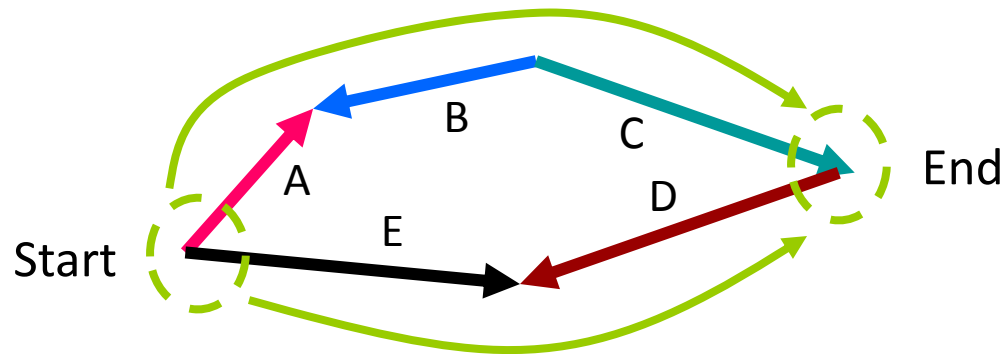


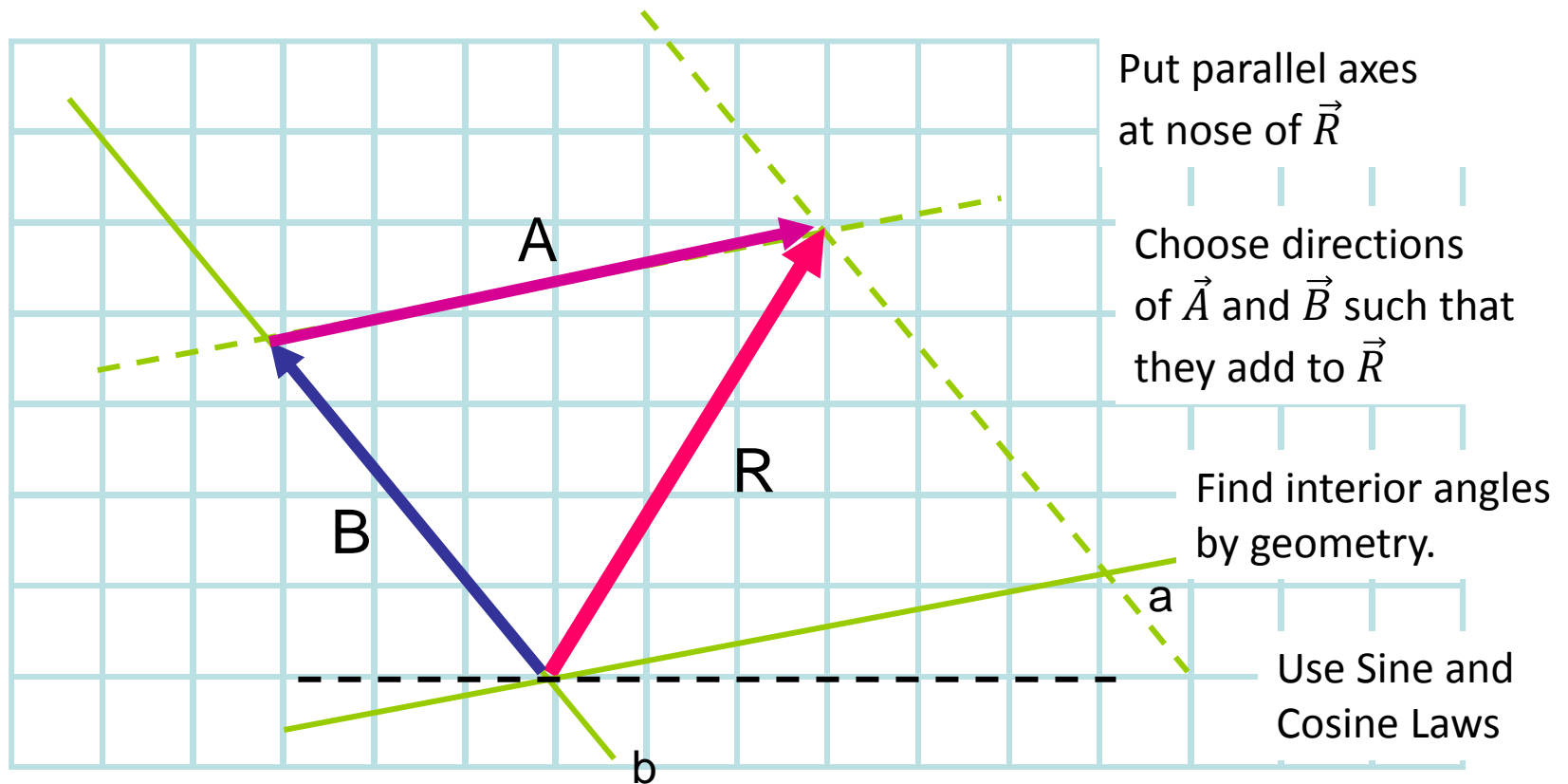
Vector Diagrams & Equations

- Pick any start and end points
- Following an arrow in same direction is +
- Follow in opposite direction is –



$$\vec{A} - \vec{B} + \vec{C} = \vec{E} - \vec{D}$$

Resolve \vec{R} along given axes such that $\vec{R} = \vec{A} + \vec{B}$



$$\text{Sine Law: } \frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}$$

$$\text{Cosine Law: } C^2 = A^2 + B^2 - 2AB \cos \gamma$$

Vector Forms

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{A} = A, \angle\alpha, \angle\beta, \angle\gamma$$

$$\vec{A} = A \hat{u}_A$$

Converting between forms

$$\hat{u}_A = \frac{\vec{A}}{A} \quad A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

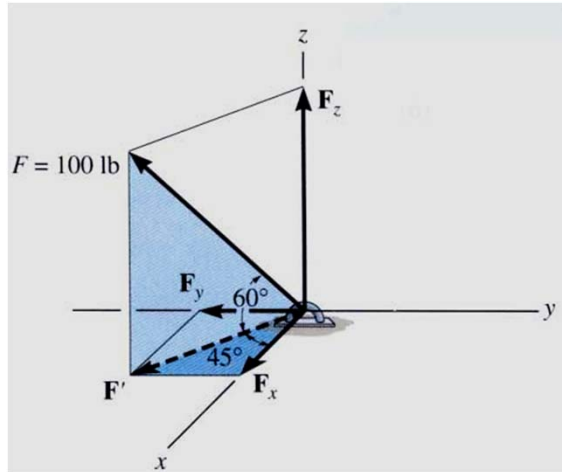
$$\hat{u}_A = \frac{A_x}{A} \hat{i} + \frac{A_y}{A} \hat{j} + \frac{A_z}{A} \hat{k}$$

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

$$\hat{u}_a = \cos\alpha \hat{i} + \cos\beta \hat{j} + \cos\gamma \hat{k}$$

$$1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$$

If we have angle information can write a vector in Cartesian form.



$$F_z = 100 \sin 60^\circ = 86.60 \text{ lb}$$

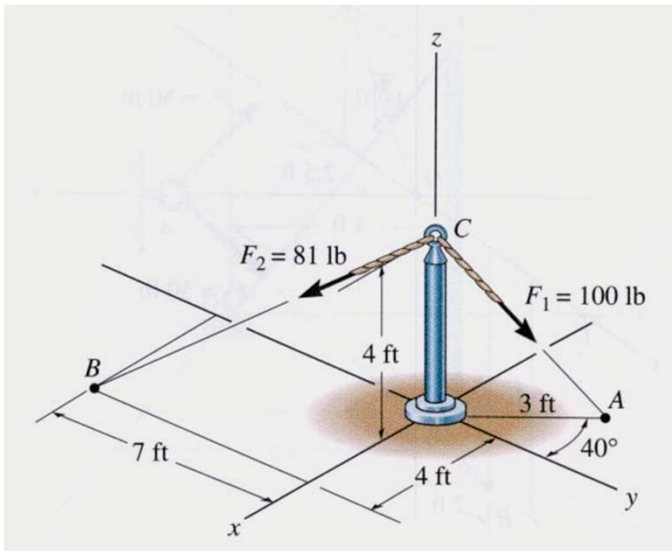
$$F' = 100 \cos 60^\circ = 50.00 \text{ lb}$$

$$F_x = 50 \cos 45^\circ = 35.36 \text{ lb}$$

$$F_y = 50 \sin 45^\circ = 35.36 \text{ lb}$$

$$\mathbf{F} = \{35.36 \mathbf{i} - 35.36 \mathbf{j} + 86.60 \mathbf{k}\} \text{ lb}$$

If we have positional information can write a vector in Cartesian form.

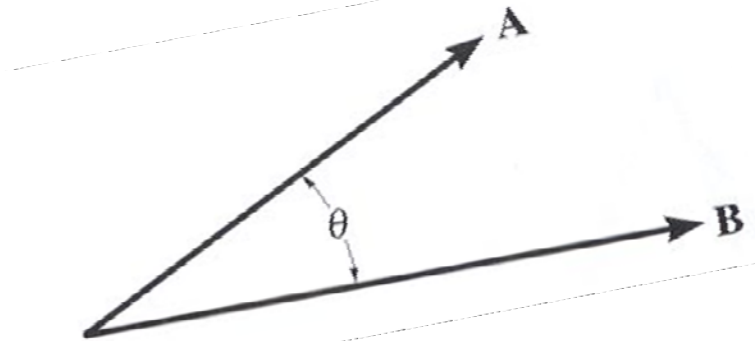


$$\mathbf{F}_2 = (81 \text{ lb}) \{ \mathbf{r}_{CB} / r_{CB} \}$$

$$\mathbf{F}_2 = (81 \text{ lb}) (4 \mathbf{i} - 7 \mathbf{j} - 4 \mathbf{k}) / 9$$

$$\mathbf{F}_2 = \{36 \mathbf{i} - 63 \mathbf{j} - 36 \mathbf{k}\} \text{ lb}$$

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS

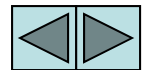


If we know two vectors in Cartesian form, finding θ is easy since we have two methods of doing the dot product; $\vec{A} \cdot \vec{B} = A B \cos\theta$ & $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$$AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

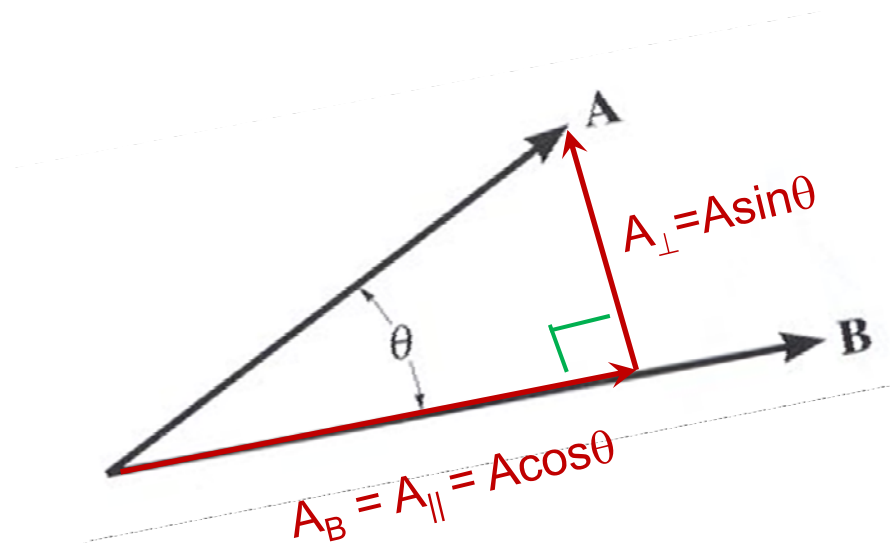
$$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2} \sqrt{B_x^2 + B_y^2 + B_z^2}}$$

More usually just written $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{A B}$ or $\cos\theta = \hat{u}_A \cdot \hat{u}_B$



Projection

How much of \vec{A} is in the same direction as \vec{B}



$$\vec{A}_B = A_B \hat{u}_B$$

$$\vec{A}_B = (\vec{A} \cdot \hat{u}_B) \hat{u}_B$$

$$\vec{A}_{\perp} = \vec{A} - \vec{A}_B$$

$$\vec{A}_B = A_B \hat{u}_B$$