane, block B will move up the other incline.

The motion of each block can be related mathematically by defining **position coordinates**, s_A and s_B . Each coordinate axis is defined from a **fixed point or datum line**, measured **positive** along each plane in the **direction of motion** of each block.



DEPENDENT MOTION

(continued)



In this example, position coordinates s_A and s_B can be defined from fixed datum lines extending from the center of the pulley along each incline to blocks A and B.

If the cord has a fixed length, the position coordinates s_A and s_B are related mathematically by the equation

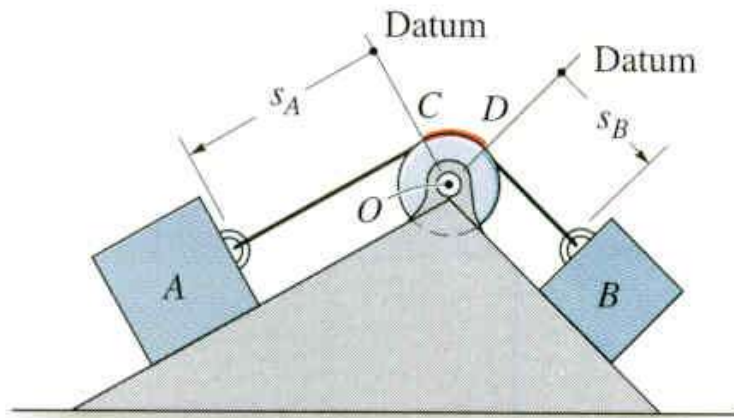
$$s_A + l_{CD} + s_B = l_T$$

Here l_T is the total cord length and l_{CD} is the length of cord passing over arc CD on the pulley.



DEPENDENT MOTION

(continued)



The **velocities** of blocks A and B can be related by **differentiating** the position equation. Note that **l_{CD} and l_T remain constant**, so $dl_{CD}/dt = dl_T/dt = 0$

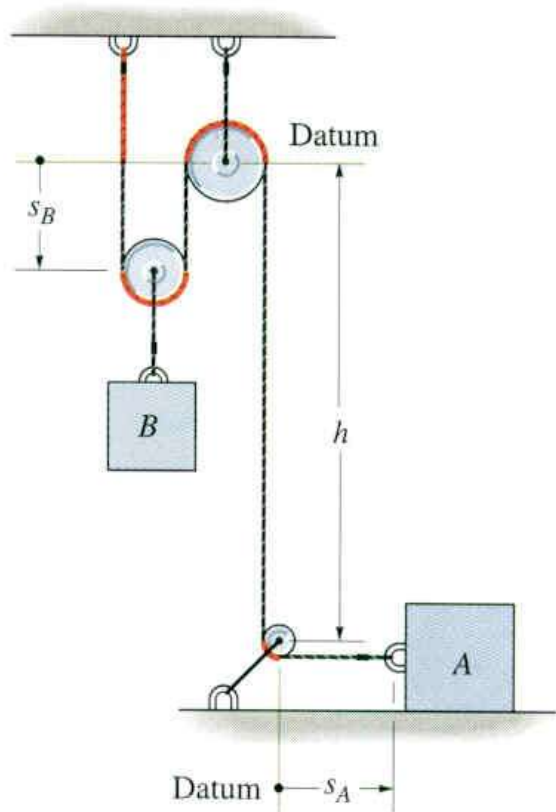
$$ds_A/dt + ds_B/dt = 0 \quad \Rightarrow \quad v_B = -v_A$$

The negative sign indicates that as A moves down the incline (positive s_A direction), B moves up the incline (negative s_B direction).

Accelerations can be found by **differentiating** the velocity expression. Prove to yourself that $a_B = -a_A$.



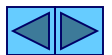
EXAMPLE



Consider a more complicated example. Position coordinates (s_A and s_B) are defined from fixed datum lines, measured along the direction of motion of each block.

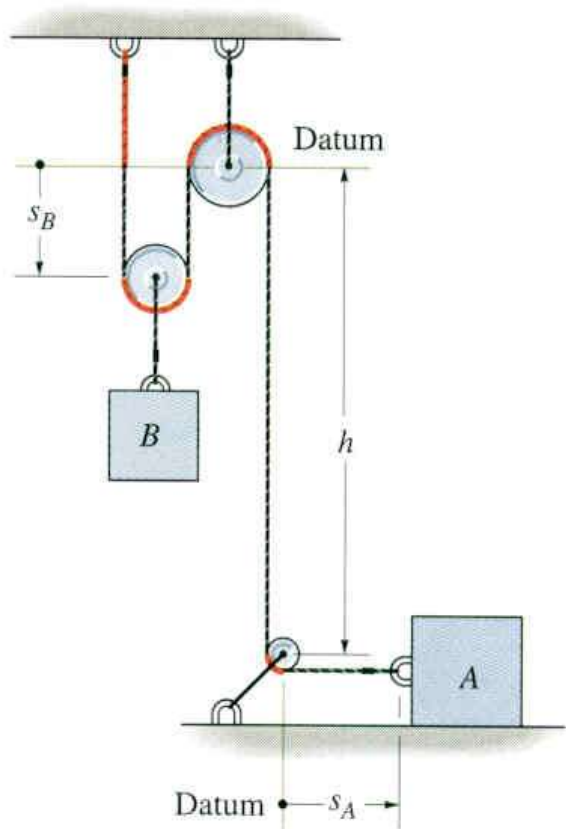
Note that s_B is only defined to the center of the pulley above block B, since this block moves with the pulley. Also, h is a constant.

The red colored segments of the cord remain constant in length during motion of the blocks.



DEPENDENT MOTION EXAMPLE

(continued)



The position coordinates are related by the equation

$$2s_B + h + s_A = l$$

Where l is the **total cord length minus the lengths of the red segments**.

Since l and h remain constant during the motion, the velocities and accelerations can be related by two successive time derivatives:

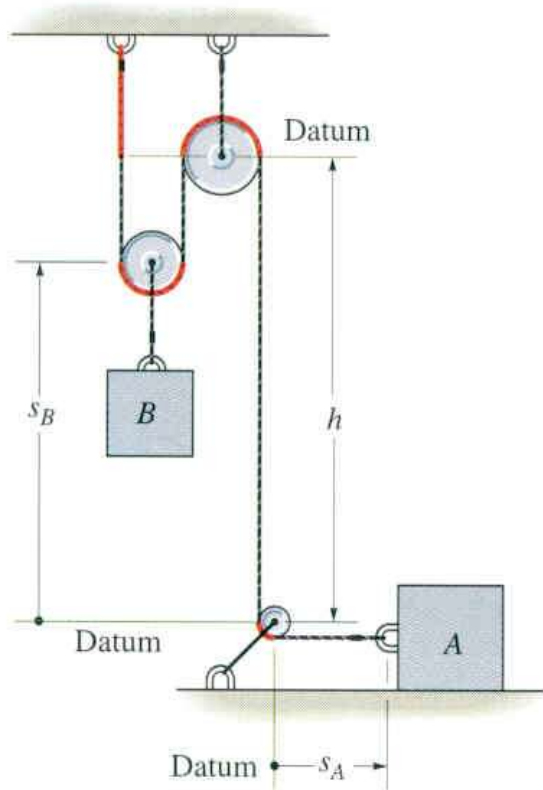
$$2v_B = -v_A \quad \text{and} \quad 2a_B = -a_A$$

When block B moves downward ($+s_B$), block A moves to the left ($-s_A$). Remember to be **consistent with the sign convention!**



DEPENDENT MOTION EXAMPLE

(continued)



This example can also be worked by defining the position coordinate for B (s_B) from the bottom pulley instead of the top pulley.

The position, velocity, and acceleration relations then become

$$2(h - s_B) + h + s_A = l$$

and $2v_B = v_A$ $2a_B = a_A$

Prove to yourself that the **results are the same**, even if the sign conventions are different than the previous formulation.



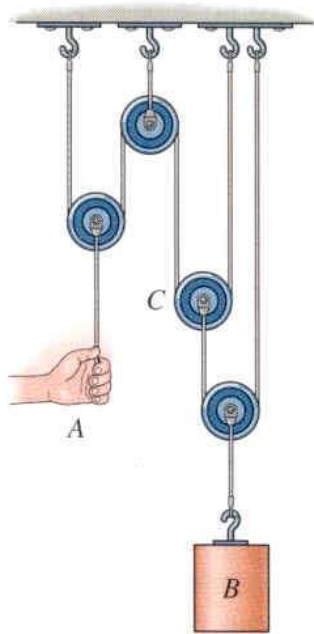
DEPENDENT MOTION: PROCEDURES

These procedures can be used to relate the **dependent motion** of particles moving along **rectilinear paths** (only the magnitudes of velocity and acceleration change, not their line of direction).

1. Define **position coordinates** from **fixed datum lines**, **along** the **path** of each particle. Different datum lines can be used for each particle.
2. Relate the position coordinates to the cord length. Segments of cord that do **not** change in length during the motion may be **left out**.
3. If a system contains more than one cord, relate the position of a point on one cord to a point on another cord. **Separate equations are written for each cord**.
4. **Differentiate** the position coordinate equation(s) to relate **velocities** and **accelerations**. Keep track of signs!



EXAMPLE II



Given: In the figure on the left, the cord at A is pulled down with a speed of 8 ft/s.

Find: The speed of block B.

Plan: There are two cords involved in the motion in this example. The position of a point on one cord must be related to the position of a point on the other cord. There will be two position equations (one for each cord).

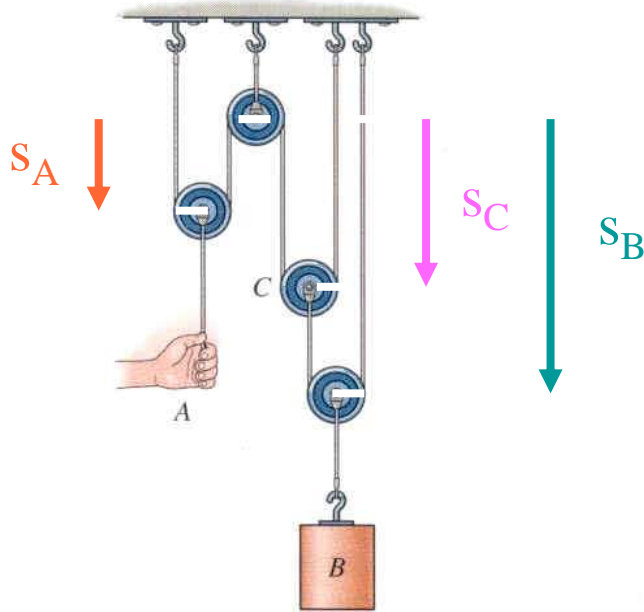


EXAMPLE

(continued)

Solution:

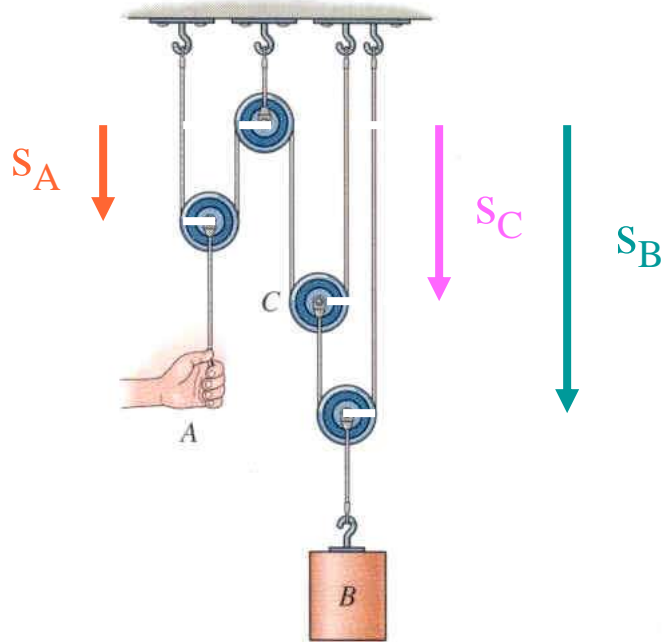
- 1) Define the position coordinates from a fixed datum line. Three coordinates must be defined: one for point A (s_A), one for block B (s_B), and one relating positions on the two cords. Note that pulley C relates the motion of the two cords.



- Define the datum line through the top pulley (which has a fixed position).
- s_A can be defined to the center of the pulley above point A.
- s_B can be defined to the center of the pulley above B.
- s_C is defined to the center of pulley C.
- All coordinates are defined as positive down and along the direction of motion of each point/object. ◀▶

EXAMPLE

(continued)



- 2) Write position/length equations for each cord. Define l_1 as the length of the first cord, minus any segments of constant length. Define l_2 in a similar manner for the second cord:

$$\text{Cord 1: } 2s_A + 2s_C = l_1$$

$$\text{Cord 2: } s_B + (s_B - s_C) = l_2$$

- 3) Eliminating s_C between the two equations, we get

$$2s_A + 4s_B = l_1 + 2l_2$$

- 4) Relate velocities by differentiating this expression. Note that l_1 and l_2 are constant lengths.

$$2v_A + 4v_B = 0 \quad \Rightarrow \quad v_B = -0.5v_A = -0.5(8) = -4 \text{ ft/s}$$

The velocity of block B is 4 ft/s up (negative s_B direction).



CONCEPT QUIZ

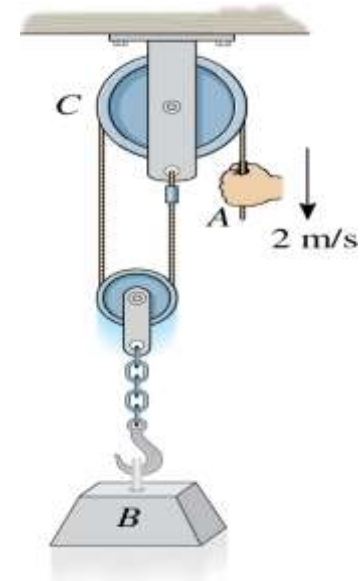
1. Determine the speed of block B.

A) 1 m/s

B) 2 m/s

C) 4 m/s

D) None of the above.



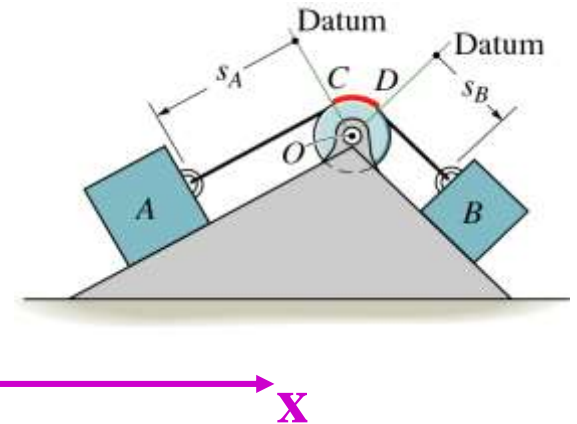
2. Two blocks are interconnected by a cable. Which of the following is correct ?

A) $v_A = -v_B$

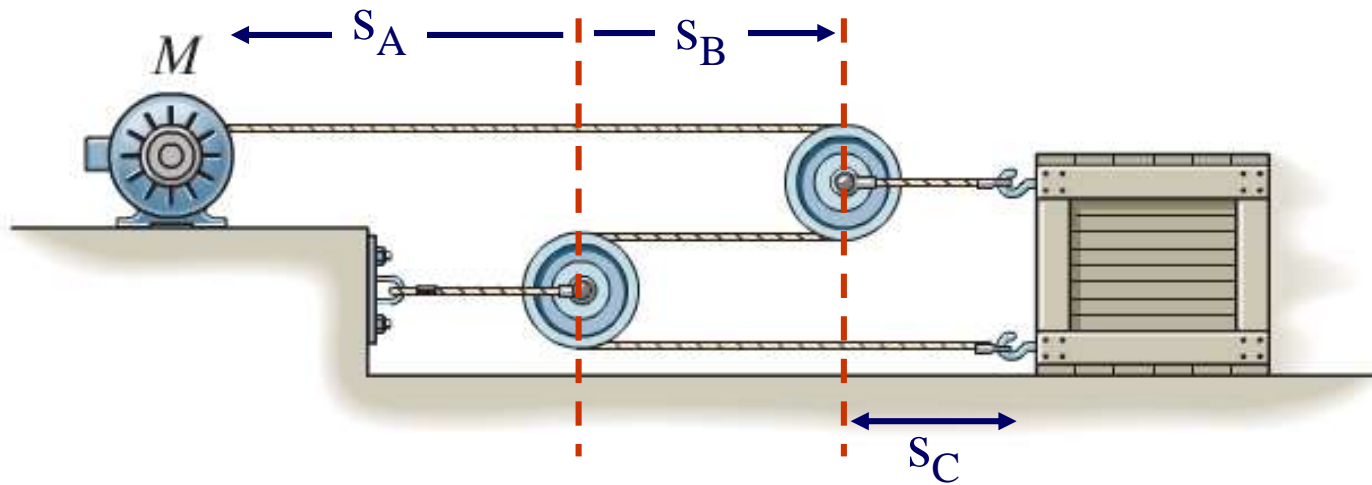
B) $(v_x)_A = - (v_x)_B$

C) $(v_y)_A = - (v_y)_B$

D) All of the above.



Cable of Varying Length



$$s_A + 3s_B + s_C = L$$

$$0 + 3v_B + 0 = v_M$$

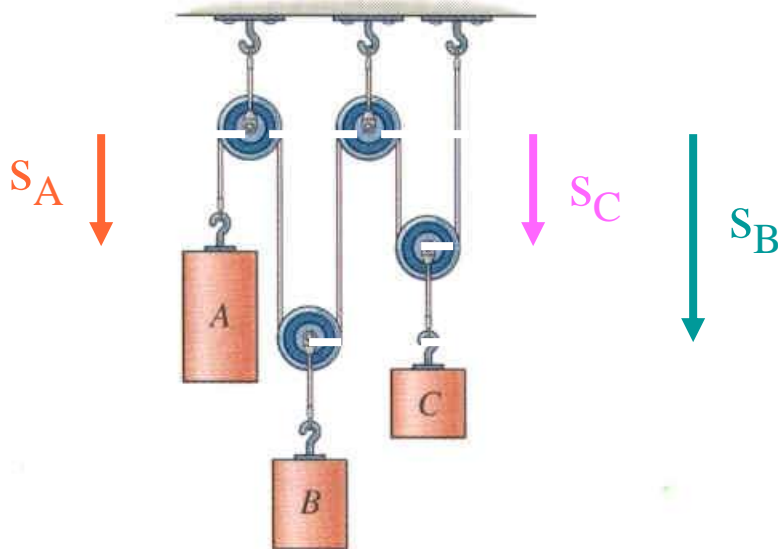
Both v_M are v_B negative since they get smaller.

GROUP PROBLEM SOLVING

(continued)

Solution:

- 1) A datum line can be drawn through the upper, fixed, pulleys and position coordinates defined from this line to each block (or the pulley above the block).



- 2) Defining s_A , s_B , and s_C as shown, the position relation can be written:

$$s_A + 2s_B + 2s_C = 1$$

- 3) Differentiate to relate velocities:

$$v_A + 2v_B + 2v_C = 0$$

$$\Rightarrow 4 + 2v_B + 2(-2) = 0$$

$$\Rightarrow v_B = 0$$



ATTENTION QUIZ

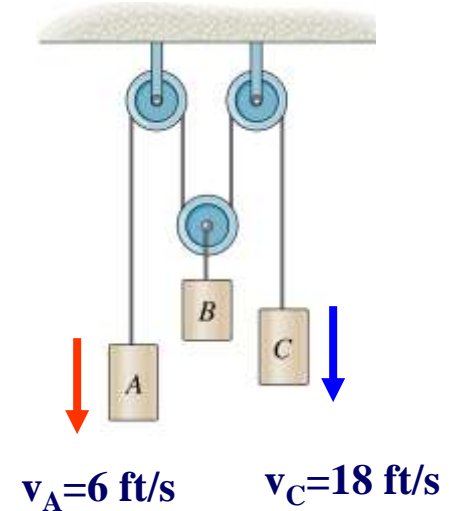
1. Determine the speed of block B when block A is moving down at 6 ft/s while block C is moving down at 18 ft/s .

A) 24 ft/s

B) 3 ft/s

C) 12 ft/s

D) 9 ft/s



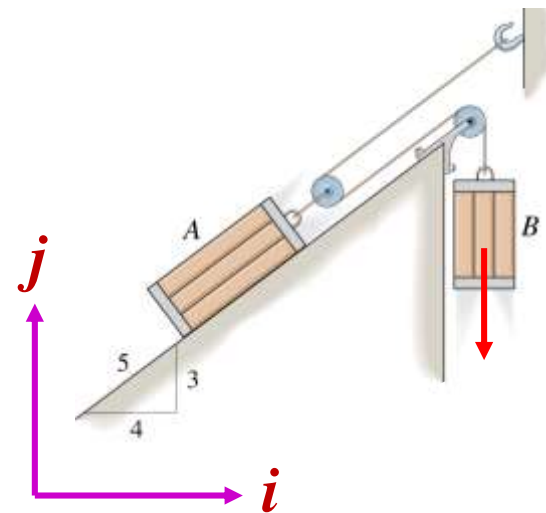
2. Determine the velocity vector of block A when block B is moving downward with a speed of 10 m/s.

A) $(8\mathbf{i} + 6\mathbf{j}) \text{ m/s}$

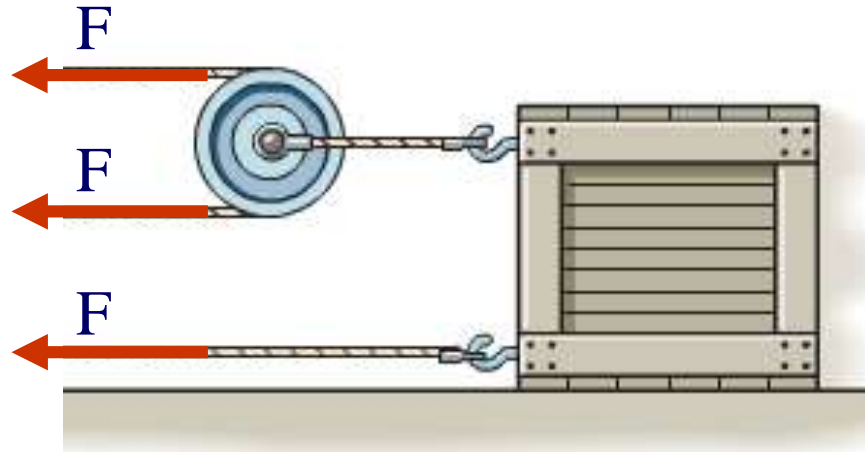
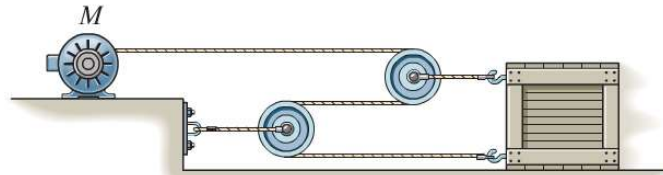
B) $(4\mathbf{i} + 3\mathbf{j}) \text{ m/s}$

C) $(-8\mathbf{i} - 6\mathbf{j}) \text{ m/s}$

D) $(3\mathbf{i} + 4\mathbf{j}) \text{ m/s}$



Pulleys are Force Multipliers



$$3F = Ma$$