

# EQUATIONS OF MOTION: CYLINDRICAL COORDINATES

## Today's Objectives:

Students will be able to:

1. Analyze the kinetics of a particle using cylindrical coordinates.



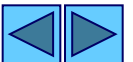
## In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Equations of Motion Using Cylindrical Coordinates
- Angle Between Radial and Tangential Directions
- Concept Quiz
- Group Problem Solving
- Attention Quiz

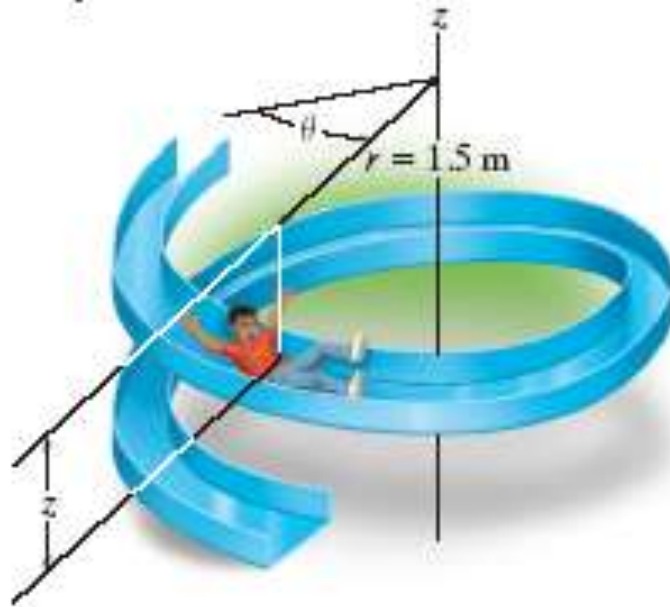


## READING QUIZ

1. The normal force which the path exerts on a particle is always perpendicular to the \_\_\_\_\_.
  - A) radial line
  - B) transverse direction
  - C) tangent to the path
  - D) None of the above.
  
2. Friction forces always act in the \_\_\_\_\_ direction.
  - A) radial
  - B) tangential
  - C) transverse
  - D) None of the above.

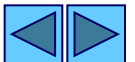


# APPLICATIONS



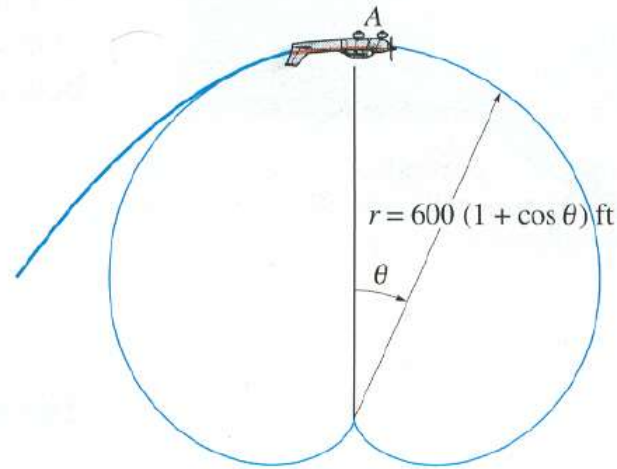
The forces acting on the 100-lb boy can be analyzed using the cylindrical coordinate system.

If the boy slides down at a constant speed of 2 m/s, can we find the frictional force acting on him?



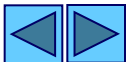
# APPLICATIONS

(continued)



When an airplane executes the vertical loop shown above, the centrifugal force causes the normal force (apparent weight) on the pilot to be smaller than her actual weight.

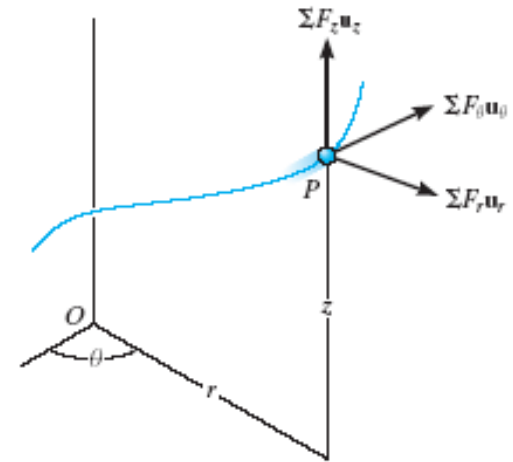
If the pilot experiences weightlessness at A, what is the airplane's velocity at A?



# CYLINDRICAL COORDINATES

(Section 13.6)

This approach to solving problems has some **external similarity** to the normal & tangential method just studied. However, the path may be more complex or the problem may have other attributes that make it desirable to use cylindrical coordinates.



Equilibrium equations or “Equations of Motion” in cylindrical coordinates (using  $r$ ,  $\theta$ , and  $z$  coordinates) may be expressed in scalar form as:

$$\sum F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\sum F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$\sum F_z = ma_z = m\ddot{z}$$



# CYLINDRICAL COORDINATES

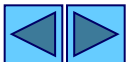
(continued)

If the particle is constrained to move only in the  $r - \theta$  plane (i.e., the  $z$  coordinate is constant), then only the first two equations are used (as shown below). The coordinate system in such a case becomes a polar coordinate system. In this case, the path is only a function of  $\theta$ .

$$\sum F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$

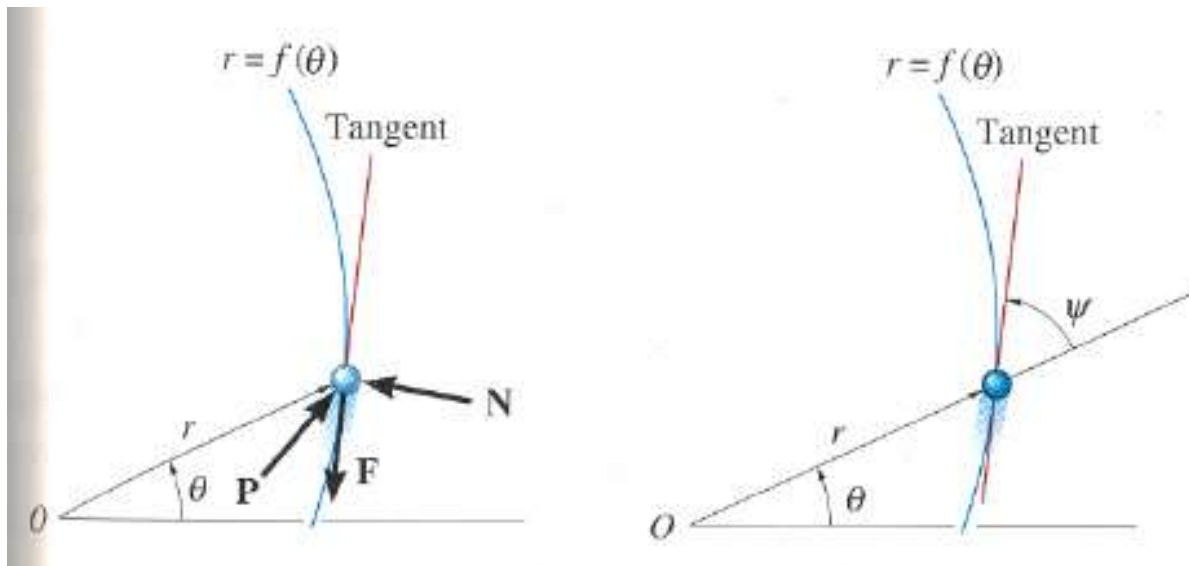
$$\sum F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Note that a fixed coordinate system is used, not a “body-centered” system as used in the  $n - t$  approach.



# TANGENTIAL AND NORMAL FORCES

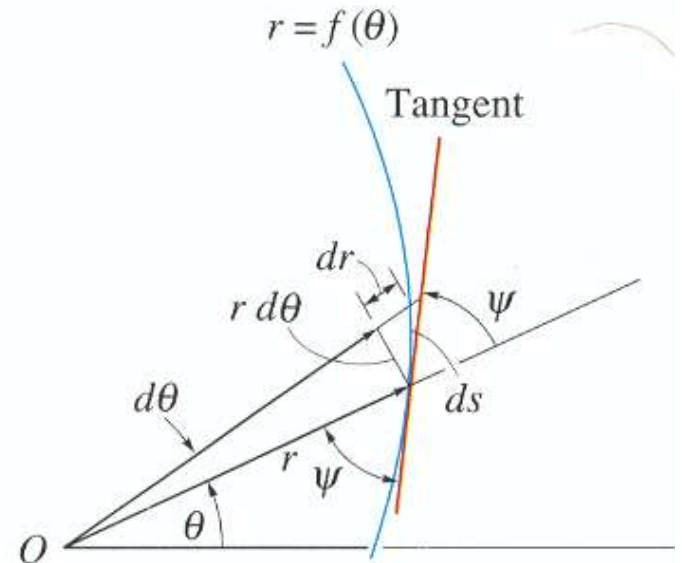
If a force  $\mathbf{P}$  causes the particle to move along a path defined by  $r = f(\theta)$ , the normal force  $\mathbf{N}$  exerted by the path on the particle is always perpendicular to the path's tangent. The frictional force  $\mathbf{F}$  always acts along the tangent in the opposite direction of motion. The directions of  $\mathbf{N}$  and  $\mathbf{F}$  can be specified relative to the radial coordinate by using angle  $\psi$ .



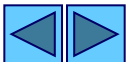
## DETERMINATION OF ANGLE $\psi$

The angle  $\psi$ , defined as the angle between the extended radial line and the tangent to the curve, can be required to solve some problems. It can be determined from the following relationship.

$$\tan \psi = \frac{r \, d\theta}{dr} = \frac{r}{dr / d\theta}$$



If  $\psi$  is positive, it is measured counterclockwise from the radial line to the tangent. If it is negative, it is measured clockwise.

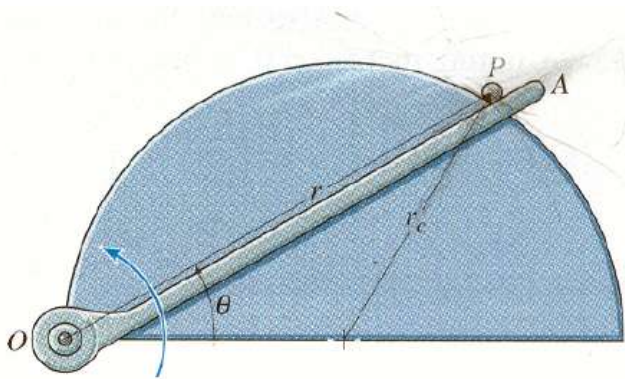




## Solution Method

- a) Do time derivatives  $\dot{\theta}$ ,  $\ddot{\theta}$ ,  $\dot{r}$ ,  $\ddot{r}$ , and  $dr/d\theta$
- b) Evaluate  $a_r = \ddot{r} - r\dot{\theta}^2$ ,  $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ , and  $\tan\psi = r/dr/d\theta$
- c) Sketch  $r$ ,  $\theta$  directions to show  $a_r$  and  $a_\theta$
- d) Sketch  $n$ ,  $t$  directions to show normal and friction.  
 $\psi$  is the angle between  $r$  and  $t$ .
- e) Sketch  $x$ ,  $y$  directions to show weight.  
 $\theta$  is the angle between  $r$  and  $x$ .
- f) Complete fbd.
- g) Write equations.

## EXAMPLE



**Given:** The ball (P) is guided along the vertical circular path.  
 $W = 0.5 \text{ lb}$ ,  $\dot{\theta} = 0.4 \text{ rad/s}$ ,  
 $\ddot{\theta} = 0.8 \text{ rad/s}^2$ ,  $r_c = 0.4 \text{ ft}$

**Find:** Force of the arm OA on the ball when  $\theta = 30^\circ$ .

**Plan:** Draw a FBD. Then develop the kinematic equations and finally solve the kinetics problem using cylindrical coordinates.

**Solution:** Notice that  $r = 2r_c \cos\theta$ , therefore:

$$\dot{r} = -2r_c \sin\theta \dot{\theta}$$

$$\ddot{r} = -2r_c \cos\theta \dot{\theta}^2 - 2r_c \sin\theta \ddot{\theta}$$



## EXAMPLE

(continued)

**Kinematics:** at  $\theta = 30^\circ$

$$r = 2(0.4) \cos(30^\circ) = 0.693 \text{ ft}$$

$$\dot{r} = -2(0.4) \sin(30^\circ)(0.4) = -0.16 \text{ ft/s}$$

$$\ddot{r} = -2(0.4) \cos(30^\circ)(0.4)^2 - 2(0.4) \sin(30^\circ)(0.8) = -0.431 \text{ ft/s}^2$$

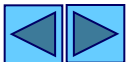
**Acceleration components are**

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.431 - (0.693)(0.4)^2 = -0.542 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.693)(0.8) + 2(-0.16)(0.4) = 0.426 \text{ ft/s}^2$$

$$\tan \psi = r / (dr/d\theta) \text{ where } dr/d\theta = -2r_c \sin\theta$$

$$\tan \psi = (2r_c \cos\theta) / (-2r_c \sin\theta) = -1/\tan\theta \quad \therefore \psi = 120^\circ$$





## EXAMPLE

(continued)

Kinetics:

$$\sum F_r = ma_r$$

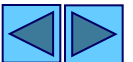
$$N_s \cos(30^\circ) - 0.5 \sin(30^\circ) = \frac{0.5}{32.2} (-0.542)$$

$$N_s = 0.279 \text{ lb}$$

$$\sum F_\theta = ma_\theta$$

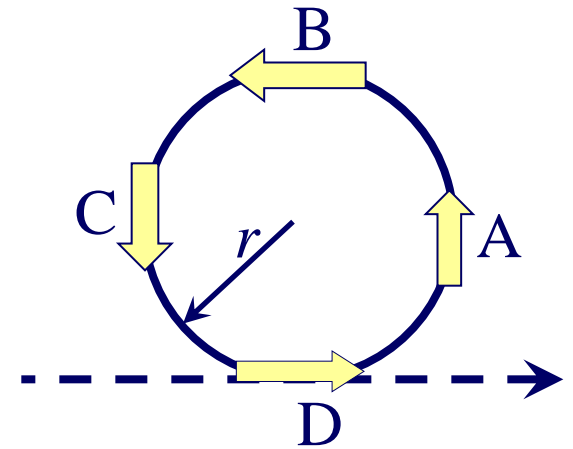
$$N_{OA} + 0.279 \sin(30^\circ) - 0.5 \cos(30^\circ) = \frac{0.5}{32.2} (0.426)$$

$$N_{OA} = 0.3 \text{ lb}$$



## CONCEPT QUIZ

1. When a pilot flies an airplane in a vertical loop of constant radius  $r$  at constant speed  $v$ , his apparent weight is maximum at



A) Point A

B) Point B (top of the loop)

C) Point C

D) Point D (bottom of the loop)

2. If needing to solve a problem involving the pilot's weight at Point C, select the approach that would be best.

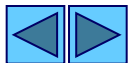
A) Equations of Motion: Cylindrical Coordinates

B) Equations of Motion: Normal & Tangential Coordinates

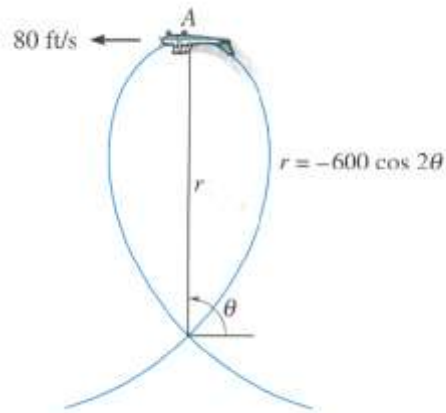
C) Equations of Motion: Polar Coordinates

D) No real difference – all are bad.

E) Toss up between B and C.



## GROUP PROBLEM SOLVING



**Given:** A plane flies in a vertical loop as shown.

$$v_A = 80 \text{ ft/s (constant)}$$

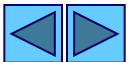
$$W = 130 \text{ lb}$$

**Find:** Normal force on the pilot at A.

**Plan:** Determine  $\dot{\theta}$  and  $\ddot{\theta}$  from the velocity at A and by differentiating  $r$ . Solve for the accelerations, and apply the equation of motion to find the force.

**Solution:**      Kinematics:       $r = -600 \cos(2\theta)$   
 $\dot{r} = 1200 \sin(2\theta) \dot{\theta}$   
 $\ddot{r} = 2400 \cos(2\theta) \dot{\theta}^2 + 1200 \sin(2\theta) \ddot{\theta}$

At A ( $\theta = 90^\circ$ )       $\dot{r} = 0$



# GROUP PROBLEM SOLVING

(continued)

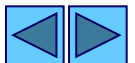
$$\text{Therefore } v_A = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} = r\dot{\theta}$$

$$\text{Since } r = 600 \text{ ft at A, } \dot{\theta} = \frac{80}{600} = 0.133 \text{ rad/s}$$

$$\text{Since } v_A \text{ is constant, } a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \Rightarrow \ddot{\theta} = 0$$

$$\ddot{r} = 2400\cos(180^\circ)\dot{\theta}^2 + 1200\sin(180^\circ)\ddot{\theta} = -2400(0.133)^2 = -42.67 \text{ ft/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -42.67 - 600(0.133)^2 = -53.33 \text{ ft/s}^2$$

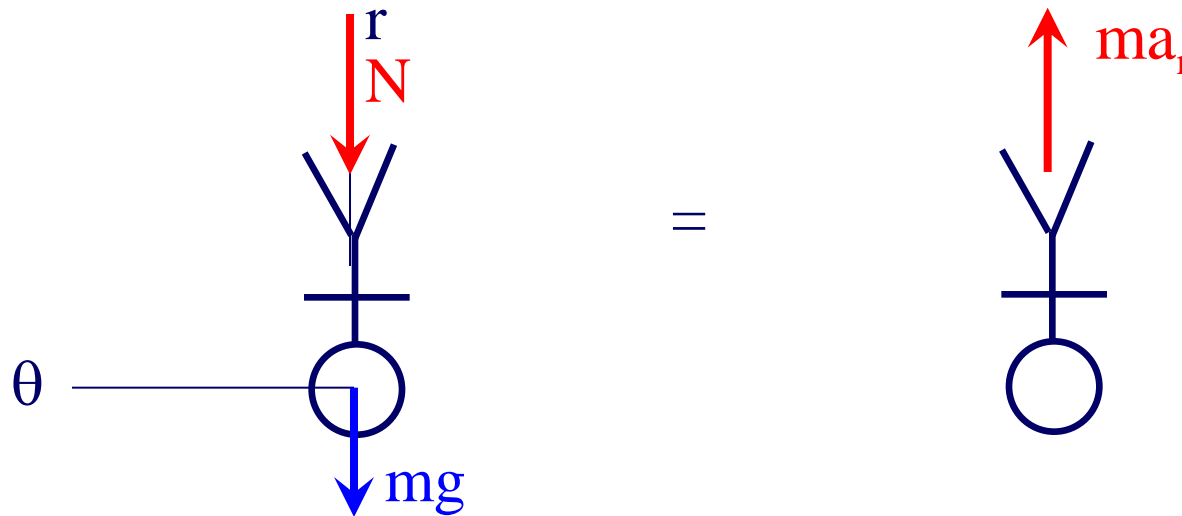




# GROUP PROBLEM SOLVING

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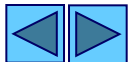
## Free Body Diagram & Kinetic Diagram



**Kinetics:**  $\sum F_r = ma_r \Rightarrow -mg - N = ma_r$

$$N = -130 - \frac{130}{32.2}(53.3) \Rightarrow N = 85.2 \text{ lb}$$

Notice that the pilot would experience weightlessness when his radial acceleration is equal to  $g$ .



## ATTENTION QUIZ

1. For the path defined by  $r = \theta^2$ , the angle  $\psi$  at  $\theta = .5$  rad is

A)  $10^\circ$

B)  $14^\circ$

C)  $26^\circ$

D)  $75^\circ$

2. If  $r = \theta^2$  and  $\theta = 2t$ , find the magnitude of  $\dot{r}$  and  $\ddot{\theta}$  when  $t = 2$  seconds.

A) 4 cm/sec, 2 rad/sec<sup>2</sup>

B) 4 cm/sec, 0 rad/sec<sup>2</sup>

C) 8 cm/sec, 16 rad/sec<sup>2</sup>

D) 16 cm/sec, 0 rad/sec<sup>2</sup>

