

# EQUATIONS OF EQUILIBRIUM & TWO- AND THREE-FORCE MEMBERS

## Today's Objectives:

Students will be able to:

- a) Apply equations of equilibrium to solve for unknowns
- b) Identify support reactions
- c) Recognize two-force members
- d) Properly and improperly aligned supports



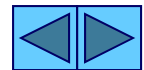
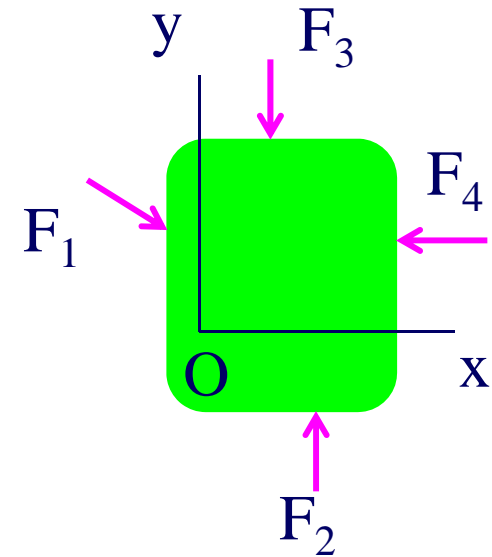
## EQUATIONS OF EQUILIBRIUM (Section 5.3)

A body is subjected to a system of forces that lie in the x-y plane. When in equilibrium, the net force and net moment acting on the body are zero (as discussed earlier in Section 5.1). This 2-D condition can be represented by the three scalar equations:

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_O = 0$$

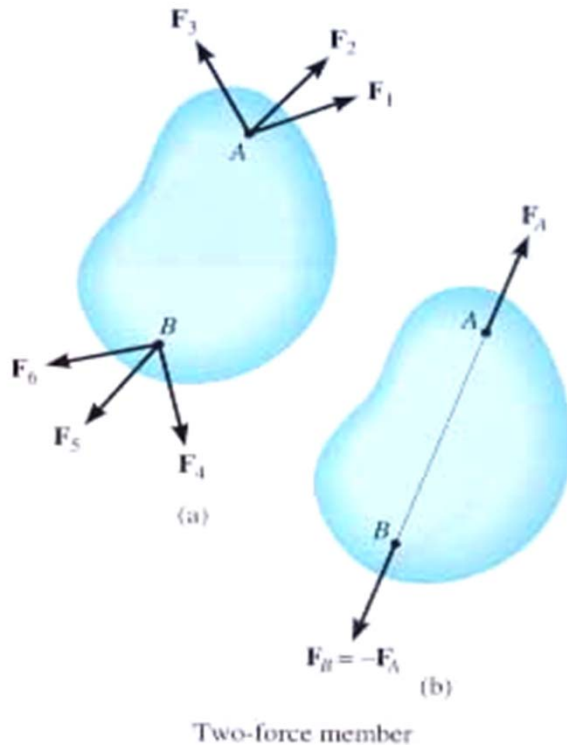
Where point O is any arbitrary point.

Please note that these equations are the ones most commonly used for solving 2-D equilibrium problems. There are two other sets of equilibrium equations that are rarely used. For your reference, they are described in the textbook.



# TWO-FORCE MEMBERS & THREE FORCE-MEMBERS

## (Section 5.4)

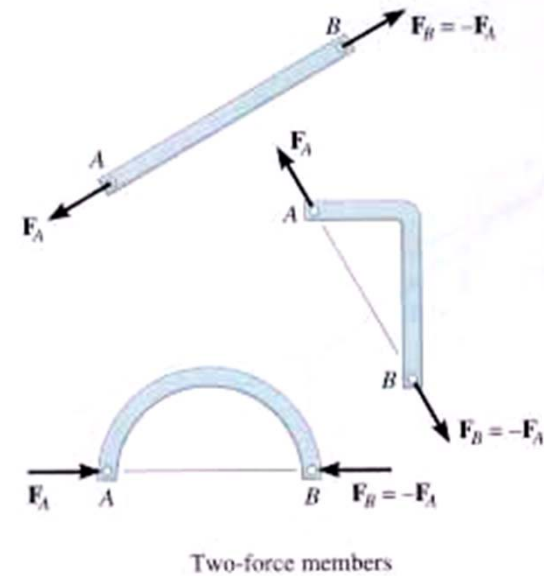


The solution to some equilibrium problems can be simplified if we recognize members that are subjected to forces at only two points (e.g., at points A and B).

If we apply the equations of equilibrium to such a member, we can quickly determine that the resultant forces at A and B must be equal in magnitude and act in the opposite directions along the line joining points A and B.



## EXAMPLE OF TWO-FORCE MEMBERS



In the cases above, members AB can be considered as two-force members, provided that their weight is neglected.

This fact simplifies the equilibrium analysis of some rigid bodies since the directions of the resultant forces at A and B are thus known (along the line joining points A and B).



## STEPS FOR SOLVING 2-D EQUILIBRIUM PROBLEMS

1. If not given, establish a suitable  $x - y$  coordinate system.
2. Draw a free body diagram (FBD) of the object under analysis.
3. Apply the three equations of equilibrium (EofE) to solve for the unknowns.

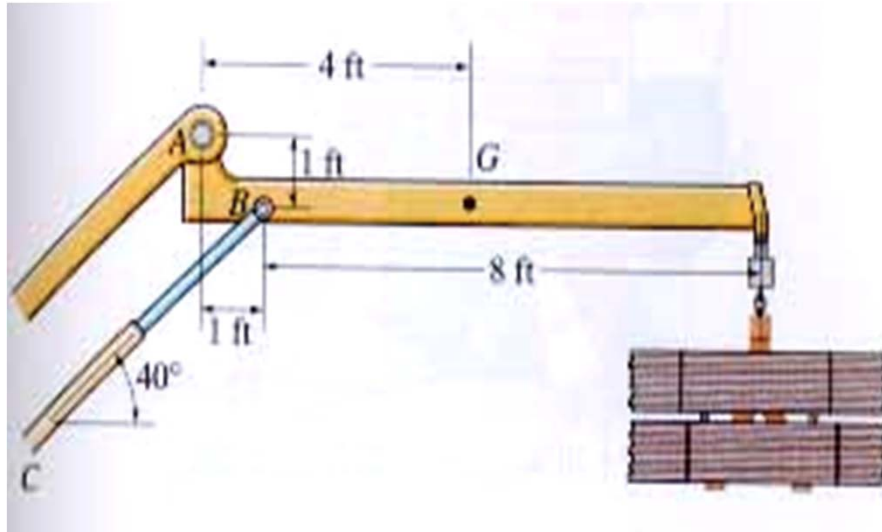


## IMPORTANT NOTES

1. If we have more unknowns than the number of independent equations, then we have a statically indeterminate situation. We cannot solve these problems using just statics.
2. The order in which we apply equations may affect the simplicity of the solution. For example, if we have two unknown vertical forces and one unknown horizontal force, then solving  $\sum F_x = 0$  first allows us to find the horizontal unknown quickly.
3. If the answer for an unknown comes out as negative number, then the sense (direction) of the unknown force is opposite to that assumed when starting the problem.
4. Problems may ignore weight of body. If mass or weight not stated, ignore it



## EXAMPLE



**Given:** Weight of the boom = 125 lb, the center of mass is at G, and the load = 600 lb.

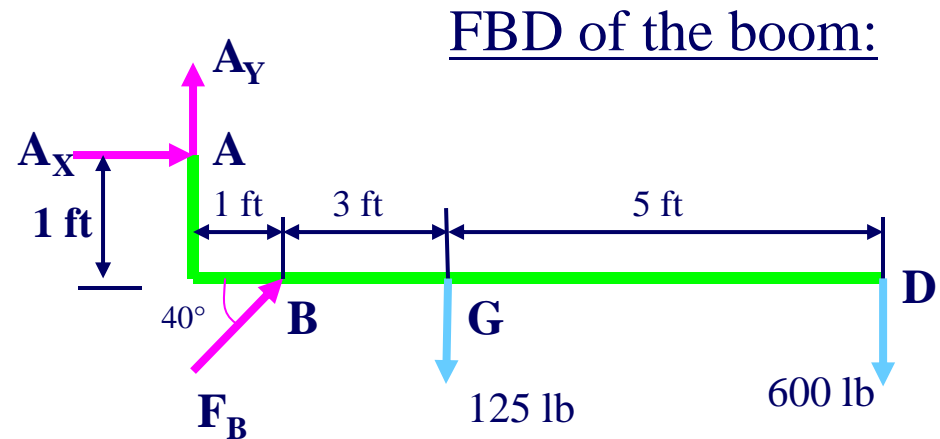
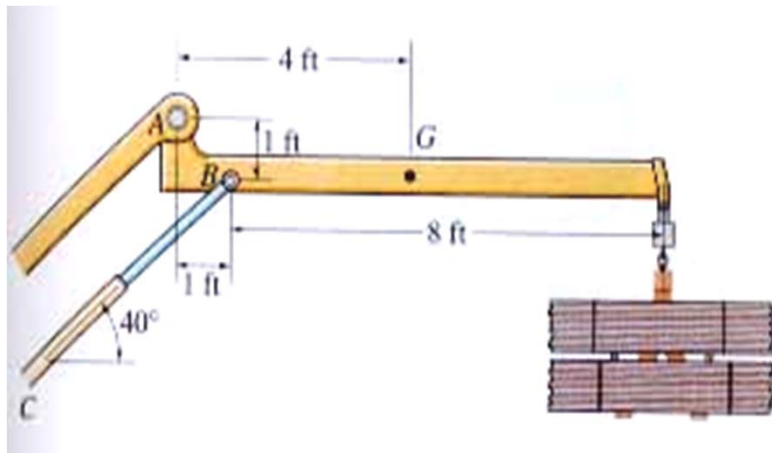
**Find:** Support reactions at A and B.

### Plan:

1. Put the x and y axes in the horizontal and vertical directions, respectively.
2. Determine if there are any two-force members.
3. Draw a complete FBD of the boom.
4. Apply the E-of-E to solve for the unknowns.



## EXAMPLE (Continued)



**Note:** Upon recognizing CB as a two-force member, the number of unknowns at B are reduced from two to one. Now, using Eof E, we get,

$$\curvearrowleft + \sum M_A = 125 * 4 + 600 * 9 - F_B \sin 40^\circ * 1 - F_B \cos 40^\circ * 1 = 0$$

$$F_B = 4188 \text{ lb or } \underline{4190 \text{ lb}}$$

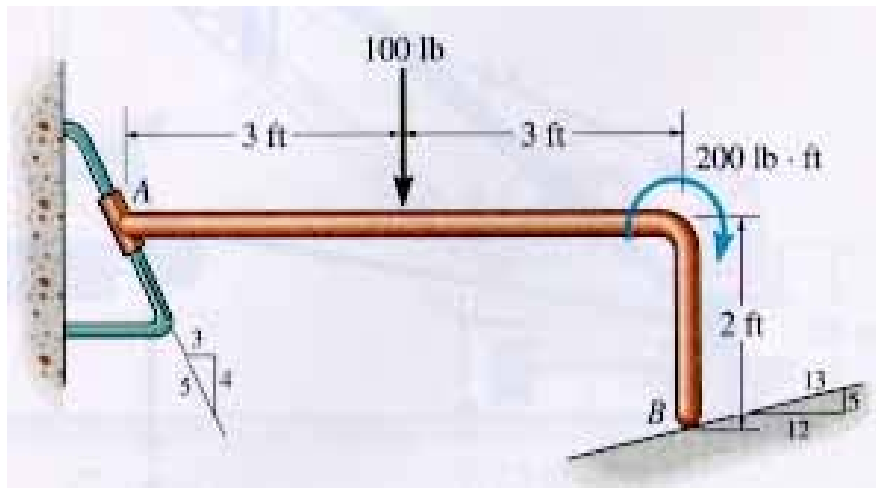
$$\rightarrow + \sum F_X = A_X + 4188 \cos 40^\circ = 0; \quad \underline{A_X = -3210 \text{ lb}}$$

$$\uparrow + \sum F_Y = A_Y + 4188 \sin 40^\circ - 125 - 600 = 0; \quad \underline{A_Y = -1970 \text{ lb}}$$





## GROUP PROBLEM SOLVING



**Given:** The load on the bent rod is supported by a smooth inclined surface at B and a collar at A. The collar is free to slide over the fixed inclined rod.

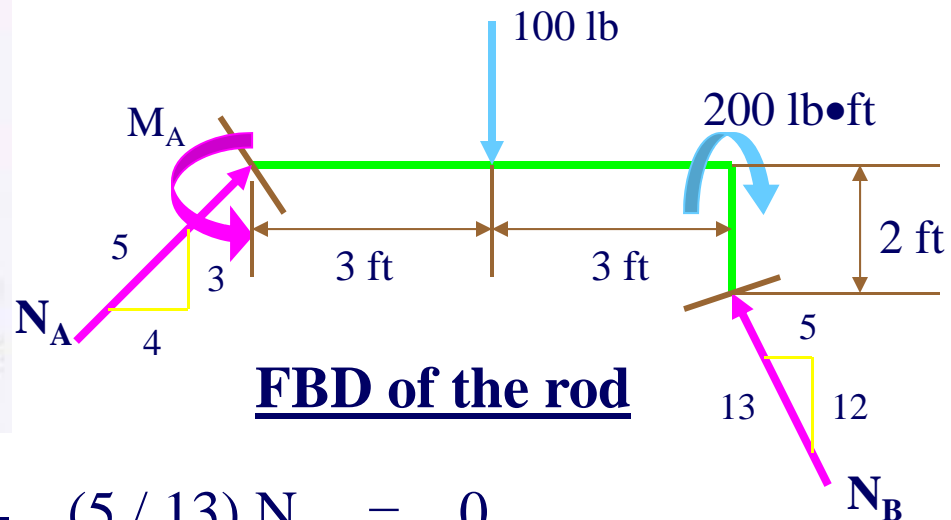
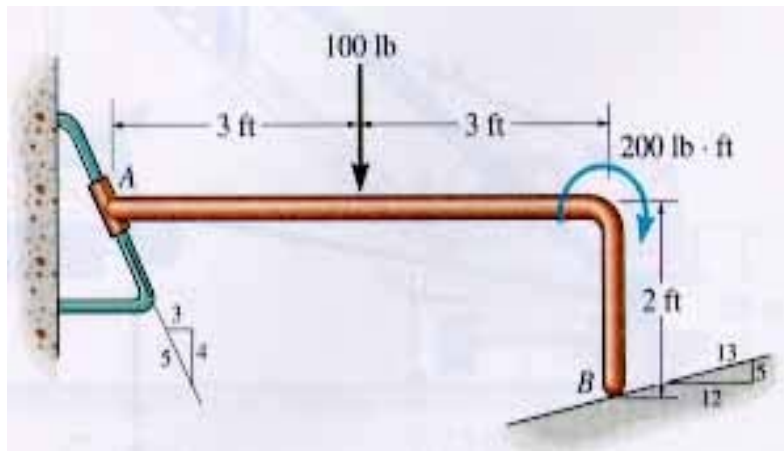
**Find:** Support reactions at A and B.

### Plan:

- Establish the  $x - y$  axes.
- Draw a complete FBD of the bent rod.
- Apply the E-of-E to solve for the unknowns.



## GROUP PROBLEM SOLVING (Continued)



$$\rightarrow + \sum F_X = (4/5) N_A - (5/13) N_B = 0$$

$$\uparrow + \sum F_Y = (3/5) N_A + (12/13) N_B - 100 = 0$$

Solving these two equations, we get

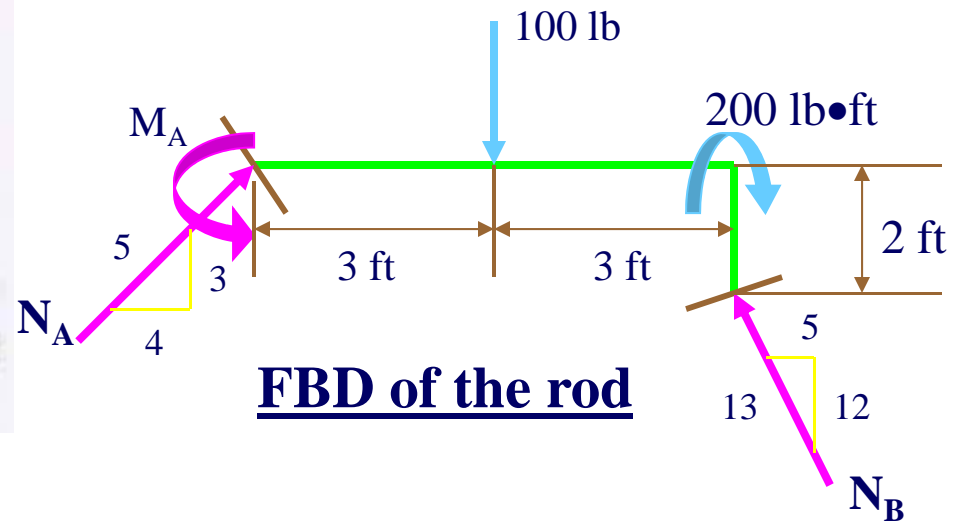
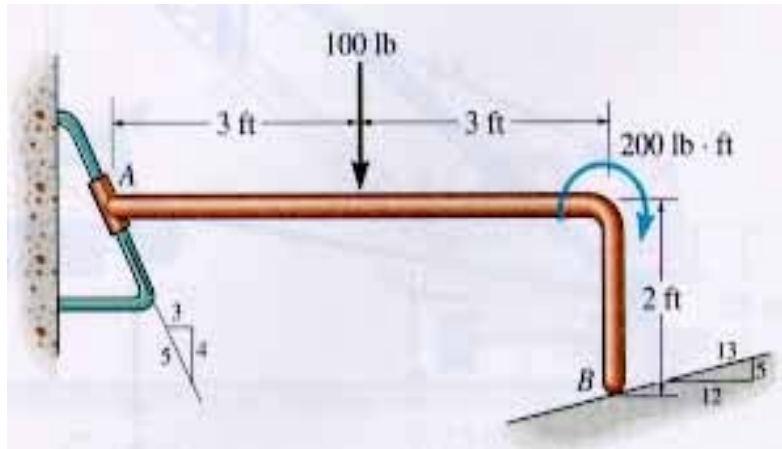
$$N_B = 82.54 \text{ or } 82.5 \text{ lb and } N_A = 39.68 \text{ or } 39.7 \text{ lb}$$

$$\curvearrowleft + \sum M_A = M_A - 100 * 3 - 200 + (12/13) N_B * 6 - (5/13) N_B * 2 = 0$$

$$M_A = 106 \text{ lb} \cdot \text{ft. (Note: no moment from } N_A \text{ but } M_A \text{ acts)}$$



## GROUP PROBLEM SOLVING (Continued)



Can do problem about point B or any other point.

$$\zeta + \sum M_B = M_A - 200 - (4/5) N_A * 2 - (3/5) N_A * 6 + 100 * 3 = 0$$

$$M_A = 106 \text{ lb} \cdot \text{ft}$$

Couple moments don't vanish!



## CONCEPT QUIZ

2. The beam AB is loaded and supported as shown: a) how many support reactions are there on the beam, b) is this problem statically determinate, and c) is the structure stable?

- A) (4, Yes, No)      B) (4, No, Yes)  
C) (5, Yes, No)      D) (5, No, Yes)

