Ratio and Proportion

Ratio:

This is a comparison of, usually, 2 quantities by division.

Example:

- Gas-to-oil mixture of 16 to 1 for a certain engine = $\frac{16\ gas}{1\ oil}$
- The slope (pitch) of a roof that has a 4 in. rise for every 12 in. run = $\frac{4\ in}{12\ in}$
- A blueprint scale of ¼ in to 1 foot = $\frac{0.25\ in}{1\ ft}$

Ratios can also be written with a colon instead of a fraction ➔ Numerator : Denominator

Example:

- $\frac{16\ gas}{1\ oil}$ can be written 16 gas : 1oil
- $\frac{4\ in}{12\ in}$ can be written 4 in : 12 in
- $\frac{0.25\ in}{1\ ft}$ can be written 0.25 in : 1 ft.

Whenever possible, ratios should be reduced to *lowest terms*.

Example: A roof that has a 4 in. rise for every 12 in. run. Express this as a ratio.

$\frac{rise}{run} = \frac{4in}{12in} = \frac{4}{12}$ This ratio is correct but can be reduced.

$\frac{4\div4}{12\div4}$ Divide top and bottom by 4.

$\frac{1}{3}$ This ratio is now reduced to *lowest terms.*

Proportion:

When two ratios can be set equal to each other.

Example: Set up a proportion with the equal ratios $\frac{2}{4}$ and $\frac{3}{6}$.

- We know that the ratios $\frac{2}{4}$ and $\frac{3}{6}$ are equal because they both reduce to $\frac{1}{2}$.
- So, $\frac{2}{4} = \frac{3}{6}$ is a true statement.

In a proportion, the product of the diagonal numbers is *always* equal.

$\frac{2}{4} \times \frac{3}{6}$ Multiplying the diagonal numbers is called *cross multiplication.*

$2 \times 6 = 3 \times 4$

$12 = 12$ The products are equal.

When 3 out of 6 quantities are known, the 4th can always be found by cross multiplication.

Example: $\frac{x}{4} = \frac{3}{6}$ This is the same proportion as above but with x instead of 2.

$\frac{x}{4} \times \frac{3}{6}$ Cross multiply.

$6x = 4 \times 3$

$6x = 12$ Now both sides divided by 6 to find x.

$x = \frac{12}{6}$

$x = 2$ This is correct because we initially replaced 2 with x.

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How to set up a proportion question:

1. When setting up a proportion question, make sure the same units are over each other.

E.g. $\frac{6\ \text{workers}}{10\ \text{workers}} = \frac{3\ days}{5\ days}$ $\frac{workers}{workers} = \frac{days}{days}$ same units top and bottom.

2. Put the smaller quantities on the tops (It is your choice to put the larger quantities on the tops instead, but make sure both sides keep the same order).

$$\frac{small}{large} = \frac{small}{large} \ or \ \frac{large}{small} = \frac{large}{small}$$

E.g. $\frac{6\ \text{workers}}{10\ \text{workers}} = \frac{3\ \text{days}}{5\ \text{days}}$ $\frac{6}{10} = \frac{3}{5}$ smaller quantities on top.

Is it directly or inversely proportional?

1. ***Directly Proportional***

A proportion is direct if *increasing* one quantity *increases* another **OR** *decreasing* one quantity *decreases* another.

Example:

More lumber will cost more

- *increasing* lumber *increases* cost

Less lumber will cost less

- *decreasing* lumber *decreases* cost

2. ***Inversely proportional***

A proportion is inverse if *increasing* one quantity *decreases* another **OR** *decreasing* one quantity *increases* another.

Example:

More workers will take less time

- *increasing* workers *decreases* time

Less workers will take more time

- *decreasing* workers *increases* time

Ratio and Proportion

Practice Problems:

1. If a team can paint $250m^2$ in 5 days, how long does it take the same team to paint a $50m^2$?

   We know: $250m^2 \longrightarrow 5\ days$ and $50m^2 \longrightarrow X\ days$

   Is it direct or inverse?

   - *Increasing* the area *increases* time or *decreasing* the area *decreases* time
   - This is directly proportional. Therefore, X will be *smaller* than 5 (this makes sense as decreasing the area that the team must paint will decrease the amount of time it takes to complete).

   $\frac{50m^2}{250m^2} = \frac{X days}{5 days}$ X goes on top because it must be smaller than 5.

   Same units over each other.

   $\frac{50}{250} = \frac{X}{5}$ Cancel the units

   $\frac{50}{250} \times \frac{X}{5}$ Cross multiply.

   $50 \times 5 = X \times 250$

   $250 = 250X$ Now divide both sides by 250.

   $X = 250/250$

   $X = 1\ day$

2. If it takes 10 painters 6 days to finish a house how long will it take 3 painters to do the same?

   We know 10 painters $\longrightarrow$ 6 days

   3 painters $\longrightarrow$ X days

   Is it direct or inverse?

   - *Increasing* workers *decreases* time (or *decreasing* workers *increases* time)
   - This is inversely proportional. Therefore, X will be *larger* than 6 (this makes sense as decreasing the number of painters should increase the time to finish painting the house).

   $\frac{small}{large} = \frac{small}{large}$

   $\frac{3\ painters}{10\ painters} = \frac{6\ days}{X\ days}$ 6 goes on top because X must be *larger.*

   Same units over each other.

   $\frac{3}{10} = \frac{6}{x}$ Cancel the units.

   $\frac{3}{10} \times \frac{6}{x}$ Cross multiply

   $x \times 3 = 6 \times 10$

   $3x = 60$ Now divide both sides by 3.

   $x = 60/3$

   $x = 20\ days$

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Word Problems:

1. If an electrician can wire 6 rooms in 3 hours, how long will it take to wire 3 rooms?

2. A car consumes 20 L of fuel in 100 km. How much fuel was consumed in the first 5km?

3. A plumber can install a pipeline system in two homes in eight hours. How long would it take for the same plumber to install systems in five homes?

4. A carpenter takes three hours to install 15 anchor bolts. How many anchor bolts can he or she install in seven hours?

5. A landscaper can mow a yard in 32 minutes with his mower at a maximum speed of 5 km/h. If this landscaper bought a new mower with a maximum speed of 8 km/h, how long would it now take the landscaper to mow the same lawn?

6. A person is planning a trip. If they travel at an average speed of 60 km/h, they will arrive in 4 hours. How long would it take them to arrive if they drive at an average speed of 80 km/h?

7. A twelve-inch gear runs at a speed of 240 rpm (revolutions per minute) meshing with an eight-inch gear. How fast would the eight-inch gear rotate?

8. Four electricians will use 600 feet of outdoor electrical wire in a day. How much electrical wire will three electricians use?

9. If it takes a team of five electricians to finish a job in 28 days, how long would it take a team of seven to finish the same job?

10. A carpentry department believes that their twenty employees will use up their stock of cable ties in 15 days. If the department hires five more employees, how long will the cable ties last?

Answers: 1. 1.5 hours, 2. 1 L, 3. 20 hours, 4. 35, 5. 20 min., 6. 3 hours, 7. 360 rpm, 8. 450 feet, 9. 20 days, 10. 12 days