Physics 1120: Simple Harmonic Motion Solutions

1. A 1.75−kg particle moves as function of time as follows:

$$x = 4\cos(1.33t+\pi/5)$$

where distance is measured in metres and time in seconds.
(a) What is the amplitude, frequency, angular frequency, and period of this motion?
(b) What is the equation of the velocity of this particle?
(c) What is the equation of the acceleration of this particle?
(d) What is the spring constant?
(e) At what next time t > 0, will the object be:

   i. at equilibrium and moving to the right,
   ii. at equilibrium and moving to the left,
   iii. at maximum amplitude, and
   iv. at minimum amplitude.

(a) First write the general expression on top of the given expression

$$x = A\cos(\omega t + \phi_0)$$
$$x = 4\cos(1.33t + \pi/5)$$

Immediately, we get A = 4 m, ω = 1.33 rad/s, and $\phi_0 = \pi/5$. Since ω = 2π / T = 2πf, we get T = 2π /1.33 rad/s = 4.724 s, and f = ω / 2π = 0.2117 s^{-1}.

(b) The velocity is given by the first derivative of position with respect to time

$$v = -\omega A\sin(\omega t + \phi_0) .$$

With the given values, we get

$$v = -5.32\sin(1.33t + \pi/5) .$$

(c) The acceleration is given by the second of position with respect to time, or the first derivative of the velocity with respect to time,

$$a = -\omega^2 A \cos(\omega t + \pi/5) .$$

With the given values, we get

$$a = -7.08\cos(1.33t + \pi/5) .$$

(d) We have the relation that ω^2 = K/m, so

$$K = \omega^2 m = (1.33 \text{ rad/s})^2(1.75 \text{ kg}) = 3.0956 \text{ N/m} .$$

(e) (i) & (ii) We know that at equilibrium $x = 0$. We also know that there are two places where this happens, one where is the velocity is positive and the object is moving to the right, and one where the velocity is negative and the object is moving to the left. So first let's set $x = 0$,

$$0 = 4\cos(\omega t + \pi/5)\ .$$

We can divide through by 4, and we get

$$0 = \cos(\omega t + \pi/5)\ .$$

Taking the inverse of both sides, the solution is

$$\omega t + \pi/5 = \cos^{-1}(0)\ ,$$

and thus,

$$t = [\cos^{-1}(0) - \pi/5] / \omega\ .$$

Now $\cos^{-1}(0)$ has many solutions, all the angles in radians for which the cosine is zero. This occurs for angles $\theta = \pi/2$, $\theta = -\pi/2$, $\theta = 3\pi/2$, $\theta = -3\pi/2$, and so on. This is usually expressed

$$\theta = n\pi/2, \qquad \text{where } n = \pm1, \pm3, \pm5, \ldots$$

So our solutions for t are in the form

$$t = [n\pi/2 - \pi/5] / \omega, \qquad \text{where } n = \pm1, \pm3, \pm5, \ldots$$

The first nonzero time when $x = 0$ occurs for $n = +1$,

$$t_1 = [\pi/2 - \pi/5] / = 0.7086 \text{ s.}$$

The second nonzero time occurs when $n = +2$,

$$t_2 = [3\pi/2 - \pi/5] / = 3.0707 \text{ s.}$$

To tell which has the object moving to the right and which to the left we examine the velocity

$$v_1 = -5.32\sin(1.33t_1 + \pi/5) = -5.32\sin(\pi/2) = -5.32 \text{ m/s},$$

$$v_2 = -5.32\sin(1.33t_2 + \pi/5) = -5.32\sin(3\pi/2) = +5.32 \text{ m/s.}$$

We see that the object is moving to the left, has negative velocity, at $t = t_1 = 0.7086$ s, and is moving to the right at $t = t_2 = 3.0707$ s.

(iii) At maximum amplitude, $x = +4$, so we have

$$4 = 4\cos(\omega t + \pi/5)\ .$$

Dividing through by 4, we get

$$1 = \cos(\omega t + \pi/5)$$

Taking the inverse of both sides, the solution is

$$\omega t + \pi/5 = \cos^{-1}(1)\ ,$$

and thus,

$$t = [\cos^{-1}(1) - \pi/5] / \omega \ .$$

Now $\cos^{-1}(1)$ has many solutions, all the angles in radians for which the cosine is plus one. This occurs for angles $\theta = 0$, $\theta = 2\pi$, $\theta = -2\pi$, $\theta = 4\pi$, $\theta = -4\pi$, and so on. This is usually expressed

$$\theta = 2n\pi, \qquad \text{where } n = 0, \pm 1, \pm 2, \pm 3\ , \ldots$$

So our solutions for t are in the form

$$t = [2n\pi - \pi/5] / \omega, \qquad \text{where } n = 0, \pm 1, \pm 2, \pm 3, \ldots$$

The first nonzero time when $x = +4$ occurs for $n = +1$,

$$t = [2\pi - \pi/5] / \omega = 4.2518 \text{ s.}$$

(iv) At minimum amplitude, $x = -4$, so we have

$$-4 = 4\cos(\omega t + \pi/5)\ .$$

Dividing through by 4, we get

$$-1 = \cos(\omega t + \pi/5)\ .$$

Taking the inverse of both sides, the solution is

$$\omega t + \pi/5 = \cos^{-1}(-1)\ ,$$

and thus,

$$t = [\cos^{-1}(-1) - \pi/5] / \omega\ .$$

Now $\cos^{-1}(-1)$ has many solutions, all the angles in radians for which the cosine is negative one. This occurs for angles $\theta = \pi$, $\theta = -\pi$, $\theta = 3\pi$, $\theta = -3\pi$, and so on. This is usually expressed

$$\theta = n\pi, \qquad \text{where } n = \pm 1, \pm 3, \pm 5, \ldots$$

So our solutions for t are in the form

$$t = [n\pi - \pi/5] / \omega, \qquad \text{where } n = \pm 1, \pm 3, \pm 5, \ldots$$

The first nonzero time when $x = -4$ occurs for $n = +1$,

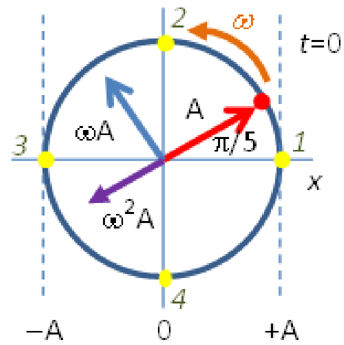
$$t = [\pi - \pi/5] / \omega = 1.8897 \text{ s.}$$

Our answers for (e) are thus

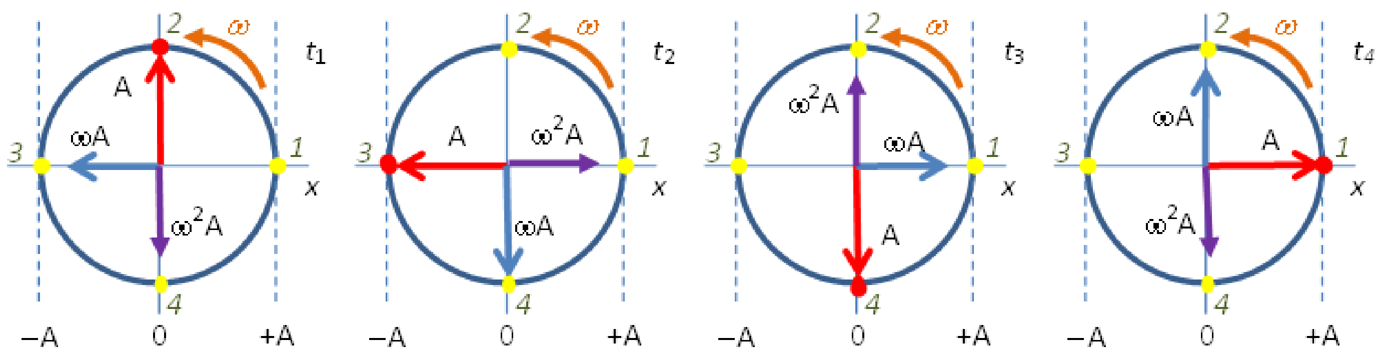
(i) $t = 3.071$ s,
(ii) $t = 0.709$ s,
(iii) $t = 4.25$ s, and
(iv) $t = 1.89$ s.

Alternate Quicker Method Using Reference Circle

An alternate way of solving this problem is to consult the reference circle for a particle undergoing uniform circular motion with radius A. The x-component of the particle's position, tangential velocity, and centripetal acceleration obey the equations of SHM. At $t = 0$, the reference circle looks like the top diagram (a) shown below. Note that the x components of the vector indicates that at t = 0, x(0) > 0, v(0) < 0, and a(0) < 0.



(a)



(b) (c) (d) (e)

Now as we rotate counterclockwise with increasing time, there are four points of interest labelled 1, 2, 3, and 4 in the diagrams. These are the points where the object is farthest to the right, moving to the left through equilibrium, farthest to the left, and moving to the right through equilibrium respectively.

The object on the spring, or equivalently the particle undergoing UCM, passes through points 2, 3, 4, and 1 in that order. These are shown as times t_1, t_2, t_3, and t_4 in diagrams (b), (c), (d), and (e) in the figure above.

The figure (b) reference circle is for $v = -\omega A$. We have rotated from the t = 0 position of $\pi/5$ to $\pi/2$ an angle of $90° - 36° = 54°$. Since a 360° rotation takes one period T, then the time required to get to here was given by the ratio $t_1/T = 54°/360°$. Solving we find $t_1 = {}^{54}/_{360} \times 4.724\text{ s} = 0.709\text{ s}$.

The figure (c) reference circle is for $x = -A$. We have rotated from the t = 0 position of $\pi/5$ to π an angle of $180° - 36° = 144°$. Since a 360° rotation takes one period T, then the time required to get to here was given by the ratio $t_2/T = 144°/360°$. Solving we find $t_2 = {}^{144}/_{360} \times 4.724\text{ s} = 1.890\text{ s}$.

The figure (d) reference circle is for $v = +\omega A$. We have rotated from the t = 0 position of $\pi/5$ to $3\pi/2$ an angle of $270° - 36° = 234°$. Since a 360° rotation takes one period T, then the time required to get to here was given by the ratio $t_3/T = 234°/360°$. Solving we find $t_3 = {}^{234}/_{360} \times 4.724\text{ s} = 3.071\text{ s}$.

The figure (e) reference circle is for $x = +A$. We have rotated from the t = 0 position of $\pi/5$ to 2π an angle of $360° - 36° = 324°$. Since a 360° rotation takes one period T, then the time required to get to here was given by the ratio $t_2/T = 324°/360°$. Solving we find $t_4 = {}^{324}/_{360} \times 4.724\text{ s} = 4.252\text{ s}$.

Our answers for (e) are thus

(i) t = 3.071 s,

(ii) t = 0.709 s,

(iii) t = 4.252 s, and

(iv) t = 1.890 s.

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2. If the amplitude in Question #1 is doubled, how would yours answers change?

   Simple Harmonic Motion is independent of amplitude. Our answers to Question #1 would not change.

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3. What are the equations for the potential and kinetic energies of the particle in Question #1? What is the total energy?

   The potential energy is spring potential energy and is given by $U = \frac{1}{2}Kx^2$, so

$$U = \tfrac{1}{2}(3.0956)[4\cos(1.33t + \pi/5)]^2 = 24.76\cos^2(1.33t + \pi/5) .$$

   The kinetic energy is given by $K = \frac{1}{2}mv^2$, so

$$K = \tfrac{1}{2}(1.75)[-5.32\sin(1.33t + \pi/5)]^2 = 24.76\sin^2(1.33t + \pi/5) .$$

   The total energy is the sum of potential and kinetic energies,

$$E = U + K = 24.76\ \text{J} .$$

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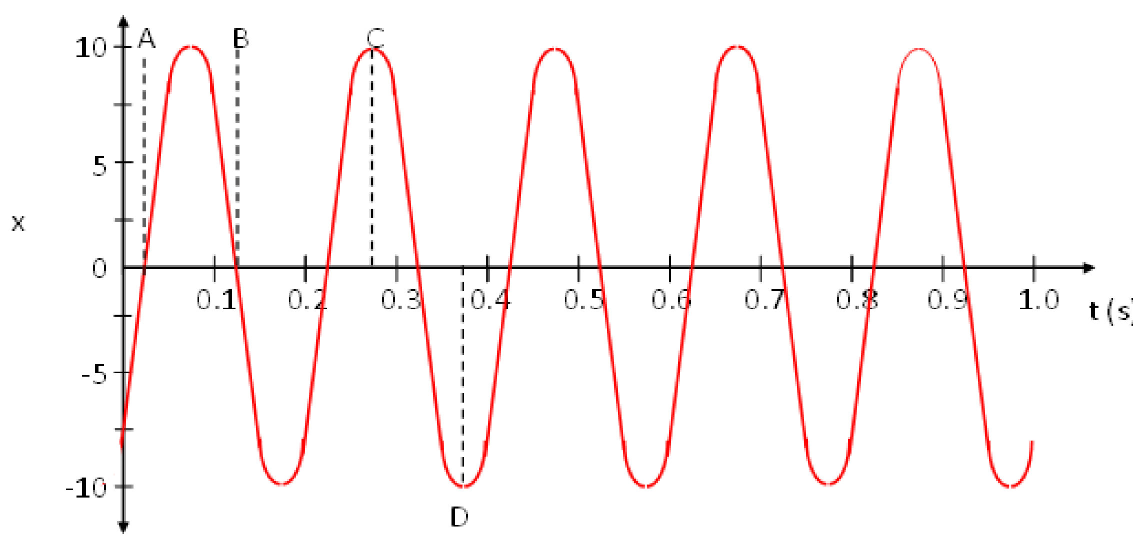
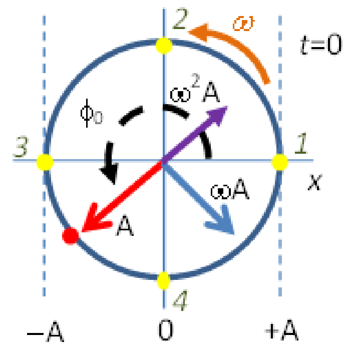
4. The diagram below shows the motion of a 2.00−kg mass on a horizontal spring. Draw the reference circle. Find the phase constant. Write down the equation of the displacement as a function of time. What is the spring constant? What is the total energy? What is the maximum speed? What is the maximum acceleration? When exactly will the mass be at equilibrium and moving to the right? When exactly will the mass be at point C?



Examining the graph we see that the largest displacement is 10 cm, so A = 0.10 m. We also see that the motion repeats every 0.2 seconds, so T = 0.2 s. Angular frequency is related to the period by $\omega = 2\pi/T$, so $\omega = 10\pi$.

At t = 0, we can read from the side of the graph that x = -7.5 cm = -0.075 m. The negative x position puts the object at either quadrant II or III of the reference circle. To decide which quadrant is correct, note that as t increases the curve goes through zero (equilibrium), labelled point A in the diagram, and it is moving from negative to positive position so it is moving to the right. This is the behaviour of the particle in quadrant III, so the reference circle looks like



We can take the general equation, $x = A\cos(\omega t + \phi_0)$ and substitute in the t = 0 values to find ϕ_0.

$$-0.075 = 0.10\cos(0 + \phi_0)\ ,$$

which reduces to

$$-0.75 = \cos(\phi_0)\ ,$$

or

$$\phi_0 = \cos^{-1}(-0.75)\ .$$

Remember that your calculator will only give you one answer but there is also a second answer 360° minus the calulator answer.

Solving yields 138.6° (2.419 rad) or 221.4° (3.864 rad). The second answer is in the third quadrant and is thus the correct phase constant.

The equation of the displacement is

$$x = 0.10\cos(10\pi t + 3.864)\ ,$$

An examination of our relationships for SHM indicates that $K = \omega^2 m$, $E = ½KA^2$, $v_{max} = \omega A$, and $a_{max} = \omega^2 A$, so

$$K = (10\pi\ /s)^2(2\ kg) = 1974\ N/m\ .$$

$$E = ½(1974\ N/m)(0.1\ m)^2 = 9.870\ J\ .$$

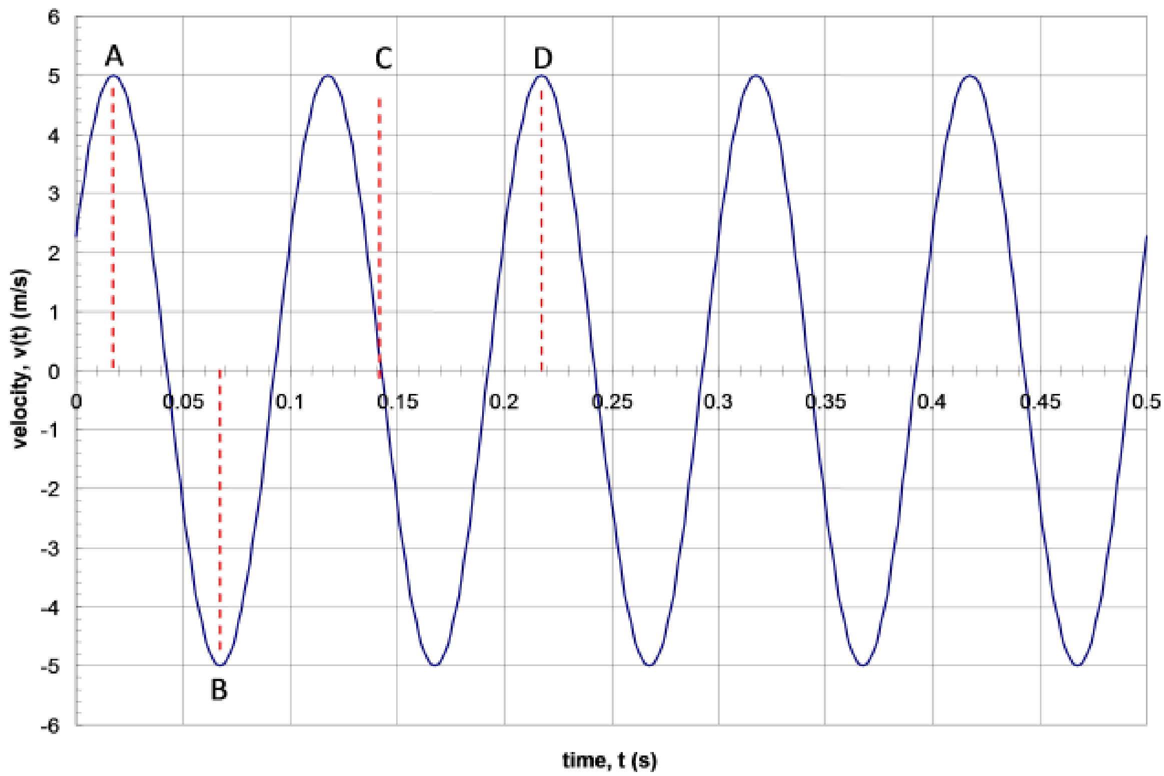
$$v_{max} = (10\pi/s)(0.1\ m) = \pi\ m/s\ .$$

$$a_{max} = (10\pi/s)^2(0.1\ m) = 9870\ m/s^2\ .$$

To find when the object is moving to the right at equilibrium (labelled A in the given x-t graph), note that this is point 3 on the reference circle given above. So the object needs to rotate to 270° from 221.4°. Since rotating 360° takes one period which here is 0.2 s, $t_A = {}^{48.6}/_{360} \times 0.2\ s = 0.027\ s$.

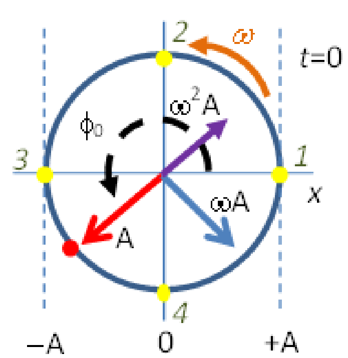
Point C in the diagram occurs at point 1 on the reference circle but note that point C is more than one period T from t = 0. We need to calculate the time to rotate to point 1 but then add one period. The angle we need to rotate through is 360° − 221.4° = 138.6°. So $t_C = {}^{138.6}/_{360} \times 0.2\ s + 0.2\ s = 0.277\ s$.

5. The diagram below shows the velocity of a 2.00-kg mass on a horizontal spring. What is the maximum amplitude of the object's displacement? What is the maximum acceleration? Draw the reference circle. What is the phase constant? Write down the equation of the displacement as a function of time. What is the spring constant? What is the total energy? When exactly will the mass have maximum positive velocity (point A)?



The equation for the velocity of an object undergoing SHM has the form $v(t) = -v_{max}\sin(\omega t+\phi_0)$, where $v_{max} = \omega A$ and $\omega = 2\pi/T$. Examining the graph, we see that the period is T = 0.1 s, so $\omega = 20\pi\ s^{-1}$. Also the maximum velocity is 5 m/s. From this we determine that $A = v_{max}/\omega = (5\ m/s)/(20\pi\ s^{-1}) = 1/(4\pi)\ m = 0.0796\ m$. Furthermore, the maximum acceleration is $a_{max} = \omega^2 A = \omega v_{max} = 100\pi\ m/s^2$.

To draw the reference circle note that the t = 0 velocity is positive (moving to the right) which only occurs when the object is in quadrants III and IV. Next note that the given v-t graph has the velocity going to a maximum, which occurs when the object passes through equilibrium. Thus the object is in the third quadrant.



We use the velocity equation, $v(t) = -v_{max}\sin(\omega t+\phi_0)$, at t = 0 to find the phase constant ϕ_0. We already found $\omega = 20\pi\ s^{-1}$. Again we look at the value of the graph at t = 0 which is v(0) = 2.40 m/s. So we have $2.40 = -5\sin(\phi_0)$, or $\sin(\phi_0) = -0.48$.

There are two angles on the unit circle that satisfy this, $\phi_0 = -28.69°$ and $\phi_0 = -151.31°$. The second angle is in the third quadrant as is necessary. As a positive angle this is $\phi_0 = 208.7° = 3.643$ rad. Our equation is $x(t) = (0.0796 \text{ m})\cos(20\pi t + 3.643)$.

The spring constant is given by $\omega^2 = K/m$, so $K = (20\pi \text{ s}^{-1})^2(2.00 \text{ kg}) = 7896$ N/m.

The total energy is $E_{total} = ½KA^2 = 25$ J.

To find when the object is at point A, maximum velocity to the right, note that the object must be at point 4 of the reference circle. So the object needs to rotate to 270° from 208.7°. Since rotating 360° takes one period which here is 0.1 s, $t_A = {}^{61.3}/_{360} \times 0.1 \text{ s} = 0.0170$ s.

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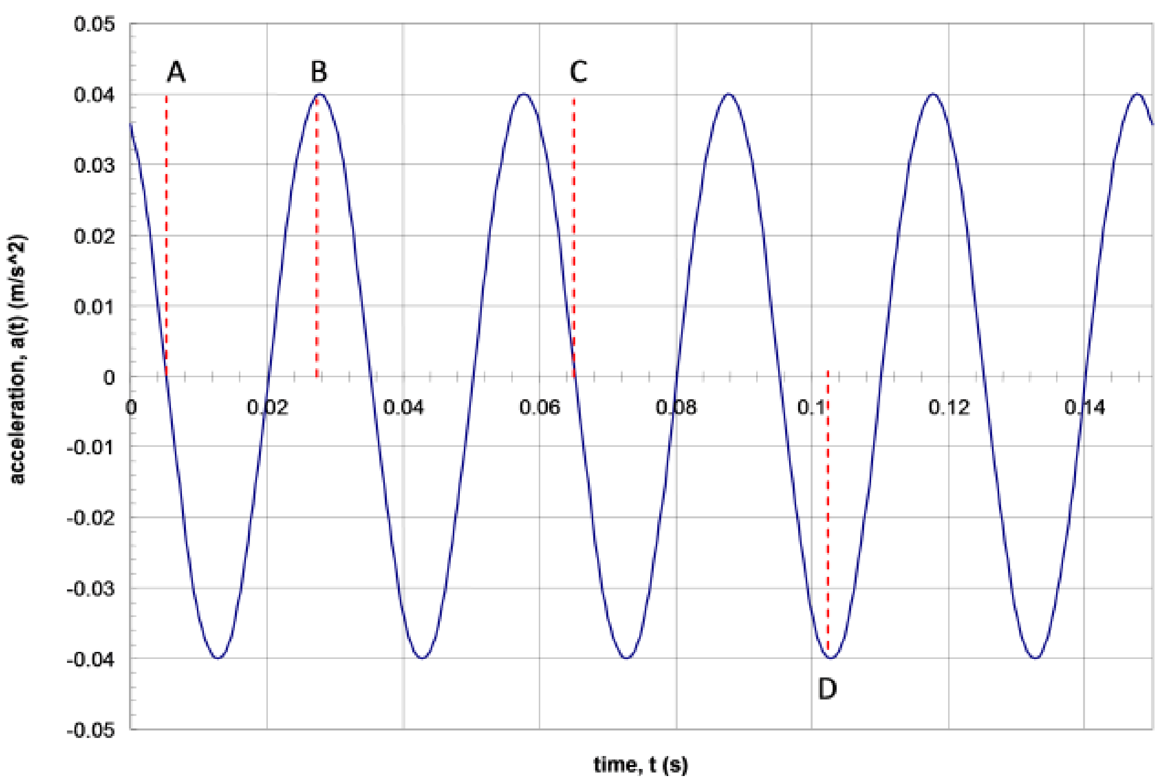
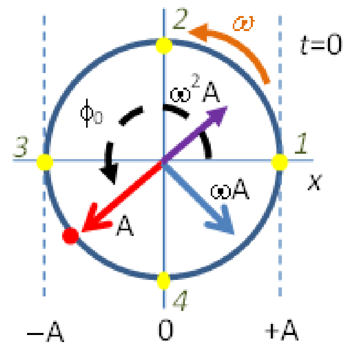
6. The diagram below shows the acceleration of a 2.00-kg mass on a horizontal spring. What is the maximum amplitude of the object's displacement? What is the maximum velocity? Draw the reference circle. Find the phase constant. Write down the equation of the displacement as a function of time. What is the spring constant? What is the total energy? When exactly will the mass first have minimum acceleration (point B)?



The equation for the acceleration of an object undergoing SHM is has the form $a(t) = -a_{max}\cos(\omega t+\phi_0)$, where $a_{max} = \omega^2 A$ and $\omega = 2\pi/T$. Examining the graph, we see that the period is $T = 0.030$ s, so $\omega = 200\pi/3 \text{ s}^{-1}$. Also the maximum acceleration is 0.040 m/s^2. From this we deduce that $A = a_{max}/\omega^2 = (0.040 \text{ m/s}^2)/(200p/3 \text{ s}^{-1})^2 = 9.119 \times 10^{-6}$ m. Furthermore, the maximum velocity is $v_{max} = \omega A = a_{max}/\omega = 1.9099 \times 10^{-4}$ m/s.

To draw the reference circle note that the t = 0 acceleration is positive (moving to the right) which only occurs when the object is in quadrants II and III. Next note that the given a-t graph has the acceleration going to zero, which occurs when the object passes through equilibrium. Thus the object is in the third quadrant.



The acceleration equation for SHM is $a(t) = -\omega^2 A\cos(\omega t + \phi_0)$. We just found ω and A so all that is left to do is to find ϕ_0. Again we look at the value of the graph at t = 0 which is a(0) = 0.035 m/s. So we have $0.035 = -0.040\cos(\phi_0)$, or $\cos(\phi_0) = -0.875$. There are two angles on the unit circle that satisfy this, $\phi_0 = 151.04°$ and $\phi_0 = -151.04°$. The second angle is in the third quadrant, so it is the correct phase constant, 209.0° or 3.648 rad.

The position equation for SHM is $x(t) = A\cos(\omega t + \phi_0)$. Using the values we found $x(t) = (9.119 \times 10^{-6}\text{ m})\cos(200\pi t/3 + 3.648)$.

The spring constant is given by $\omega^2 = K/m$, so $K = (200\pi/3\text{ s}^{-1})^2(2.00\text{ kg}) = 87730$ N/m.

The total energy is $E_{total} = ½KA^2 = 3.65 \times 10^{-6}$ J.

Point B is maximum positive acceleration, that occurs when the object is compressing the spring the most which is at point 3 on the reference circle diagram above. So the object needs to rotate counterclockwise to 180° from 209.0° which is 331.0°. Since rotating 360° takes one period which here is 0.030 s, $t_B = {}^{331}/_{360} \times 0.030\text{ s} = 0.0276$ s.

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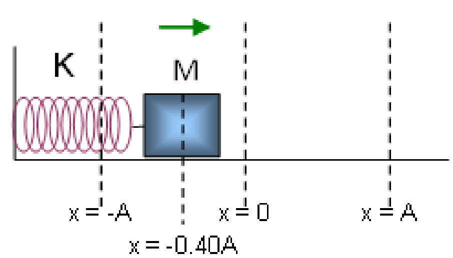
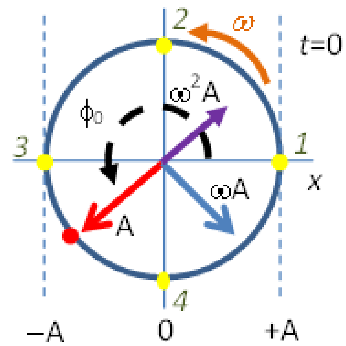
7. The diagram below shows a block oscillating back and forth at t = 0. Sketch the reference circle. Find the phase constant ϕ_0 in degrees. Sketch the x-t and v-t graphs.

t = 0



To draw the reference circle note that the t = 0 position is negative and the velocity is positive (moving to the right) which only occurs when the object is in quadrants III.



We can take the general equation, $x = A\cos(\omega t + \phi_0)$ and substitute in the t = 0 values to find ϕ_0.

$$-0.40A = A\cos(0 + \phi_0) ,$$

which reduces to

$$-0.40 = \cos(\phi_0) ,$$

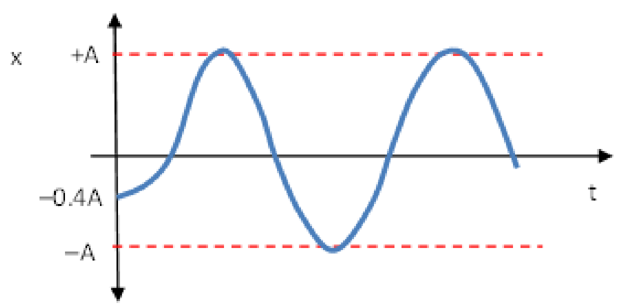
or

$$\phi_0 = \cos^{-1}(-0.40) .$$

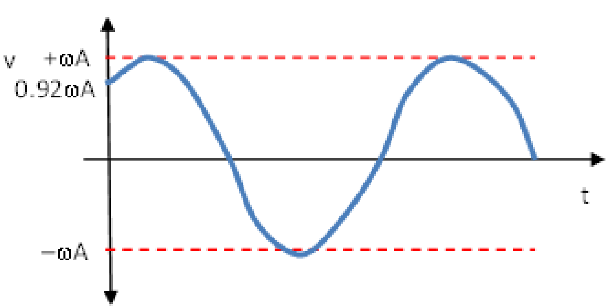
Remember that your calculator will only give you one answer but there is also a second answer 360° minus the calulator answer.

Solving yields 113.6° (1.983 rad) or 246.4° (4.300 rad). The second answer is in the third quadrant and is thus the correct phase constant.

To sketch the x-t graph first note that the t = 0 position is -0.40A and that the velocity is positive $v = \omega A$and that the object on the reference circle will rotate next to point 4, which has x = 0. So graph must rise through x = 0.



To sketch the v-t graph first note that the t = 0 velocity is positive and has value $v = -\omega A\sin(\phi_0) = 0.9164\omega A$ .When the object rotates to position 4, equilibrium velocity will be a maximum. So the velocity has risen.



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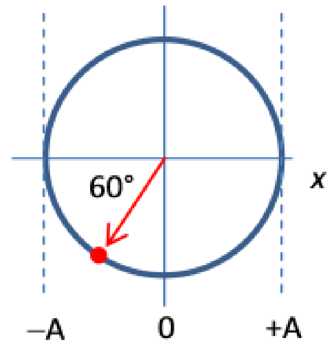
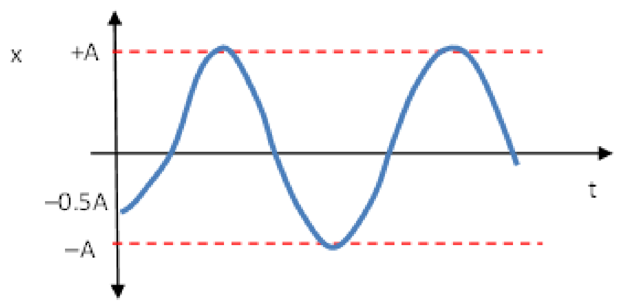
8. The t = 0 reference circle shown is for a 4.0 kg block on a spring with stiffness K = 900 N/m. What is the phase constant ϕ_0? Sketch the x-t and v-t graphs. When exactly will the block reach equilibrium?

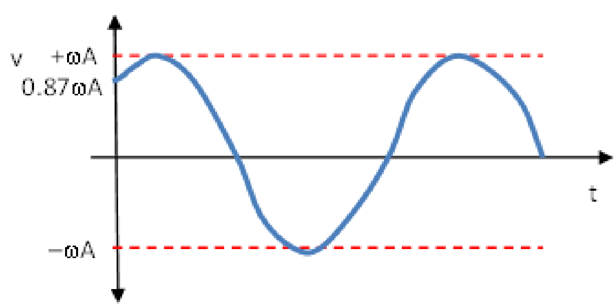


The phase constant ϕ_0 is measured from the positive x axis. Thus $\phi_0 = 240° = 4\pi/3$.

To sketch the x-t graph, we need to know the t = 0 position. Here $x(0) = A\cos(240°) = -½A$. To determine the behaviour for t > 0, note that the object in the reference circle rotates counterclockwise and will reach equilibrium, x = 0.



To sketch the v-t graph, we need to know the t = 0 velocity. Here $v(0) = -\omega A\sin(240°) = +0.866\omega A$. To determine the behaviour for t > 0, now that the object in the reference circle rotates counterclockwise and will reach equilibrium, x = 0, where the velocity is positive maximum $v = +\omega A$.



The object has to rotate through 30° counterclockwise to reach equilibrum. Since a rotation of 360° takes one period T, this object will reach equilibrium in time $t = {}^{30}/_{360} \times T = T/12$. We just need to know the period T. Since $\omega^2 = K/M$, $\omega = 15$ rad/s. Now $T = 2\pi/\omega = 0.419$ s. So the object reaches equilibrium in t = (= 0.419 s)/12 = 0.0349 s.

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9. A horizontal spring with k = 200N/m has an attached mass of 0.150 kg. It is stretched and released. As the mass passes through the equilibrium point, its speed is 5.25 m/s. What was the amplitude of the motion?

An examination of our relationships for SHM indicates that the velocity of a particle is at a maximum as it goes through equilibrium. So the problem has given use the maximum velocity. As well we have $v_{max} = \omega A$, and $\omega = [K/m]^{½}$, so

$$A = v_{max} / [K/m]^{½} = 5.25 \text{ m/s} / [(200 \text{ N/m}) / (0.15 \text{ kg})]^{½} = 0.144 \text{ m} .$$

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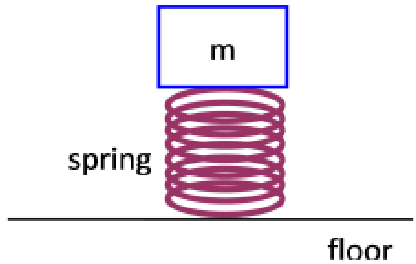
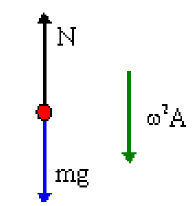
10. A stiff spring k = 400 N/m has be attached to the floor vertically. A mass of 6.00 kg is placed on top of the spring as shown below and it finds a new equilibrium point. If the block is pressed downward and released it oscillates. If the compression is too big, however, the block will lose contact with the spring at the maximum vertical extension. Draw a free body diagram and find that extension at which the block loses contact with the spring.



The problem tells us that the block loses contact with the spring. That means the normal acting on the block goes to zero. Typically, for problems involving forces we draw the FBD and apply Newton's Second Law. However, we have always avoided doing this with springs because we are dealing with a non−constant force and a non−constant acceleration. We don't have this difficulty in this problem because we are told that the spring is at maximum extension, and thus we know that the acceleration is $a_{max} = \omega^2 A$ downwards.



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$$\Sigma F_y = ma_y$$

$$N - mg = -m\omega^2 A$$

The equation we have is

$$N - mg = -m\omega^2 A .$$

Setting N = 0, and rearranging yields

$$A = g / \omega^2 .$$

Our relationships for SHM tell us that $\omega = [K/m]^{½}$, so

$$A = g / [K/m] = (9.81 \text{ m/s}^2) / [(400 \text{ N/m}) / (6 \text{ kg})] = 14.7 \text{ cm} .$$

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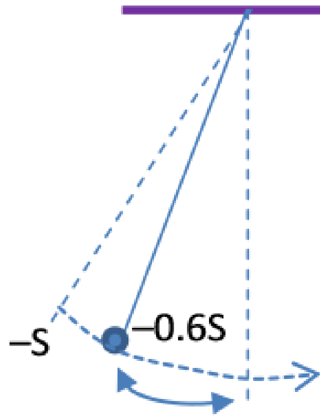
11. A 100-g ball hangs from a metre-long string. Its swing is shown at t = 0 and it is moving to the right. Sketch the reference circle. Find the phase constant ϕ_0 in degrees. Sketch the x-t and v-t graphs. When will the ball pass through equilibrium?



The object has a negative position and is moving to the right so it must be in quadrant III on the reference circle.

insert diagram

We can take the general equation, $s = S\cos(\omega t + \phi_0)$ and substitute in the t = 0 values to find ϕ_0.

$$-0.6S = S\cos(0 + \phi_0) ,$$

which reduces to

$$-0.60 = \cos(\phi_0) ,$$

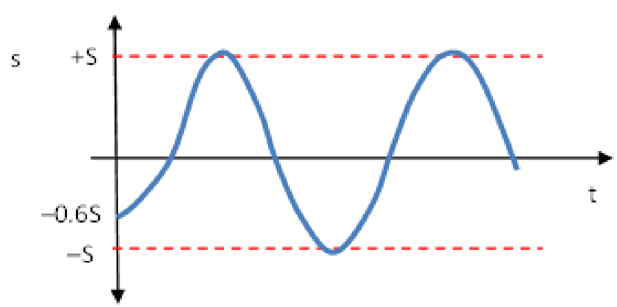
or

$$\phi_0 = \cos^{-1}(-0.60) .$$

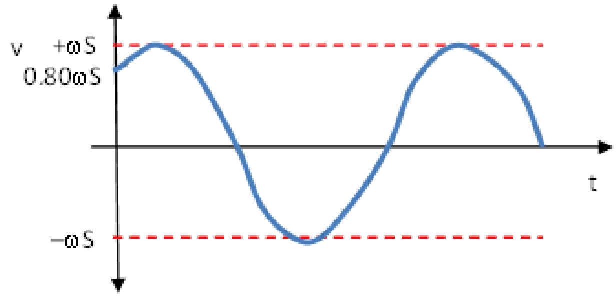
Remember that your calculator will only give you one answer but there is also a second answer 360° minus the calulator answer.

Solving yields 126.9° (2.214 rad) or 233.4° (4.069 rad). The second answer is in the third quadrant and is thus the correct phase constant.

To sketch the x-t graph first note that the t = 0 position is -0.60S and that the velocity is positive $v = \omega A$ and that the object on the reference circle will rotate next to point 4, which has x = 0. So graph must rise through x = 0.



To sketch the v-t graph first note that the t = 0 velocity is positive and has value $v = -\omega S\sin(\phi_0) = 0.8028\omega S$. When the object rotates to position 4, equilibrium velocity will be a maximum. So the velocity has risen.



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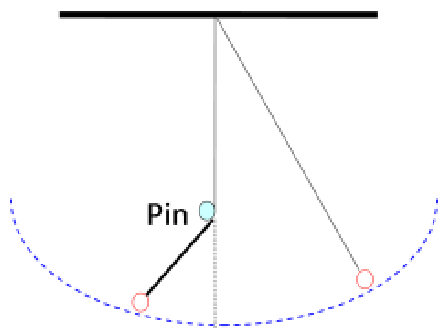
12. In the diagram below, a mass on a string of length L encounters a nail positioned a distance L/n from the bottom of the string when the string hangs vertical. What is the period of this "interrupted" pendulum?



Examining the motion we see that the object swings for half the motion on a string of length L, and the other half as a string of length L/n. From the period of a pendulum, we know how long each of these motion are. Therefore the total time is

$$T = \tfrac{1}{2}T_1 + \tfrac{1}{2}T_2 = \pi\{[L/g]^{1/2} + [L/(ng)]^{1/2}\} \ .$$

My thanks to C. Kufazvinei and F. Z. Mavhunga for pointing out a previous typographical error in the above equation.

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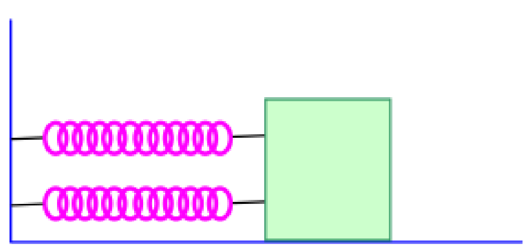
13. A block of mass M is on a frictionless surface as shown below. It is attached to a wall by two springs with the same constant K. Initially the block is at rest and the springs unstretched. The block is pulled a distance A and then released.
(a) What is the speed of the block as it passes through equilibrium?
(b) What is the angular frequency ω of the motion?
(c) If the two springs were replaced by one spring so that ω remains the same, what would its spring constant have to be?
(d) If the two springs were kept, what would the mass have to be so that it has the same ω as a system comprised of a single block of mass M and spring with stiffness K?



(a) The fact that we are looking for ω indicates an SHM problem. It is not the simple one spring and block system we studied since there are two springs. However energy methods let us handle everything since we know $E_f = E_i$, hence we have

$$2 \times \frac{1}{2}KA^2 = \frac{1}{2}Mv^2$$

Solving for v yields $v = \sqrt{\frac{2K}{M}}A$.

(b) Now this is the maximum velocity of the block and we know for SHM that $v_{max} = \omega A$. So $\omega = \sqrt{\frac{2K}{M}}$.

(c) For a single spring we know $\omega = \sqrt{\frac{K_{effective}}{M}}$. Since ω and M are the same, $K_{effective} = 2K$. We replace the two springs with a single spring with twice the spring constant.

(d) We found $\omega = \sqrt{\frac{2K}{M}}$ for the two parallel springs and block but we want the same ω as for the single spring and block result $\omega = \sqrt{\frac{K}{M}}$. If we double the mass from M to 2M for the two spring system, the 2's will cancel and we will have the same ω.

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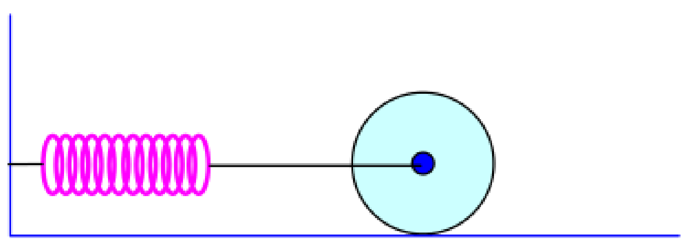
14. A disk of mass M is on a surface as shown below. It is attached to a wall by a spring of constant K. Initially the disk is at rest and the spring is unstretched. The disk is pulled a distance A and then released.
(a) What is the speed ofthe block as it passes through equilibrium?
(b) What is the angular frequency ω of the motion?
(c) What would spring constant be for this system to have the same ω as a system comprised of a single block of mass M and spring with stiffness K?
(d) What would the mass of the disk have to be for this system to have the same ω as a system comprised of a single block of mass M and spring with stiffness K?



(a) The fact that we are looking for ω indicates an SHM problem. It is not that simple a spring and block system since there is also rolling. However energy methods let us handle everything since we know $E_f = E_i$, hence we have

$$\frac{1}{2}KA^2 = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega_{rolling}^2$$

For a disk, $I = \frac{1}{2}Mr^2$. Since it is rolling, $\omega_{rolling} = v/r$. The above equation reduces to

$$\frac{1}{2}KA^2 = \frac{3}{4}Mv^2$$

Solving for v yields $v = \sqrt{\frac{2K}{3M}}A$.

(b) Now this is the maximum velocity of the block and we know for SHM that $v_{max} = \omega A$. So $\omega = \sqrt{\frac{2K}{3M}}$.

(c) We found that $\omega = \sqrt{\frac{2K}{3M}}$ for the single spring and rolling disk. If we had the spring and a block the result would have been $\omega = \sqrt{\frac{K}{M}}$. We would need to increase the spring constant in the spring-disk system to increase ω to the same value as the spring-block system. To get rid of the fraction $^2/_3$ in our spring-disk system we need $K_{new} = {}^3/_2K$.

(d) We found that $\omega = \sqrt{\frac{2K}{3M}}$ for the single spring and rolling disk. If we had the spring and a block the result would have been $\omega = \sqrt{\frac{K}{M}}$. We would need to decrease the mass in the spring-disk system to increase ω to the same value as the spring-block system. To get rid of the fraction $^2/_3$ in our spring-disk system we need $M_{new} = {}^2/_3M$.

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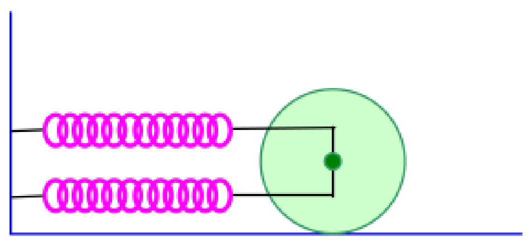
15. A solid sphere of mass M is on a surface as shown below. It is attached to a wall by two springs with the same constant K. Initially the sphere is at rest and the springs unstretched. The sphere is pulled a distance A and then released.
(a) What is the speed of the sphere as it passes through equilibrium?
(b) What is the angular frequency ω of the motion?
(c) What would the spring constants have to be for this system to have the same ω as a system comprised of a single block of mass M and spring with stiffness K?
(d) What would the mass of the sphere have to be for this system to have the same ω as a system comprised of a single block of mass M and spring with stiffness K?



(a) The fact that we are looking for ω indicates an SHM problem. It is not a simple single spring and block system since there is also rolling. However energy methods let us handle everything since we know $E_f = E_i$, hence we have

$$2 \times \tfrac{1}{2}KA^2 = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega_{rolling}^2$$

For a solid sphere, $I = {}^2/_5Mr^2$. Since it is rolling, $\omega_{rolling} = v/r$. The above equation reduces to

$$KA^2 = {}^7/_{10}Mv^2$$

Solving for v yields $v = \sqrt{\frac{10K}{7M}}A$.

(b) Now this is the maximum velocity of the block and we know for SHM that $v_{max} = \omega A$. So $\omega = \sqrt{\frac{10K}{7M}}$.

(c) We found that $\omega = \sqrt{\frac{10K}{7M}}$ for the double spring and rolling disk. If we had a spring and a block system, the result would have been $\omega = \sqrt{\frac{K}{M}}$. We would need to decrease the spring constant in the double spring-disk system to decrease ω to the same value as the spring-block system. To get rid of the fraction $^{10}/_{7}$ in our double spring-disk system equation we need $K_{new} = {}^{7}/_{10}K$.

(d) We found that $\omega = \sqrt{\frac{10K}{7M}}$ for the double spring and rolling disk. If we had a spring and a block system, the result would have been $\omega = \sqrt{\frac{K}{M}}$. We would need to increase the mass in the double spring-disk system to decrease ω to the same value as the spring-block system. To get rid of the fraction $^{10}/_{7}$ in our double spring-disk system equation we need $M_{new} = {}^{10}/_{7}M$.

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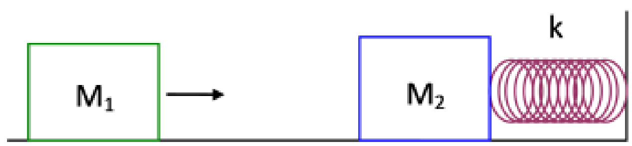
16. A block with a mass M_1 = 5.00 kg is sliding to the right with a speed of 15.0 m/s when it hits and sticks to a block of mass, M_2 = 3.00 kg, attached to a spring of spring constant k = 3000 N/m. The second block was at the equilibrium position and the spring uncompressed at the time of the collision. The horizontal surface is frictionless.
(a) Find the angular frequency of vibration for the system.
(b) Find the maximum compression of the spring.
(c) Find the maximum acceleration of the joined blocks.



First we have a totally inelastic collision, so

$$(M_1 + M_2)v_f = M_1 v\ .$$

This tells us that

$$v_f = M_1 v /(M_1 + M_2) = (5)(15\text{ m/s})/(5 + 3) = 9.375\text{ m/s}\ .$$

This is the maximum velocity of the double−block SHM.

(a) From our relations for SHM, we know that $\omega = [K/m]^{1/2}$, so

$$\omega = [(3000\text{ N/m}) / (5\text{ kg} + 3\text{ kg})]^{1/2} = 19.36\text{ rad/s}\ .$$

(b) From the SHM relations, we also know $v_{max} = \omega A$, so the maximum compression is

$$A = v_{max} / \omega = (9.375\text{ m/s}) / (19.36\text{ rad/s}) = 0.4842\text{ m}\ .$$

(c) The maximum acceleration is given by $a_{max} = \omega^2 A$, so

$$a_{max} = (19.36\text{ rad/s})^2(0.4842\text{ m}) = 181.5\text{ m/s}^2\ .$$

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