EQUATIONS OF MOTION: RECTANGULAR COORDINATES

Today's Objectives:

Students will be able to:

1. Apply Newton's second law to determine forces and accelerations for particles in rectilinear motion.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Equations of Motion Using Rectangular (Cartesian) Coordinates
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

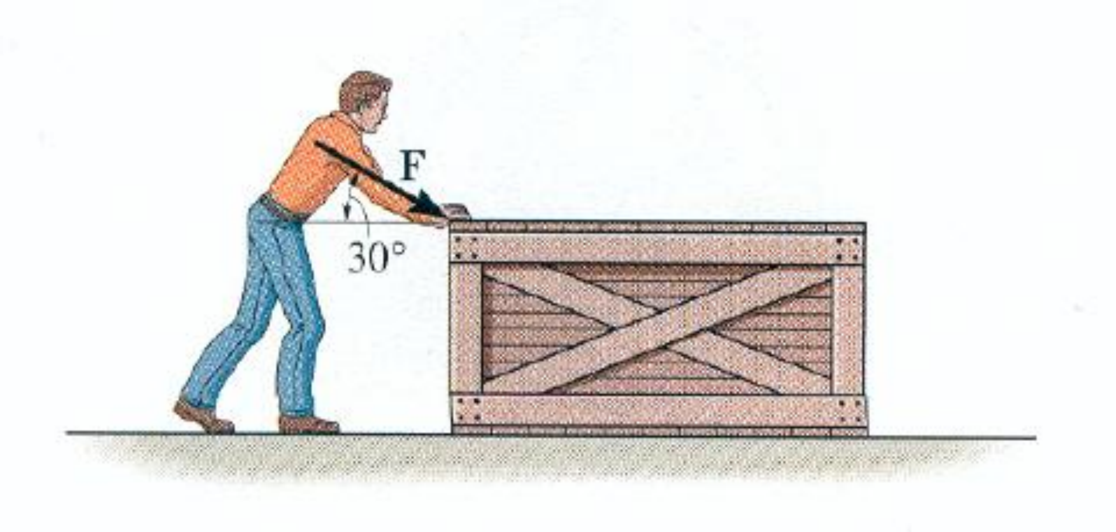
1. In dynamics, the friction force acting on a moving object is always

   A) in the direction of its motion.
   B) a kinetic friction.
   C) a static friction.
   D) zero.

2. If a particle is connected to a spring, the elastic spring force is expressed by $F = ks$. The "s" in this equation is the

   A) spring constant.
   B) un-deformed length of the spring.
   C) difference between deformed length and un-deformed length.
   D) deformed length of the spring.

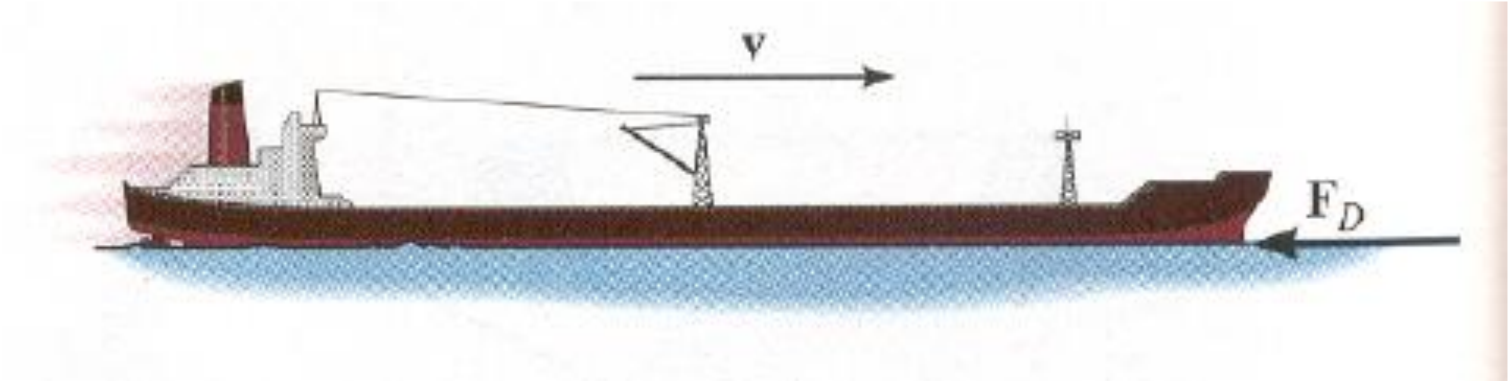
APPLICATIONS



If a man is pushing a 100 lb crate, how large a force F must he exert to start moving the crate?

What would you have to know before you could calculate the answer?

APPLICATIONS
(continued)



Objects that move in any fluid have a drag force acting on them. This drag force is a function of velocity.

If the ship has an initial velocity v_o and the magnitude of the opposing drag force at any instant is half the velocity, how long it would take for the ship to come to a stop if its engines stop?

RECTANGULAR COORDINATES
(Section 13.4)

The equation of motion, $\mathbf{F} = m\,\mathbf{a}$, is best used when the problem requires finding forces (especially forces perpendicular to the path), accelerations, velocities or mass. Remember, unbalanced forces cause acceleration!

Three scalar equations can be written from this vector equation. The equation of motion, being a vector equation, may be expressed in terms of its three components in the Cartesian (rectangular) coordinate system as

$$\sum \mathbf{F} = m\mathbf{a} \quad \text{or} \quad \sum F_x\,\mathbf{i} + \sum F_y\,\mathbf{j} + \sum F_z\,\mathbf{k} = m(a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\,\mathbf{k})$$

or, as scalar equations, $\sum F_x = ma_x$, $\sum F_y = ma_y$, and $\sum F_z = ma_z$.

PROCEDURE FOR ANALYSIS

- **Free Body Diagram**

Establish your coordinate system and draw the particle's free body diagram showing only external forces. These external forces usually include the weight, normal forces, friction forces, and applied forces. Show the 'm***a***' vector (sometimes called the inertial force) on a separate diagram.

Make sure any friction forces act opposite to the direction of motion! If the particle is connected to an elastic spring, a spring force equal to *ks* should be included on the FBD.

PROCEDURE FOR ANALYSIS
(continued)

- **Equations of Motion**

If the forces can be resolved directly from the free-body diagram (often the case in 2-D problems), use the scalar form of the equation of motion. In more complex cases (usually 3-D), a Cartesian vector is written for every force and a vector analysis is often best.

A Cartesian vector formulation of the second law is

$$\sum \boldsymbol{F} = m\boldsymbol{a} \quad \text{or}$$

$$\sum F_x \boldsymbol{i} + \sum F_y \boldsymbol{j} + \sum F_z \boldsymbol{k} = m(a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k})$$

Three scalar equations can be written from this vector equation. You may only need two equations if the motion is in 2-D.

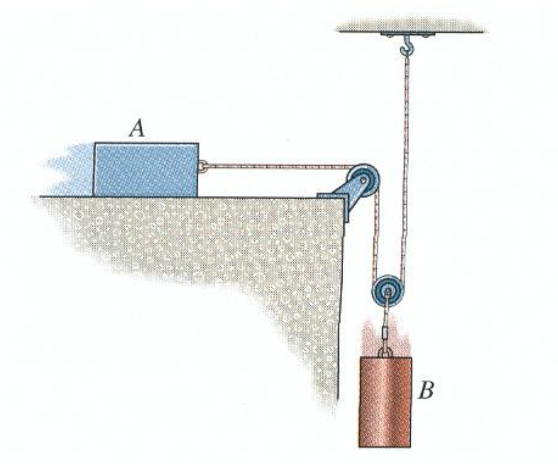
PROCEDURE FOR ANALYSIS
(continued)

- **Kinematics**

The second law only provides solutions for forces and accelerations. If velocity or position have to be found, kinematics equations are used once the acceleration is found from the equation of motion.

Any of the tools learned in Chapter 12 may be needed to solve a problem. Make sure you use consistent positive coordinate directions as used in the equation of motion part of the problem!

EXAMPLE



Given: $W_A = 10$ lb
$W_B = 20$ lb
$v_{oA} = 2$ ft/s →
$\mu_k = 0.2$

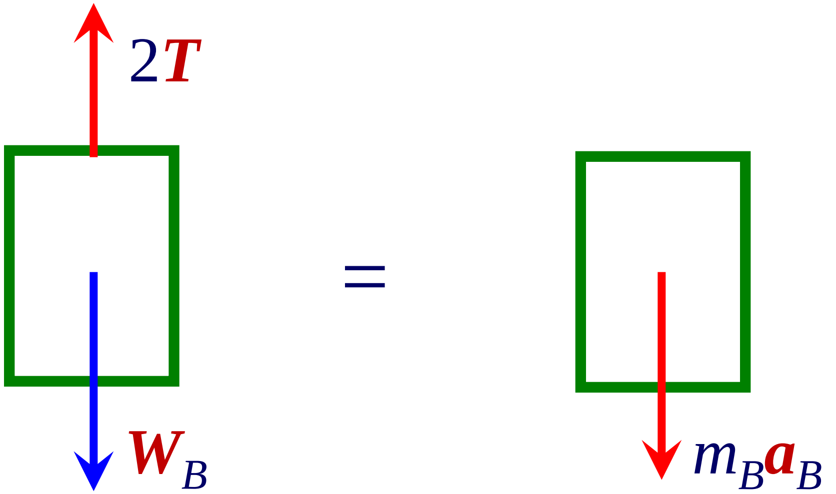
Find: v_A when A has moved 4 feet.

Plan: Since both forces and velocity are involved, this problem requires both the equation of motion and kinematics. First, draw free body diagrams of A and B. Apply the equation of motion . Using dependent motion equations, derive a relationship between $\boldsymbol{a}_A$ and $\boldsymbol{a}_B$ and use with the equation of motion formulas.

EXAMPLE
(continued)

Solution:

Free-body and kinetic diagrams of B:



Apply the equation of motion to B:

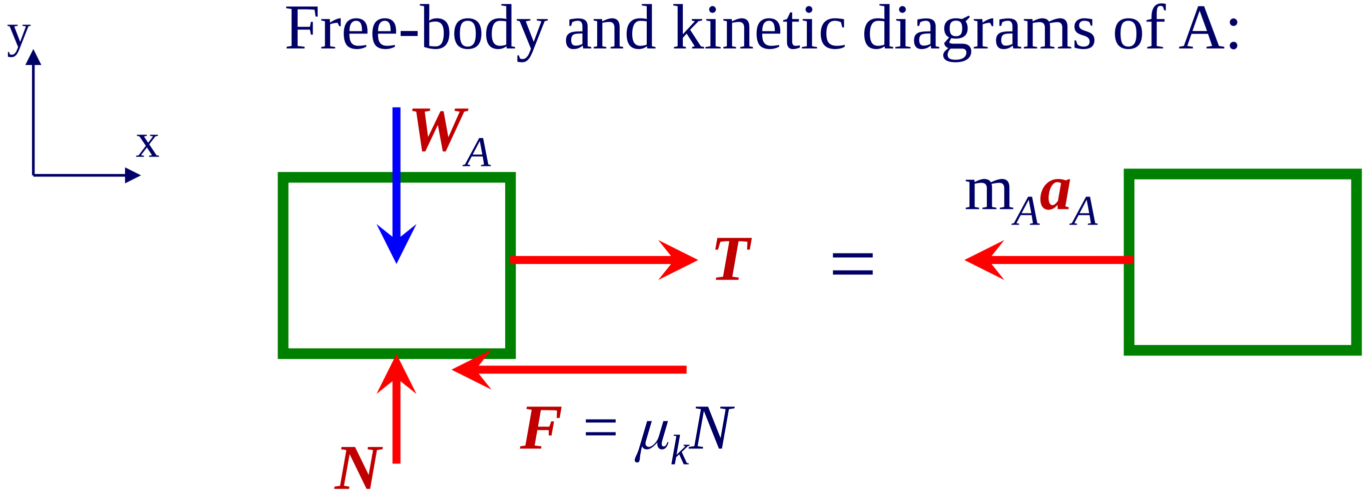
$$+\downarrow \sum F_y = m a_y$$

$$W_B - 2T = m_B a_B$$

$$20 - 2T = \frac{20}{32.2} a_B \qquad (1)$$

EXAMPLE
(continued)

Free-body and kinetic diagrams of A:



Apply the equations of motion to A:

$$+\uparrow \sum F_y = ma_y = 0$$

$$N = W_A = 10\ lb$$

$$F = \mu_k N = 2\ lb$$

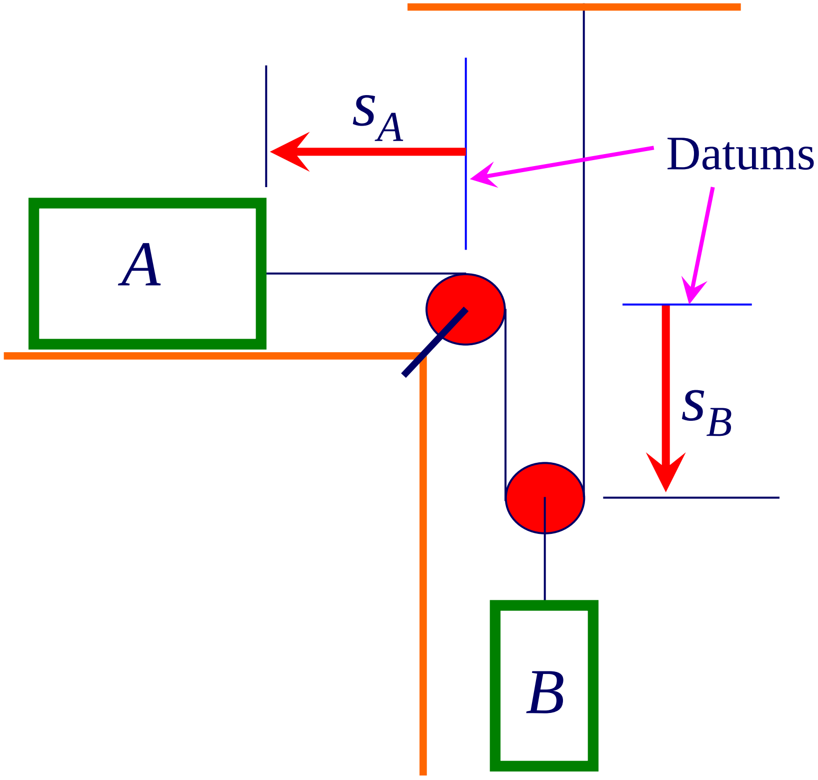
$$\overset{+}{\leftarrow} \sum F_x = ma_x$$

$$F - T = m_A a_A$$

$$2 - T = \frac{10}{32.2} a_A \qquad (2)$$

EXAMPLE
(continued)

Now consider the kinematics.



Constraint equation:

$$s_A + 2\, s_B = \text{constant}$$

or

$$v_A + 2\, v_B = 0$$

Therefore

$$a_A + 2\, a_B = 0$$

$$a_A = -2\, a_B \qquad (3)$$

(Notice a_A is considered positive to the left and a_B is positive downward.)

EXAMPLE
(continued)

Now combine equations (1), (2), and (3).

$$T = \frac{22}{3} = 7.33\ lb$$

$$a_A = -17.16\ ft/s^2 = 17.16\ ft/s^2 \rightarrow$$

Now use the kinematic equation:

$$v_A{}^2 = v_{oA}{}^2 + 2a_A(s_A - s_{oA})$$

$$v_A{}^2 = 2^2 + 2(17.16)(4)$$

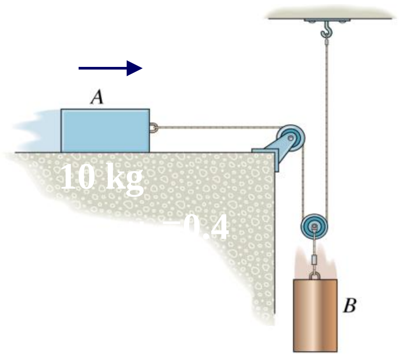
$$v_A = 11.9\ ft/s \rightarrow$$

CONCEPT QUIZ

1. If the cable has a tension of 3 N, determine the acceleration of block B.

   A) 4.26 m/s^2 ↑ B) 4.26 m/s^2 ↓

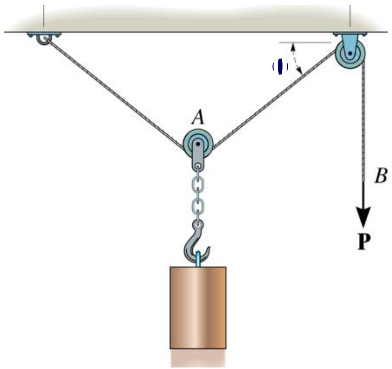
   C) 8.31 m/s^2 ↑ D) 8.31 m/s^2 ↓



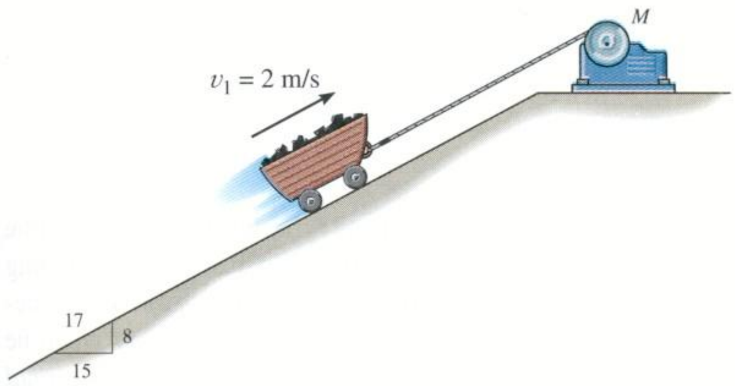
2. Determine the acceleration of the block.

   A) 2.20 m/s^2 ↑ B) 3.17 m/s^2 ↑

   C) 11.0 m/s^2 ↑ D) 4.26 m/s^2 ↑



GROUP PROBLEM SOLVING



Given: The 400 kg mine car is hoisted up the incline. The force in the cable is $F = (3200t^2)$ N. The car has an initial velocity of $v_i = 2$ m/s at t = 0.

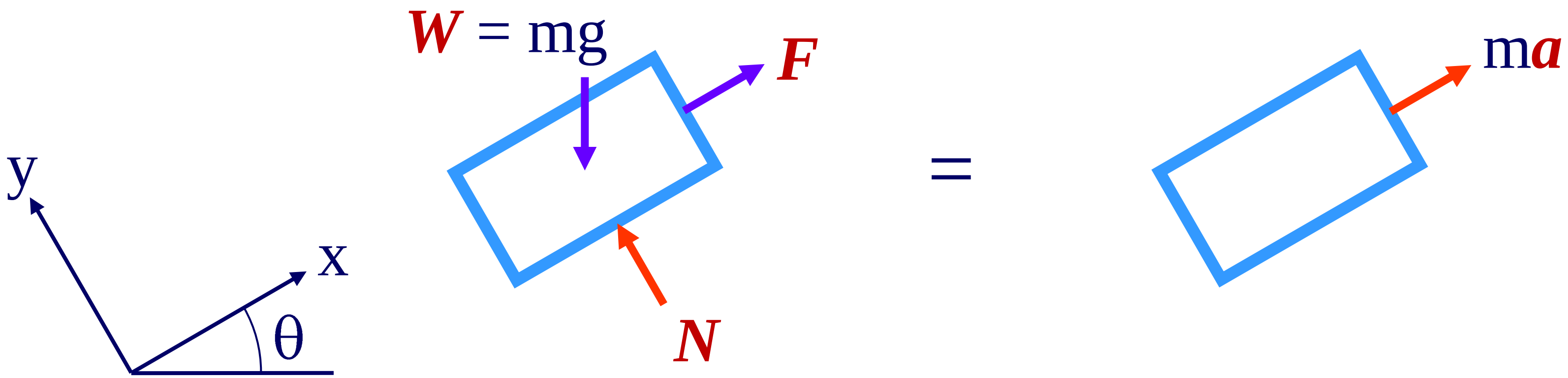
Find: The velocity when t = 2 s.

Plan: Draw the free-body diagram of the car and apply the equation of motion to determine the acceleration. Apply kinematics relations to determine the velocity.

GROUP PROBLEM SOLVING
(continued)

Solution:

1) Draw the free-body and kinetic diagrams of the mine car:



Since the motion is up the incline, rotate the x-y axes.

$\theta = \tan^{-1}(8/15) = 28.07$

Motion occurs only in the x-direction.

GROUP PROBLEM SOLVING
(continued)

2) Apply the equation of motion in the x-direction:

$$+\nearrow \sum F_x = ma_x \quad \Rightarrow \quad F - mg(\sin\theta) = ma_x$$

$$\Rightarrow \quad 3200t^2 - (400)(9.81)(\sin 28.07\ ) = 400a$$

$$\Rightarrow \quad a = (8t^2 - 4.616)\ m/s^2$$

3) Use kinematics to determine the velocity:

$$a = dv/dt \quad \Rightarrow \quad dv = a\,dt$$

$$\int_{v_1}^{v} dv = \int_{0}^{t} (8t^2 - 4.616)\,dt, \quad v_1 = 2\ m/s,\ t = 2\ s$$

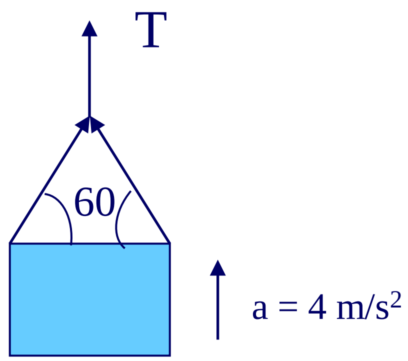
$$v - 2 = (8/3t^3 - 4.616t)\Big|_0^2 = 12.10 \quad \Rightarrow \quad v = 14.1\ m/s$$

ATTENTION QUIZ

1. Determine the tension in the cable when the 400 kg box is moving upward with a 4 m/s² acceleration.

   A) 2265 N   B) 3365 N

   C) 5524 N   D) 6543 N



2. A 10 lb particle has forces of $\boldsymbol{F}_1 = (3\boldsymbol{i} + 5\boldsymbol{j})$ lb and $\boldsymbol{F}_2 = (-7\boldsymbol{i} + 9\boldsymbol{j})$ lb acting on it. Determine the acceleration of the particle.

   A) $(-0.4\,\boldsymbol{i} + 1.4\,\boldsymbol{j})$ ft/s²   B) $(-4\,\boldsymbol{i} + 14\,\boldsymbol{j})$ ft/s²

   C) $(-12.9\,\boldsymbol{i} + 45\,\boldsymbol{j})$ ft/s²   D) $(13\,\boldsymbol{i} + 4\,\boldsymbol{j})$ ft/s²